Approximate Matching

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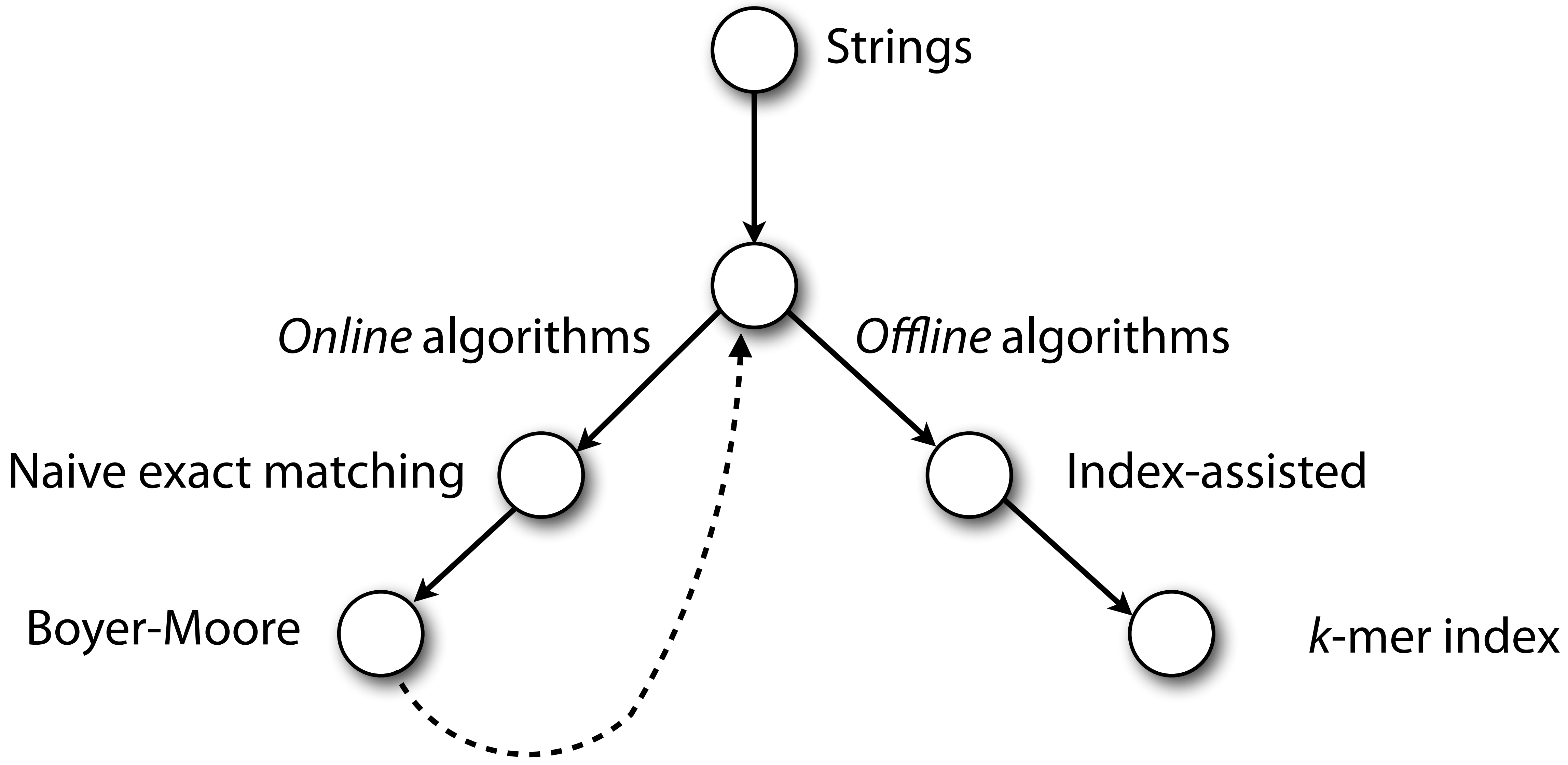
Approximate matching



We have focused on *exact* matching...

... in reality, we have to deal with *differences*

Read

```
CTCAAACTCCTGACCTTTGGTGATCCACCCGCCTNGGCCTTC
```

Reference

```
GATCACAGGTCTATCACCCTATTAACCACTCACGGGAGCTCTCCATGCATTTGGTATTTT
CGTCTGGGGGGTATGCACGCGATAGCATTGCGAGACGCTGGAGCCGGAGCACCCTATGTC
GCAGTATCTGTCTTTGATTCCTGCCTCATCCTATTATTTATCGCACCTACGTTCAATATT
ACAGGCGAACATACTTACTAAAGTGTGTTAATTAATTAATGCTTGTAGGACATAATAATA
ACAATTGAATGTCTGCACAGCCACTTTCCACACAGACATCATAACAAAAAATTTCCACCA
AACCCCCCCTCCCCCGCTTCTGGCCACAGC[illegible]TAAACACA[illegible]TCTGCCAAACCCCAAAA
ACAAAGAACCCTAACACCAGCCTAACCA[illegible]ATTTCAAATTTTATC[illegible]TTTGGCGGTATGCAC
TTTTAACAGTCACCCCCCAACTAACA[illegible]ATTATTTTCCCCTCCCAC[illegible]CATACTACTAAT
CTCATCAATACAACCCCCGCCCATC[illegible]TACCCAGCACACACACACCG[illegible]CTAACCCCATA
CCCCGAACCAACCAAACCCCAAAG[illegible]CACCCCCCACAGTTTATGTAGC[illegible]ACCTCCTCAAA
GCAATACACTGACCCGCTCAAAC[illegible]CCTGGATTTTGGATCCACCCAGCG[illegible]TTGGCCTAAA
CTAGCCTTTCTATTAGCTCTTAG[illegible]AAGATTACACATGCAAGCATCCCCG[illegible]TCCAGTGAGT
TCACCCTCTAAATCACCACGATC[illegible]AAAGGAACAAGCATCAAGCACGCAG[illegible]AATGCAGCTC
AAAACGCTTAGCCTAGCCACACC[illegible]CCACGGGAAACAGCAGTGATTAAC[illegible]TTAGCAATAA
ACGAAAGTTTAACTAAGCTATACT[illegible]ACCCCAGGGTTGGTCAATTTCGT[illegible]CCAGCCACCGC
GGTCACACGATTAACCCAAGTCAAT[illegible]GAAGCCGGCGTAAAGAGTGTT[illegible]TAGATCACCCCC
TCCCCAATAAAGCTAAAACTCACCTGA[illegible]TTGTAAAAAACTCCAGT[illegible]ACACAAAATAGAC
TACGAAAGTGGCTTTAACATATCTGAACA[illegible]ACAATAGCTAAGA[illegible]TGGGATTAGA
TACCCCACTATGCTTAGCCCTAAACCTCAACAG[illegible]CAACAA[illegible]CGCCAGAA
CACTACGAGCCACAGCTTAAAACTCAAAGGACCTGGCGGTGCTTCATA[illegible]TAGAGG
AGCCTGTTCTGTAATCGATAAACCCCGATCAACCTCACCACCTCTTGCT[illegible]TATA
CCGCCATCTTCAGCAAACCCTGATGAAGGCTACAAAGTAAGCGCAAGTACC[illegible]AG
ACGTTAGGTCAAGGTGTAGCCCATGAGGTGGCAAGAAATGGGCTACATTTTCT[illegible]
AAAACTACGATAGCCCTTATGAAACTTAAGGGTCGAAGGTGGATTTAGCAGTAAA[illegible]
AGTAGAGTGCTTAGTTGAACAGGGCCCTGAAGCGCGTACACACCGCCCGTCACCCT[illegible]
AAGTATACTTCAAAGGACATTTAACTAAAACCCCTACGCATTTATATAGAGGAGACAA[illegible]
CGTAACCTCAAACTCCTGCCTTTGGTGATCCACCCGCCTTGGCCTACCTGCATAATGAAG
AAGCACCCAACTTACACTTAGGAGATTTCAACTTAACTTGACCGCTCTGAGCTAAACCTA
GCCCCAAACCCACTCCACCTTACTACCAGACAACCTTAGCCAAACCATTTACCCAAATAA
AGTATAGGCGATAGAAATTGAAACCTGGCGCAATAGATATAGTACCGCAAGGGAAAGATG
AAAAATTATAACCAAGCATAATATAGCAAGGACTAACCCCTATACCTTCTGCATAATGAA
TTAACTAGAAATAACTTTGCAAGGAGAGCCAAAGCTAAGACCCCCGAAACCAGACGAGCT
ACCTAAGAACAGCTAAAAGAGCACACCCGTCTATGTAGCAAAATAGTGGGAAGATTTATA
GGTAGAGGCGACAAACCTACCGAGCCTGGTGATAGCTGGTTGTCCAAGATAGAATCTTAG
TTCAACTTTAAATTTGCCCACAGAACCCTCTAAATCCCCTTGTAAATTTAACTGTTAGTC
CAAAGAGGAACAGCTCTTTGGACACTAGGAAAAAACCTTGTAGAGAGAGTAAAAAATTTA
ACACCCATAGTAGGCCTAAAAGCAGCCACCAATTAAGAAAGCGTTCAAGCTCAACACCCA
CTACCTAAAAAATCCCAAACATATAACTGAACTCCTCACACCCAATTGGACCAATCTATC
ACCCTATAGAAGAACTAATGTTAGTATAAGTAACATGAAAACATTCTCCTCCGCATAAGC
CTGCGTCAGATTAAAACACTGAACTGACAATTAACAGCCCAATATCTACAATCAACCAAC
AAGTCATTATTACCCTCACTGTCAACCCAACACAGGCATGCTCATAAGGAAAGGTTAAAA
AAAGTAAAAGGAACTCGGCAAATCTTACCCCGCCTGTTTACCAAAAACATCACCTCTAGC
ATCACCAGTATTAGAGGCACCGCCTGCCCAGTGACACATGTTTAACGGCCGCGGTACCCT
AACCGTGCAAAGGTAGCATAATCACTTGTTCCTTAAATAGGGACCTGTATGAATGGCTCC
```

