

Constrained Control for Surgical Assistant Robots

Ankur Kapoor, Ming Li and Russell H. Taylor



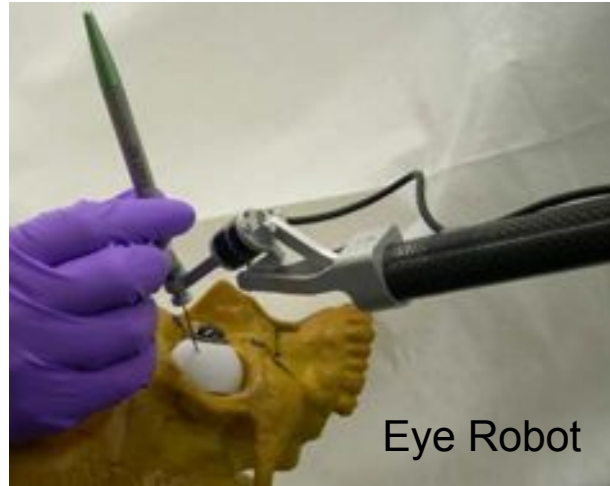
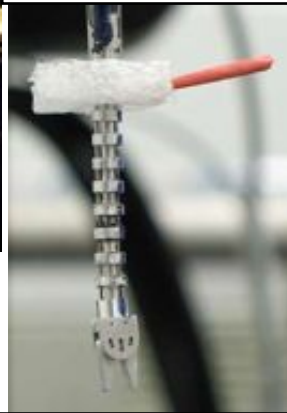
*Department of Computer Science
The Johns Hopkins University*



