1. (20 points) Show that a language can be decided by a TM (i.e., can be recognized by some TM that halts on all inputs), if and only if some machine can enumerate the language (i.e., enumerate all strings in this language) in lexicographic order. You may assume the alphabet is \{0, 1\}.

2. (20 points) Let \( T = \{\langle M \rangle | M \text{ is a TM that accepts } \alpha^R \text{ whenever it accepts } \alpha \} \). Show that \( T \) is undecidable. Note: \( \alpha^R \) is the reverse string of \( \alpha \). You may assume the alphabet is \{0, 1\}.

3. (20 points) Define the language \( C_{TM} = \{\langle M_1, M_2 \rangle | M_1, M_2 \text{ are two Turing machines such that } L(M_1) \subseteq L(M_2) \} \).
   Show that \( C_{TM} \) is undecidable.

4. (20 points) Prove that the following language is in \( P \).
   **Two coloring:** \( 2COL = \{G : \text{ graph } G \text{ has a coloring with 2 colors} \} \).
   Here a coloring of \( G \) with \( c \) colors is an assignment of a number in \( [c] = \{1, 2, \ldots, c\} \) to each vertex such that no adjacent vertices get the same number.

5. (20 points) Suppose \( L_1, L_2 \in NP \). Then is \( L_1 \cup L_2 \in NP \)? What about \( L_1 \cap L_2 \)? Prove your answers.