Sequence differences occur because of...

1. Sequencing error
2. Genetic variation

Approximate matching

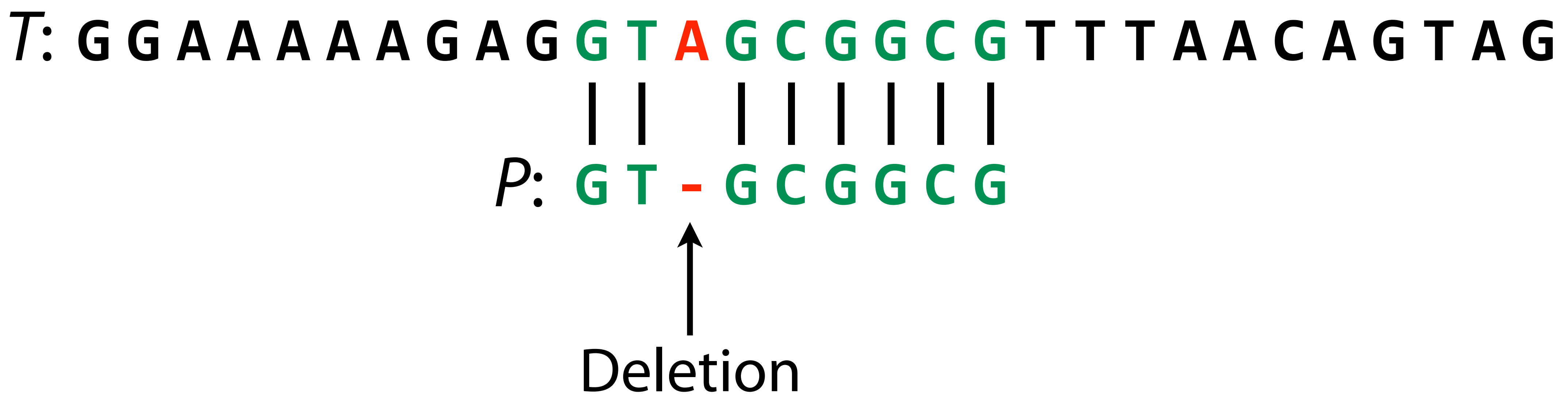
```
T: GGAAAAAGAGGTAGCGGCGTTTAACAGTAG
             ||| |||||
          P: GTAACGGCG
                ↑
```

Mismatch
(Substitution)

Approximate matching

```
T: G G A A A A A G A G G T A G C - G C G T T T A A C A G T A G
                       | | | | |   | | |
P:                     G T A G C G G C G
                                 ↑
                                 |
                             Insertion
```

Approximate matching



Hamming distance

For *X* & *Y* where |*X*| = |*Y*|, *hamming distance* = minimum # substitutions needed to turn one into the other

```
X: G A G G T A G C G G C G T T
   |   | | | |   | | |   | | |
Y: G T G G T A A C G G G G T T
```

Hamming distance = 3

Edit distance

(AKA Levenshtein distance)

For *X* & *Y*, *edit distance* = minimum # edits (substitutions, insertions, deletions) needed to turn one into the other

```
X: T G G C C G C G C A A A A A C A G C
   | |   | | | | | | | | | |   | | | |      Edit distance = 2
Y: T G A C C G C G C A A A A - C A G C

X: G C G T A T G C G G C T A - A C G C
   | |   | | | | | | | | | |   | | |        Edit distance = 2
Y: G C - T A T G C G G C T A T A C G C
```

Approximate matching

Like exact matching, but *pattern P* may be within a certain *distance* (usually Hamming or edit) of *T*. Each such place is an *approximate match*.