Context: teleoperated and cooperatively controlled surgical robots



4mm Snake-Like Robot for throat



Eye Robot



DaVinci

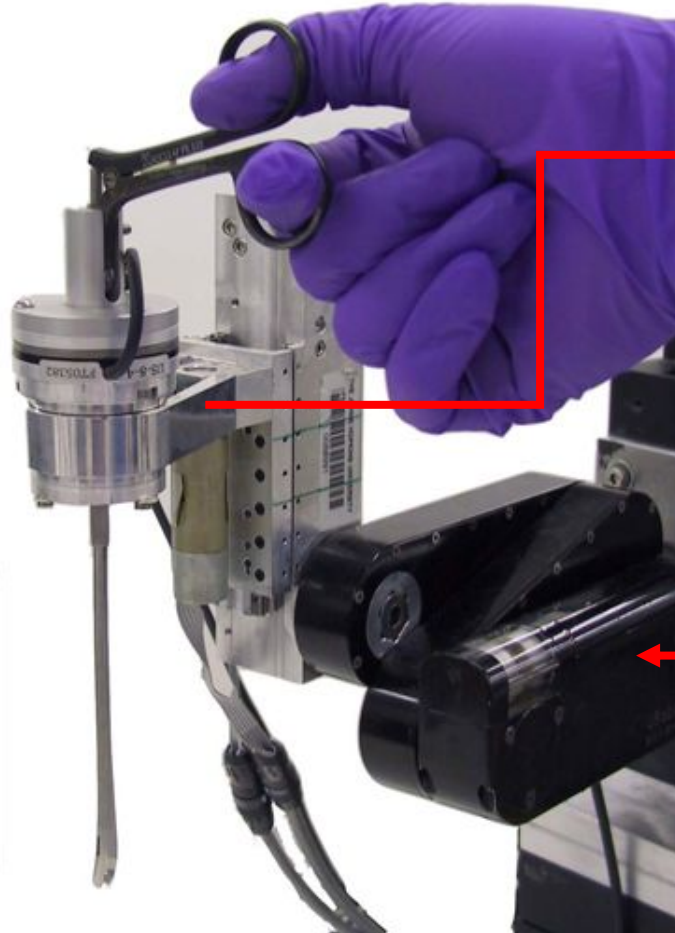


Steady Hand Robot

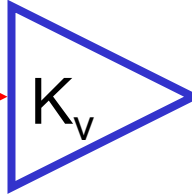


Steady Hand Robot

Admittance Control



Handle Force



Joint Velocities

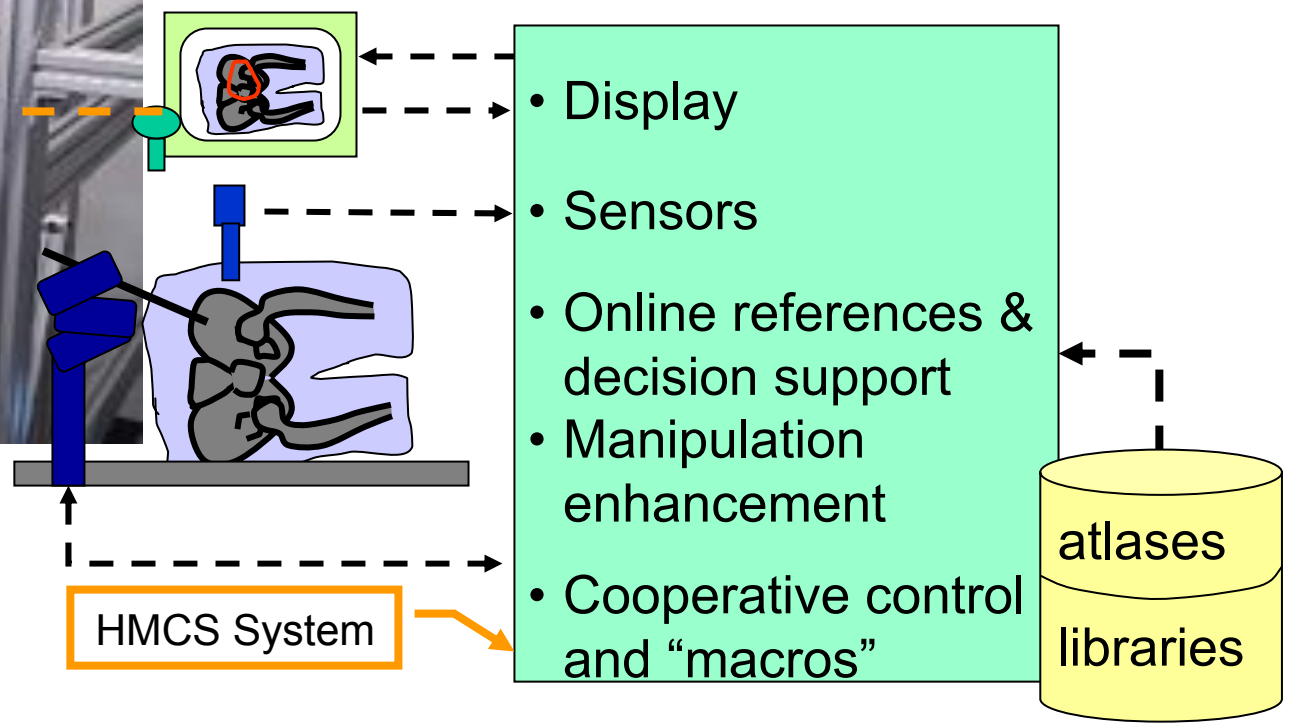


Kumar et al., MICCAI '99



Human-machine cooperative manipulation in surgery

Situation assessment
 Task strategy & decisions
 Sensory-motor coordination

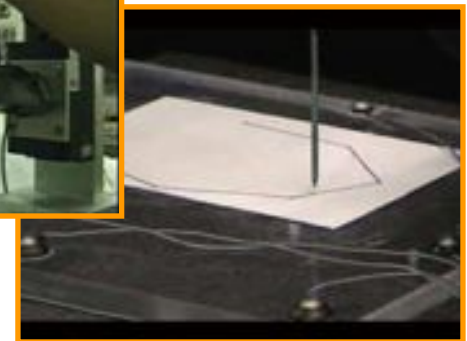
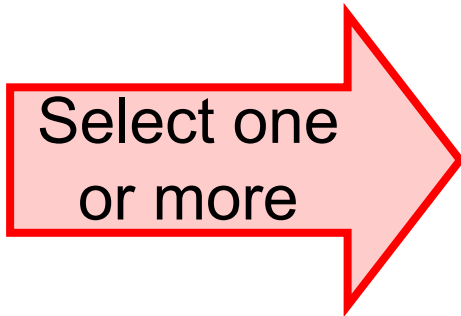
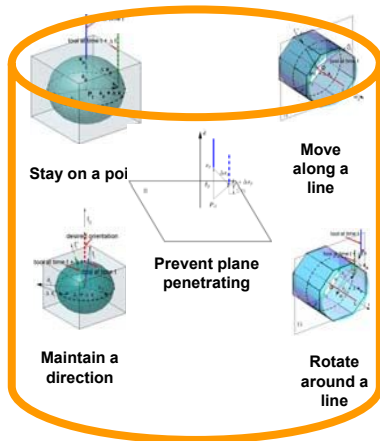


