1. (25 points) Let $\text{DOUBLE-SAT} = \{\phi | \phi$ is a CNF that has at least two satisfying assignments$\}$. Show that $\text{DOUBLE-SAT}$ is NP-complete.

2. (25 points) A subset of the nodes of a graph $G$ is a dominating set if every other node of $G$ is adjacent to some node in the subset. Let

$$\text{DOMINATING-SET} = \{\langle G, k \rangle | G \text{ has a dominating set with } k \text{ nodes} \}.$$ 

Show that it is NP-complete by giving a reduction from VERTEX-COVER.

3. (25 points) Show that if $P = \text{NP}$, then $\text{NP} = \text{coNP}$.

4. (25 points) A language is called unary if every string in it is of the form $1^i$ (the string of $i$ ones) for some $i > 0$. Show that if there exists an NP-complete unary language then $P = \text{NP}$.

Hint: If there is a $n^c$ time reduction from 3SAT to a unary language $L$, then this reduction can only map size $n$ instances of 3SAT to some string of the form $1^i$ where $i \leq n^c$. Use this observation to obtain a polynomial-time algorithm for SAT using the search to decision reduction techniques we showed in class.