Allowing edits is more challenging than just allowing mismatches

We'll return to edits

Approximate matching

```
def naive(p, t):
    occurrences = []
    for i in range(len(t) - len(p) + 1):  # loop over alignments
        match = True
        for j in range(len(p)):           # loop over characters
            if t[i+j] != p[j]:            # compare characters
                match = False             # mismatch; reject alignment
                break
        if match:
          occurrences.append(i)           # all chars matched; record
    return occurrences
```

Approximate matching

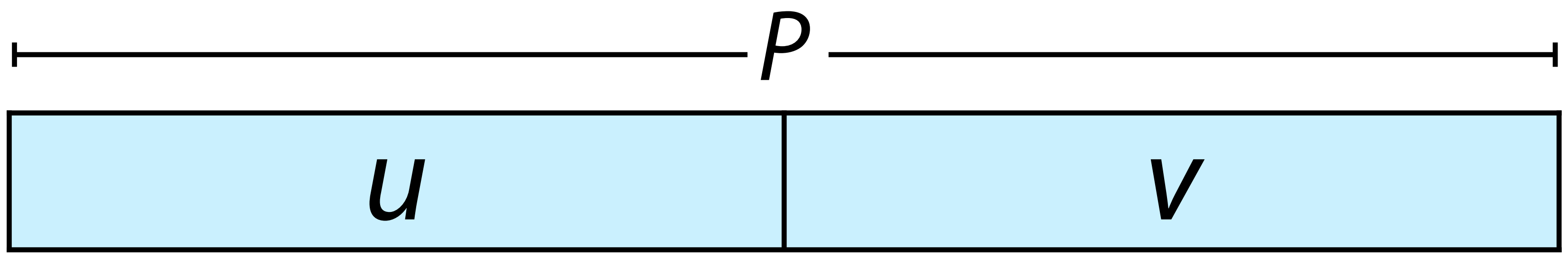
```
def naive_approx_hamming(p, t, maxDistance):
    occurrences = []
    for i in range(len(t) - len(p) + 1):  # loop over alignments
        nmm = 0
        for j in range(len(p)):           # loop over characters
            if t[i+j] != p[j]:            # compare characters
                nmm += 1                  # mismatch
                if nmm > maxDistance:
                    break                 # exceeded max hamming dist
        if nmm <= maxDistance:
          occurrences.append(i)           # approximate match
    return occurrences
```

http://bit.ly/CG_NaiveApprox

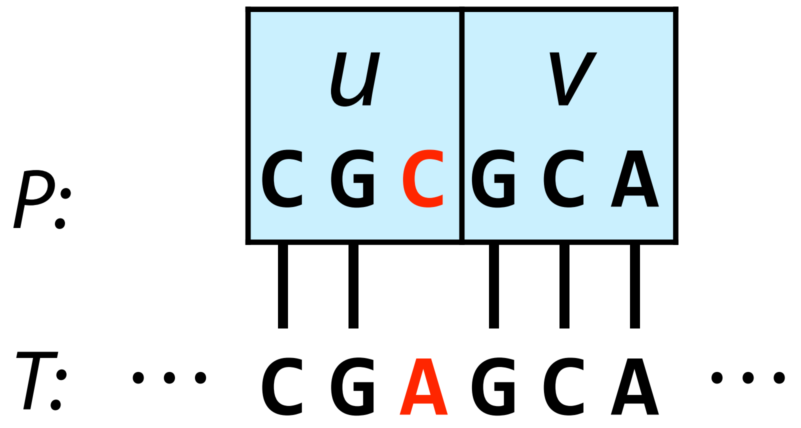
Approximate matching

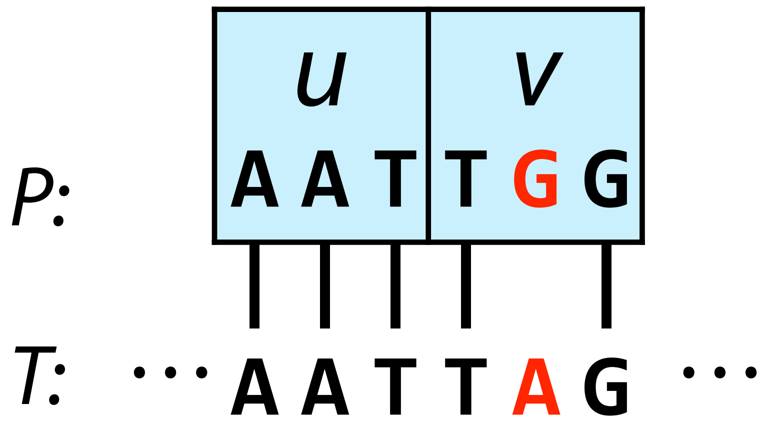
Wanted: way to apply exact matching algorithms to approximate matching problems