R. Taylor



Contribution

- Method for combining human-understandable task primitives into sophisticated task-specific behaviors
- Covers the implementation of guidance virtual fixtures, and forbidden region virtual fixtures, with both “hard” and “soft” constraints.
- Mathematical formulations based on constrained quadratic optimization and nonlinear optimization
- Independent of manipulator, low-level controller, or form of human interaction



Customized virtual fixtures

Library of primitives



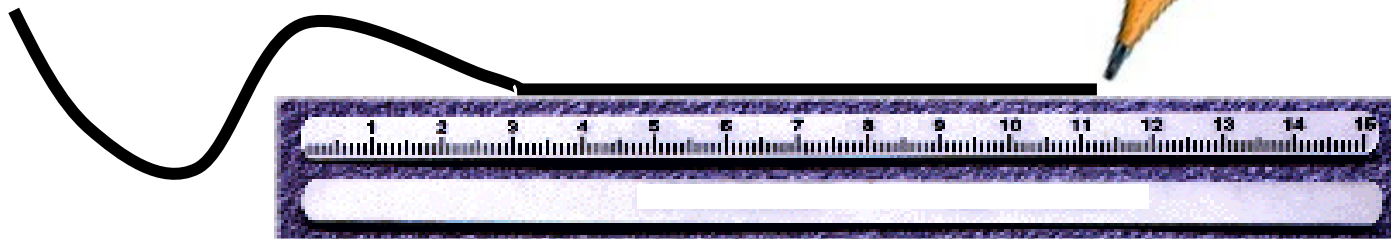
Related Works

- Virtual fixtures: perceptual overlays designed to enhance performance
 - **Active Compliance**
 - Rosenberg
 - Stanisic et al.
 - Davies et al.
 - Park et al
 - **Vision Based**
 - Hager & Okamura group
 - **Based on Constrained Control**
 - Funda et al
 - Li et al



Virtual Fixtures




- Bridge the gap between autonomous robots and direct human control.
- Assist the human operator in safer, faster, and more accurate task completion.
- Designed to maintain a degree of operator control.

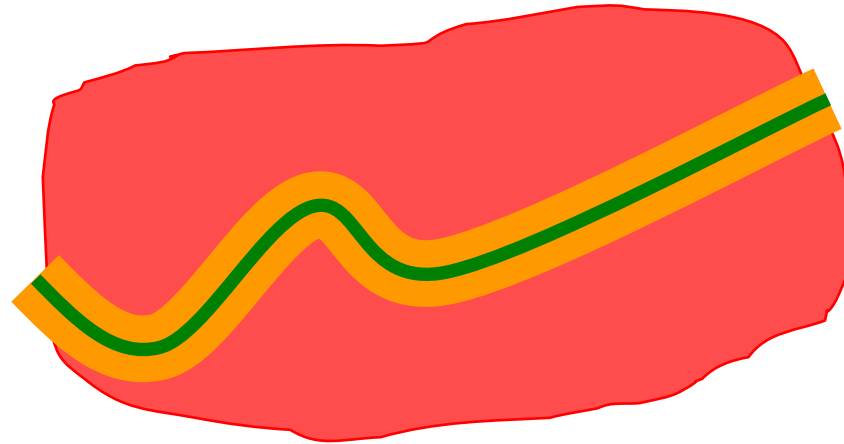


- But surgical tasks have uncertainty and non-rigidity, examples
 - Registration errors
 - Variations in anatomy
 - Unexpected changes during procedures



