Note: For problems 2, 3, 4, 5 you need to formally prove your construction works. Simply giving a state diagram or a 5-tuple description of a DFA/NFA is not enough.

1. (20 points) Give an NFA (both a state diagram and a formal description) recognizing the language $0^*1^*0^+$ with three states. The alphabet is $\{0, 1\}$.

2. (20 points) Say that string $x$ is a prefix of string $y$ if a string $z$ exists where $xz = y$ and that $x$ is a proper prefix of $y$ if in addition $x \neq y$. Let $A$ be a regular language. Show that the class of regular languages is closed under the following operation.

$$\text{NOEXTEND}(A) = \{w \in A | w \text{ is not the proper prefix of any string in } A\}$$

3. (20 points) Let $C_n = \{x | x \text{ is a binary number that is a multiple of } n\}$. Show that for each $n \geq 1$, the language $C_n$ is regular.

4. (20 points) For languages $A$ and $B$, let the perfect shuffle of $A$ and $B$ be the language

$$\{w | w = a_1b_1 \cdots a_kb_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}$$

Show that the class of regular languages is closed under perfect shuffle.

5. (20 points) For languages $A$ and $B$, let the shuffle of $A$ and $B$ be the language

$$\{w | w = a_1b_1 \cdots a_kb_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma^*\}$$

Show that the class of regular languages is closed under shuffle.