Approximate matching

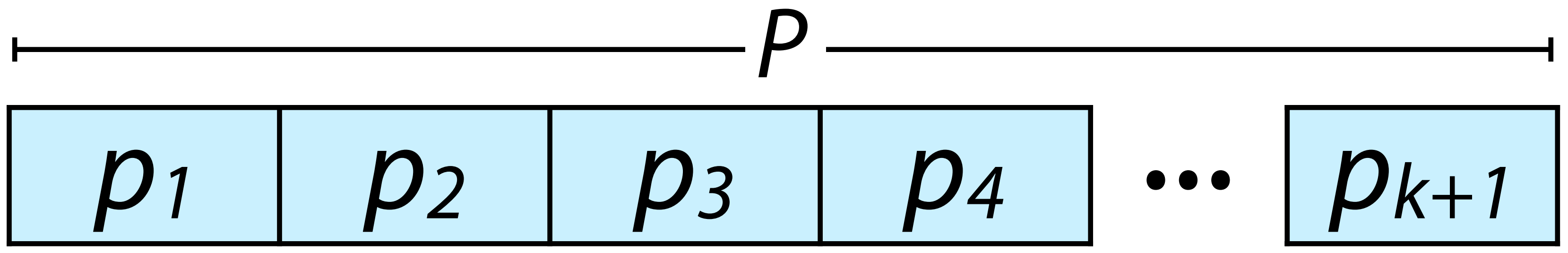


If *P* occurs in *T* with 1 edit, then *u* or *v* appears with no edits



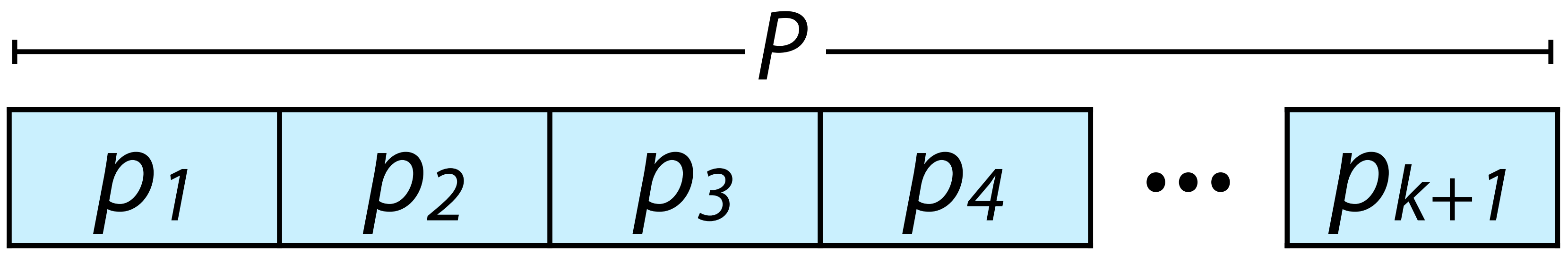


Approximate matching



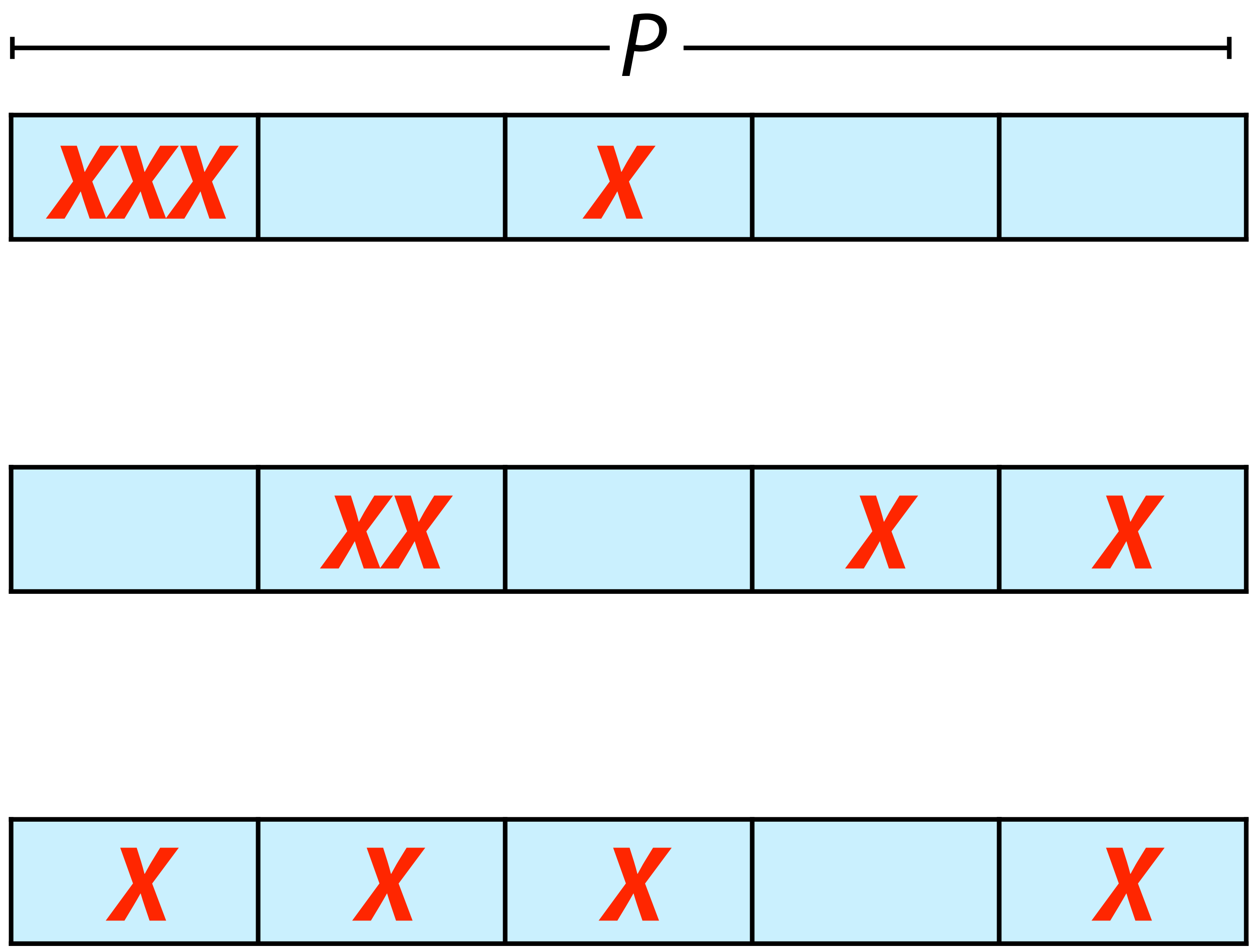
If P occurs in T with up to k edits…

Approximate matching



If P occurs in T with up to k edits, then at least one of p_1, p_2, ..., p_{k+1} must appear with 0 edits

Approximate matching



5 partitions
4 edits (X)

Approximate matching

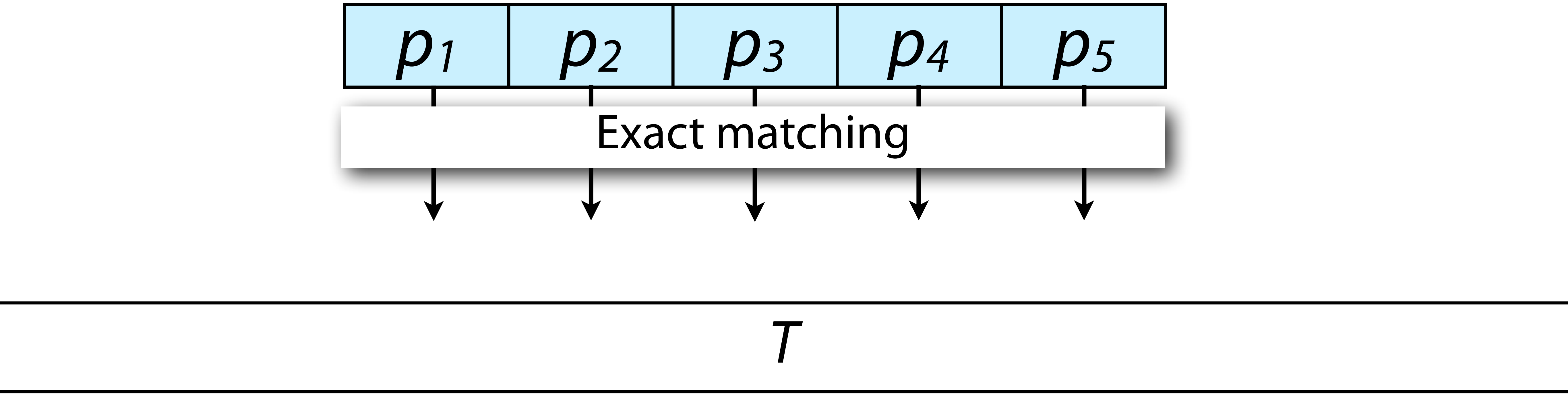
Pigeonhole principle: *k+1* pigeons, *k* holes.
At least one has >1 pigeon!

Approximate matching

We have *k* pigeons, *k*+1 holes, at least one...

...is *empty*

Pigeonhole principle



What algorithm can we use?

