Note: For problems 3, 4 you need to formally prove that your construction works, i.e., show that your construction recognizes the desired language by proving both directions. However, for these two problems you don’t need to write the formal description of a finite automaton, you can just describe your construction informally (e.g., using a state diagram).

1. (25 points) Give the state diagram of a finite automaton recognizing the following language. The alphabet is \{0, 1\}.

   \{w | w has length exactly 3 and its last symbol is different from its first symbol\}

2. (25 points) Give a finite automaton (both a state diagram and a formal description) recognizing the following language. The alphabet is \{0, 1\}.

   \{w | w is not the empty string and every odd position of w is a 1\}

3. (25 points) Show that the following language is regular, where the alphabet is \{0, 1\}.

   \{w | w contains an equal number of occurrences of the substrings 01 and 10\}

   For example, 010 is in this language since it has one occurrence of each substring, but 0101 is not in this language since it has two 01’s but only one 10.

   Hint: First find an equivalent and simpler characterization of the language.

4. (25 points) For any string \(w = w_1w_2 \cdots w_n\), the reverse of \(w\), written as \(w^R\), is the string \(w\) in reverse order, \(w_n \cdots w_2w_1\). For any language \(A\), let \(A^R = \{w^R | w \in A\}\).

   Show that if \(A\) is regular, so is \(A^R\).