1. We define an operation BACKANDFORWARD such that:

\[
BACKANDFORWARD(A) = \{ w \in A | w \in A \text{ and } w^R \in A \}.
\]

That is, given any language \( A \), \( BACKANDFORWARD(A) \) is a new language containing all elements in \( A \) whose reverse is also in \( A \). Prove that the class of regular languages is closed under the operation \( BACKANDFORWARD \).

2. (a) Prove that the class of regular languages is closed under set difference. That is, if \( A \) is regular and \( B \) is regular, show that \( A \setminus B = \{ w \in A | w \notin B \} \) is regular.
   
   (b) Show that the class of regular languages is closed under symmetric set difference, defined as \( A \Delta B = \{ w | w \in A \text{ or } w \in B \text{ but not both} \} \).

3. Prove that the class of regular languages is closed under \( PREFIX \) where \( PREFIX(A) = \{ w | \exists x \in \Sigma^* \text{ such that } wx \in A \} \).

4. Convert the following regular expression into an NFA:

\((0 \cup 11)^*01(01 \cup 1)^*\)

5. Let \( \Sigma = \{0, 1\} \). Give an equivalent regular expression for each of the languages below.

   (a) \( L_1 = \{ w | w \text{ begins with a 1 and ends with a 0} \} \)
   (b) \( L_2 = \{ w | w \text{ contains at least three 1s} \} \)
   (c) \( L_3 = \{ w | w \text{ contains the substring 0101} \} \)
   (d) \( L_4 = \{ w | w \text{ has length at least three and its third symbol is 0} \} \)
   (e) \( L_5 = \{ w | w \text{ starts with a 0 and has odd length, or starts with a 1 and has even length} \} \)
   (f) \( L_6 = \{ w | w \text{ does not contain the substring 011} \} \)