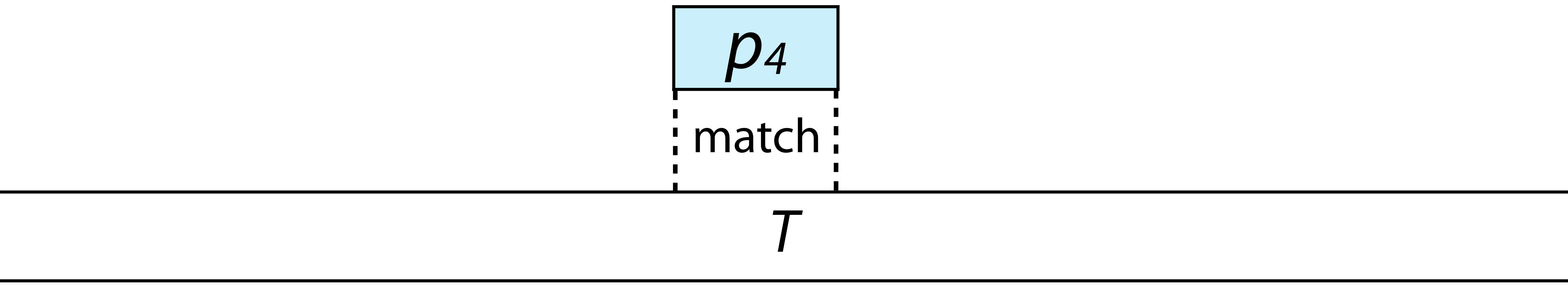
Any exact matching algorithm

If we have a k-mer index, we can use that

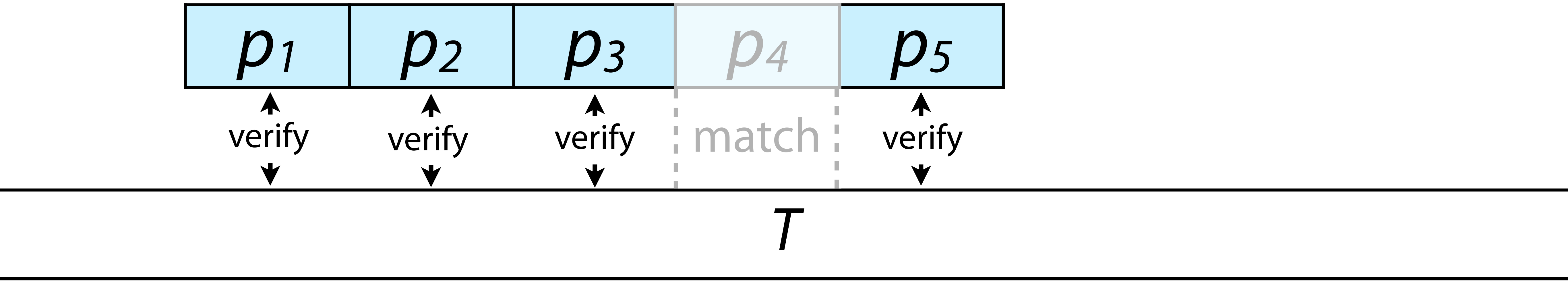
Naive exact matching

Boyer-Moore

Pigeonhole principle

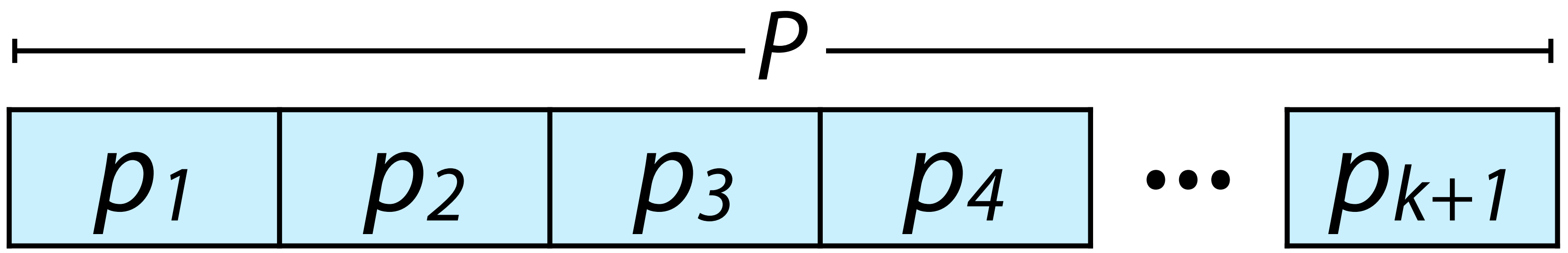


Pigeonhole principle



For Hamming distance, verification is essentially just the inner loop of **naive_approx_hamming** from before

Pigeonhole principle



Advantages

Reuse favorite exact matching algos; fast and easy

Flexible; works for Hamming and edit distance*

Disadvantages

Large *k* yields small partitions matching many times by chance; lots of verification work

k+1 exact matching problems, one per partition

* we don't know how to do edit distance verification yet

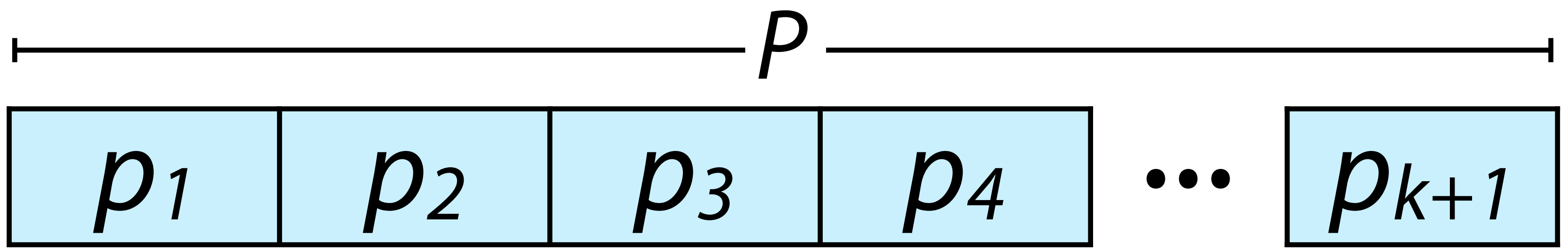
Implementation of pigeonhole principle with Boyer-Moore as exact matching algorithm: http://j.mp/CG_ApproxBM

	Boyer-Moore, exact			Boyer-Moore, ≤1 mismatch with pigeonhole			Boyer-Moore, ≤2 mismatches with pigeonhole		
	# character comparisons	wall clock time	# matches	# character comparisons	wall clock time	# matches	# character comparisons	wall clock time	# matches
P: "tomorrow" **T**: Shakespeare's complete works	786 K	1.91s	17	3.05 M	7.73 s	24	6.98 M	16.83 s	382
P: 50 nt string from Alu repeat* **T**: Human reference (hg19) chromosome 1	32.5 M	67.21 s	336	107 M	209 s	1,045	171 M	328 s	2,798

* GCGCGGTGGCTCACGCCTGTAATCCCAGCACTTTGGGAGGCCGAGGCGGG

Generalizing pigeonhole, part 1

If P occurs in T with up to k edits, then at least one of $p_1, p_2, ..., p_{k+1}$ must appear with 0 edits



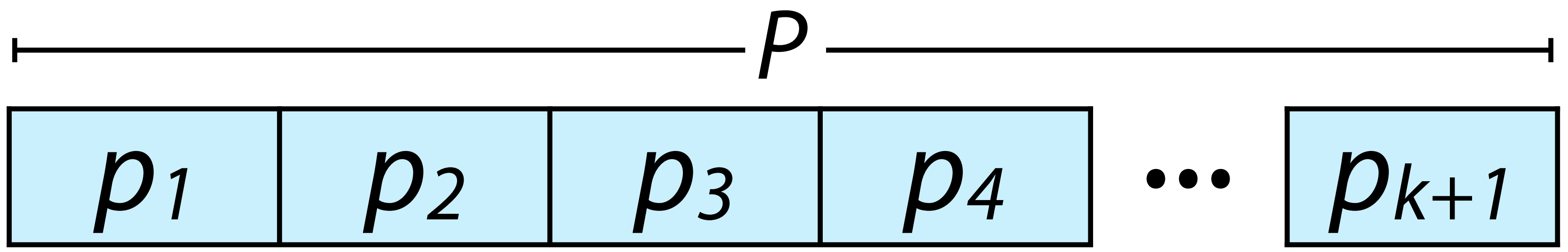
But doesn't *have to* be "at least one of" . . .

what would we have to change for "at least two of"?

