Theory of Computation — Problem Set No. 4
Instructor: Xin Li — Due in class on Dec. 3

1. (25 points) An undirected graph is *bipartite* if its nodes may be divided into two sets so that all edges go from a node in one set to a node in the other set. Show that a graph is bipartite if and only if it does't contain a cycle that has an odd number of nodes. Let $BIPARTITE = \{\langle G\rangle | G \text{ is a bipartite graph}\}$. Show that $BIPARTITE \in \mathsf{NL}$.

2. (25 points) Let $\varphi$ be a 2CNF formula with exactly two literals per clause. Let $x_1, \cdots, x_n$ be the variables in $\varphi$. Associate with $\varphi$ a directed graph $G_\varphi = (V, E)$, where

$$V = \{x_1, \overline{x_1}, x_2, \overline{x_2}, \cdots, x_n, \overline{x_n}\}$$

   (i.e., $V$ is the set of all literals that may appear in $\varphi$), and a pair $(t_1, t_2)$ is an edge in $G_\varphi$ iff $(\overline{t_1} \vee t_2)$ is a clause in $\varphi$.

   (a) Show that $\varphi$ is unsatisfiable iff there is a directed cycle in $G_\varphi$ in which both $x_i$ and $\overline{x_i}$ appear, for some variable $x_i$.

   (b) Use part a) to show that 2SAT is in $\mathsf{NL}$.

   (c) Show that $\overline{\text{PATH}} \leq_\ell$ 2SAT. Use this and part b) to show that 2SAT is $\mathsf{NL}$-complete.

3. (15 points) Problem 4.10 in Required Textbook.

4. (15 points) Problem 4.12 in Required Textbook.