Theory of Computation — Problem Set No. 2

Instructor: Xin Li — Due by 12pm Oct. 15

You may email or hand in your homework to either the TA or me. You do not need to write the exact description of a Turing machine. Just describe what the TM does in your words.

1. (20 points) Prove that the following languages are not regular.

   (a) $\{0^n 1^m 0^n | m, n \geq 0\}$

   (b) $\{0^m 1^n | m \neq n\}$

2. (20 points)

   (a) Let $B = \{1^k y | y \in \{0,1\}^*$ and y contains at least k 1s, for any integer $k \geq 1\}$. Show that B is a regular language.

   (b) Let $C = \{1^k y | y \in \{0,1\}^*$ and y contains at most k 1s, for any integer $k \geq 1\}$. Show that C is not a regular language.

3. (20 points) Define a *two-dimensional* Turing machine to be a TM where each of its tapes is an infinite grid (and the machine can move not only Left and Right but also Up and Down). Show that for every (time-constructible) $T : \mathbb{N} \rightarrow \mathbb{N}$ and every Boolean function f, if f can be computed in time $T(n)$ using a two-dimensional TM, then it can also be computed by a standard (one-dimensional) TM in time $O(T(n)^2)$. **Note:** You may assume that the tapes of the two-dimensional TM start at $(0,0)$ and can only access points with non-negative integer coordinates.

4. (20 points) Let $T = \{\langle M \rangle | M$ is a TM that accepts α^R whenever it accepts $\alpha\}$. Show that T is undecidable. **Note:** α^R is the reverse string of α. You may assume the alphabet is $\{0,1\}$.