If P occurs in T with up to k edits, then at least two of ____________ must appear with 0 edits

Generalizing pigeonhole, part 1

If P occurs in T with up to k edits, then at least one of $p_1, p_2, ..., p_{k+1}$ must appear with 0 edits



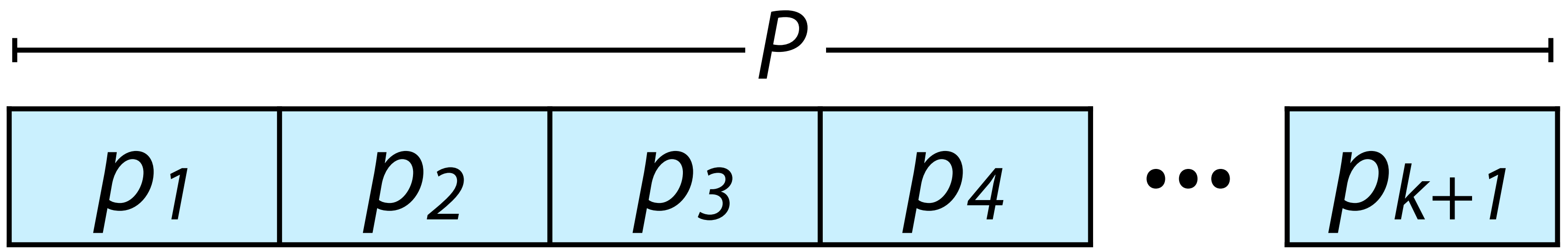
But doesn't *have to* be “at least one of” …

what would we have to change for “at least two of”?

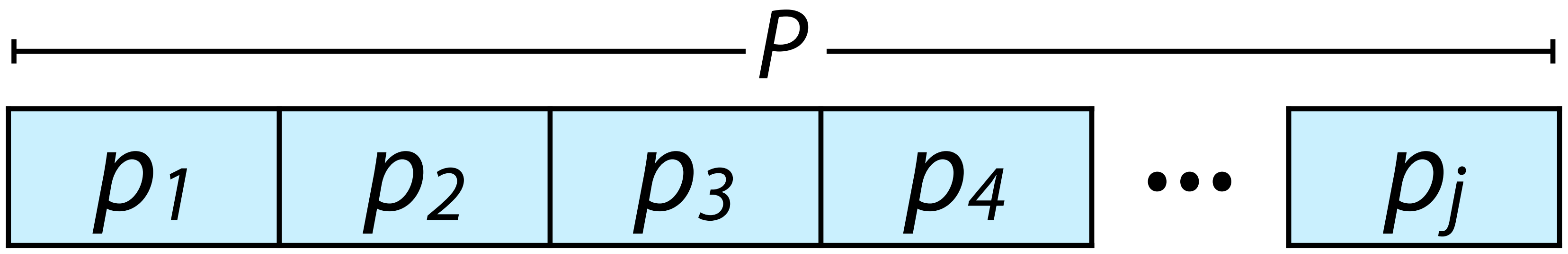
If P occurs in T with up to k edits, then at least two of $p_1, p_2, ..., p_{k+2}$ must appear with 0 edits

Generalizing pigeonhole, part 2

If P occurs in T with up to k edits, then at least one of $p_1, p_2, ..., p_{k+1}$ must appear with 0 edits

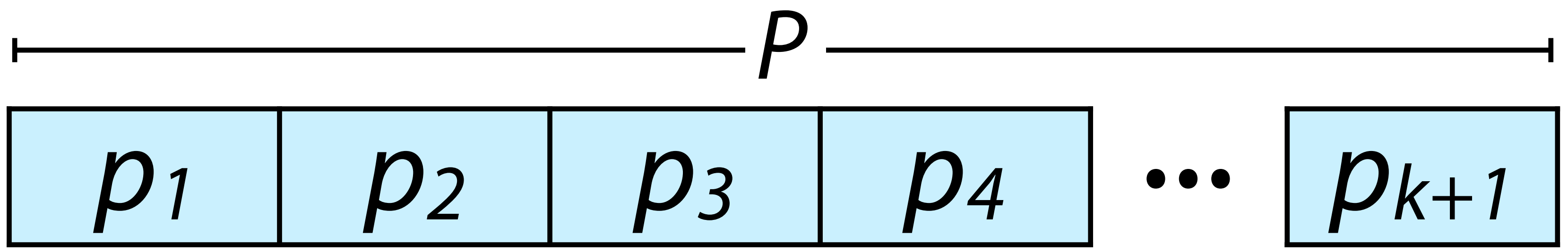


Let $p_1, p_2, ..., p_j$ be a partitioning of P. If P occurs with up to k edits, then at least one of $p_1, p_2, ..., p_j$ must occur with ≤ ??? edits.

