1. (25 points) Define a two-dimensional Turing machine to be a TM where each of its tapes is an infinite grid (and the read/write head can move not only Left and Right but also Up and Down). Show that for every $T : \mathbb{N} \rightarrow \mathbb{N}$, any two-dimensional TM that runs in time $T(n)$ can be simulated by a standard (one-dimensional) TM in time $O(T(n)^2)$. **Note:** You may assume that the tapes of the two-dimensional TM start at $(0, 0)$ and can only access points with non-negative integer coordinates. The function $T(n)$ is not known in advance.

2. (25 points) Show that a language can be decided by a TM (i.e., can be recognized by some TM that halts on all inputs), if and only if some machine can enumerate the language (i.e., enumerate all strings in this language) in lexicographic order. You may assume the alphabet is \{0, 1\}.

3. (25 points) Let $T = \{<M> | M$ is a TM that accepts $\alpha^R$ whenever it accepts $\alpha \}$. Show that $T$ is undecidable. **Note:** $\alpha^R$ is the reverse string of $\alpha$. You may assume the alphabet is \{0, 1\}.

4. (25 points) Define the language $C_{TM} = \{<M_1, M_2> | M_1, M_2$ are two Turing machines such that $L(M_1) \subseteq L(M_2) \}$. Show that $C_{TM}$ is undecidable.