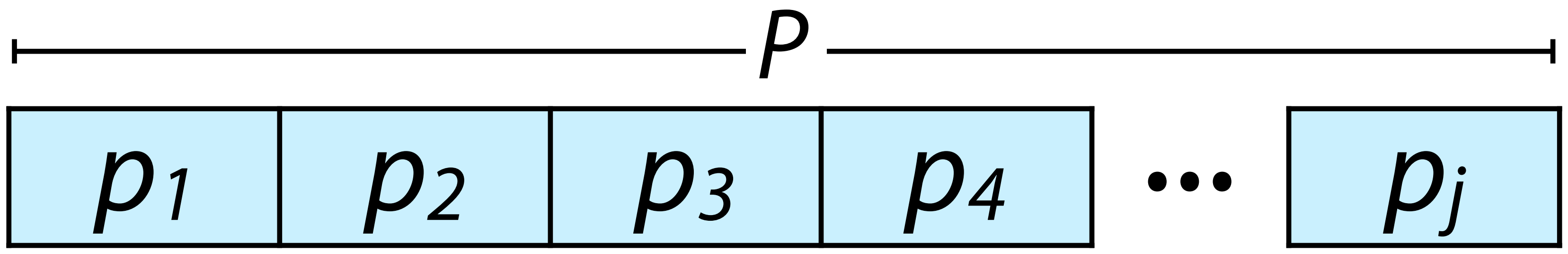


Generalizing pigeonhole, part 2

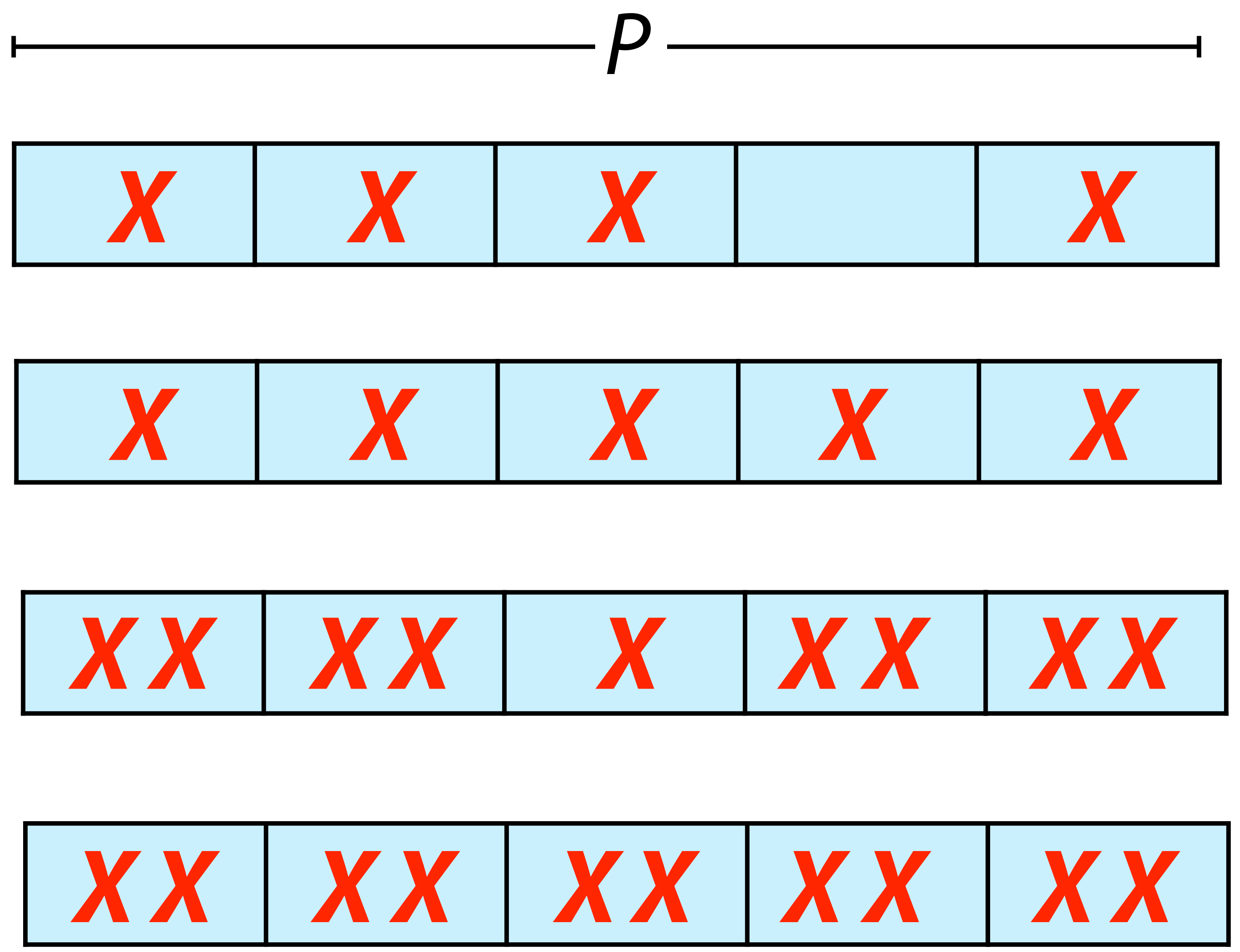
If P occurs in T with up to k edits, then at least one of $p_1, p_2, ..., p_{k+1}$ must appear with 0 edits



Let $p_1, p_2, ..., p_j$ be a partitioning of P. If P occurs with up to k edits, then at least one of $p_1, p_2, ..., p_j$ must occur with $\leq$ *floor*(k / j) edits.



Generalizing pigeonhole, part 2



At least one of $p_1, p_2, ..., p_5$ occurs with...

k = 4 edits	≤0 edits
k = 5 edits	≤1 edits
k = 9 edits	≤1 edits
k = 10 edits	≤2 edits

etc

Generalizing pigeonhole, part 2

General

Pigeonhole principle

If P occurs in T with up to k edits, then at least one of $p_1, p_2, ..., p_j$ must appear with *floor*(k / j) edits

Specific

Pigeonhole principle with $j = k + 1$

If P occurs in T with up to k edits, then at least one of $p_1, p_2, ..., p_{k+1}$ must appear with 0 edits

Let $j = k + 1$

Why?

Smallest value s.t. *floor*(k / j) = 0

Why make floor(k / j) = 0?

So we can use exact matching

Why is smaller j good?

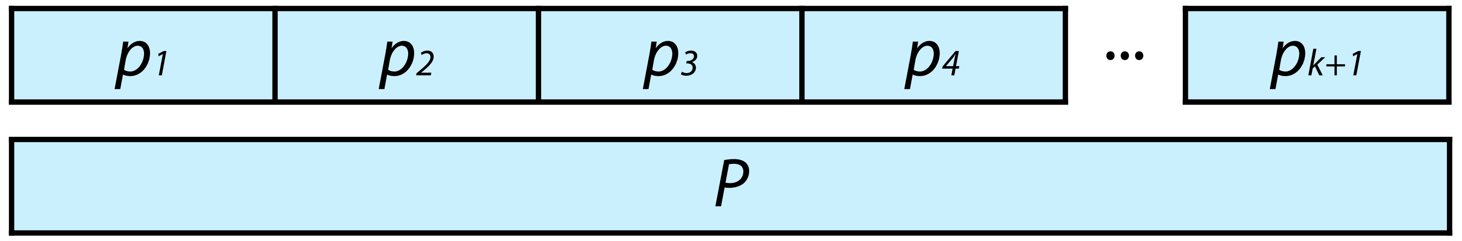
Yields fewer, longer partitions

Why are long partitions good?

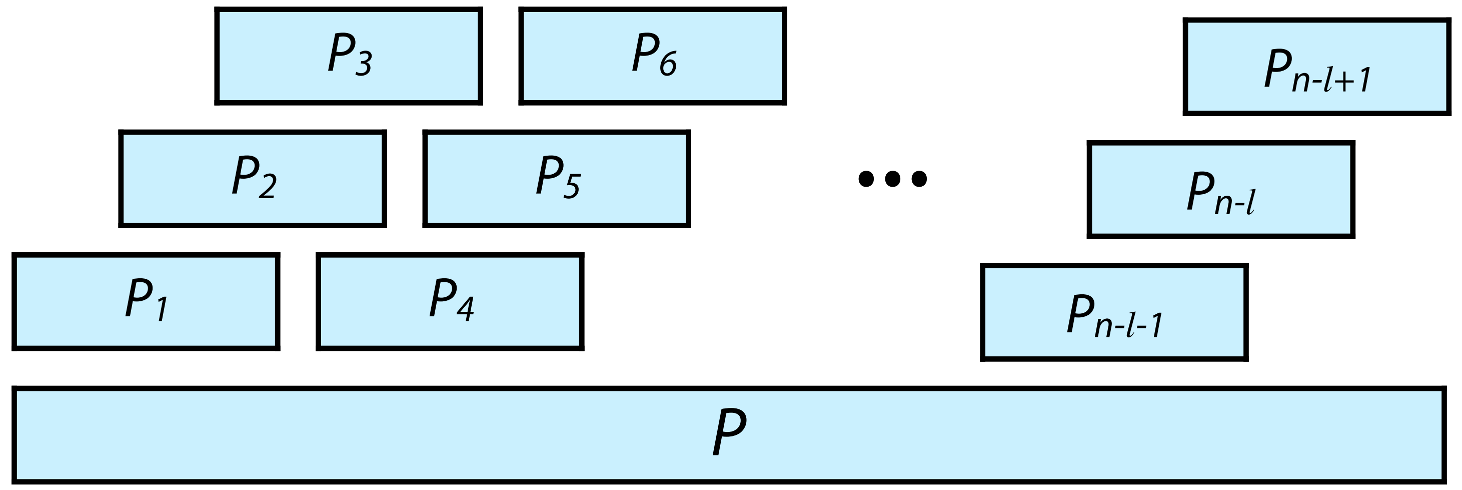
Makes exact-matching filter more specific, minimizing # candidates

A different principle

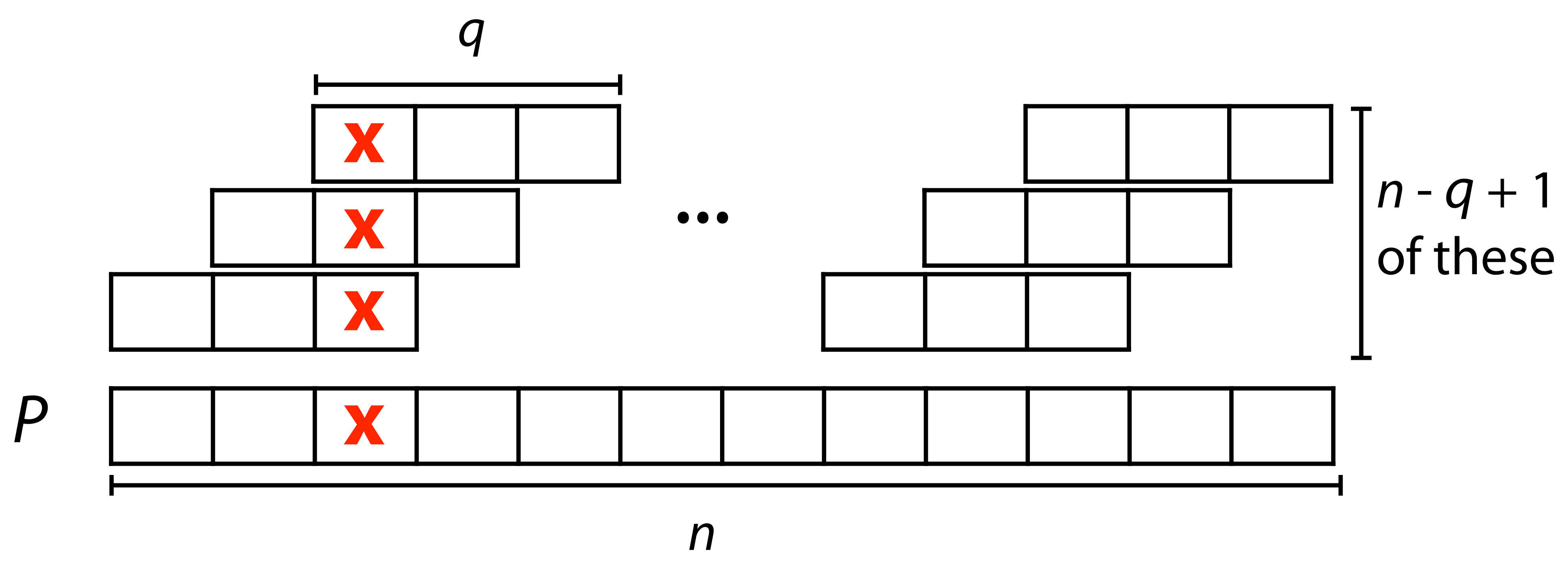
We partitioned *P* into non-overlapping substrings



Now consider *overlapping* substrings



Approximate string matching: more principles



Say substrings are length *q*. There are *n* - *q* + 1 such substrings.

1 edit to *P* changes *at most q* substrings

kq is *worst case; could be < kq*

Minimum # of length-*q* substrings unedited after *k* edits? $n - q + 1 - \boxed{kq}$

q-gram lemma: if *P* occurs in *T* with up to *k* edits, alignment must contain *t* exact matches of length *q*, where $t \geq n - q + 1 - kq$

Approximate string matching: more principles

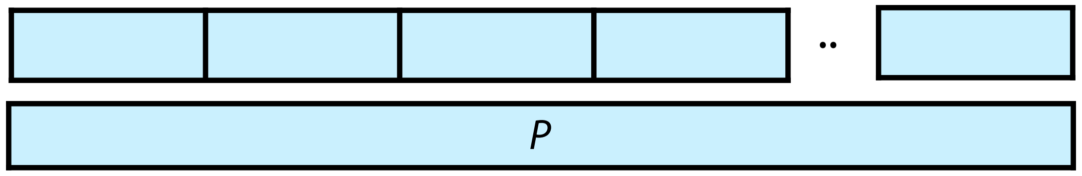
If P occurs in T with up to k edits, alignment contains an exact match of length q, where $q \geq \mathit{floor}(n / (k + 1))$

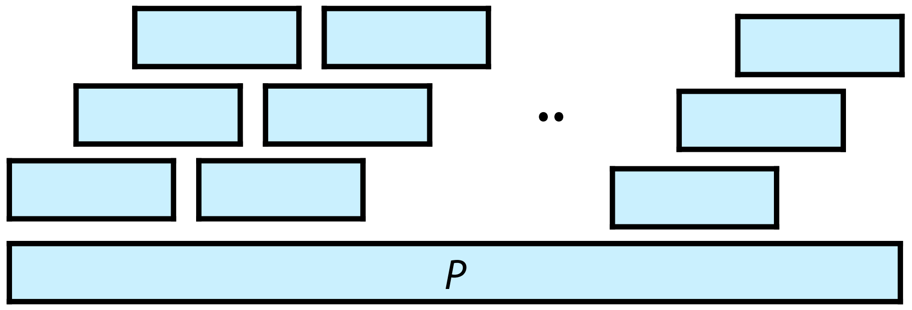
Obtained by solving for q: $n - q + 1 - kq \geq 1$

Exact matching filter: find matches of length $\mathit{floor}(n / (k + 1))$ between T and *any* substring of P. Check vicinity for full match.

Approximate matching principles

	Non-overlapping substrings	Overlapping substrings
General	*Pigeonhole principle* $p_1, p_2, ..., p_j$ is a partitioning of P. If P occurs with $\leq k$ edits, at least one partition matches with $\leq$ *floor*(k / j) edits.	*q-gram lemma* If P occurs with $\leq k$ edits, alignment contains t exact matches of length q, where $t \geq n - q + 1 - kq$
Specific	*Pigeonhole principle with $j = k + 1$* $p_1, p_2, ..., p_{k+1}$ is a partitioning of P. If P occurrs in T with $\leq k$ edits, at least one partition matches exactly.	*q-gram lemma with $t = 1$* If P occurs with $\leq k$ edits, alignment contains an exact match of length q where $q \geq$ *floor*(n / (k + 1))



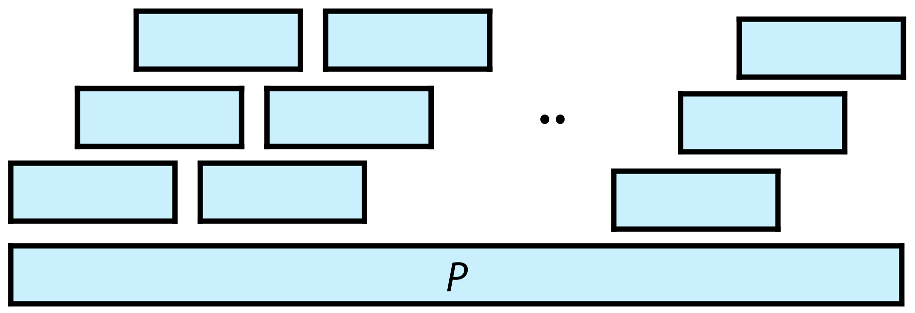


Sensitivity

Sensitivity = fraction of "true" approximate matches discovered by the algorithm

Lossless algorithm finds all of them, *lossy* algorithm doesn't necessarily

We've seen *lossless* algorithms. Most everyday tools are *lossy*. Lossy algorithms are usually much speedier & still acceptably sensitive.



Example lossy algorithm: pick $q > \textit{floor}(n \,/\, (k + 1))$