Virtual fixtures for surgical tasks

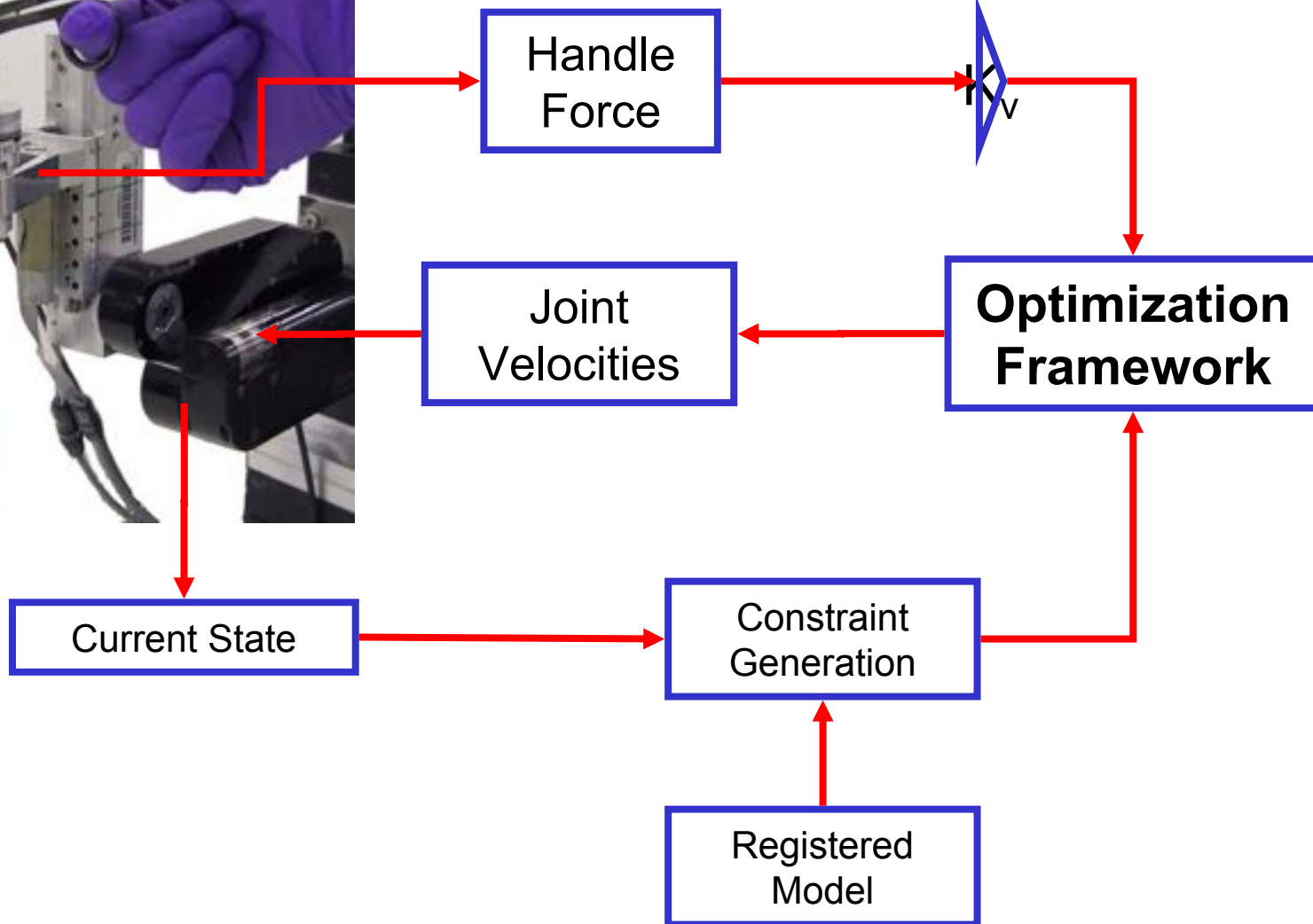
-  Preferred region
-  Safety region
-  Forbidden region



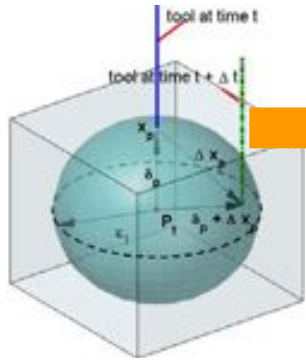
- Constraints on the task can be “hard” or “soft”
- The relative sizes depend on the procedure, ranging from micros to tenths of millimeter.
- Allows the controller to accommodate uncertainties inherent in surgical procedures.

Steady Hand Robot

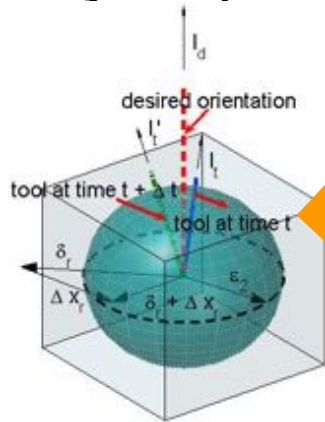
High Level Constrained Control



5 Basic Geometric Constraints (Virtual fixture library)



Stay on a point



Maintain a direction

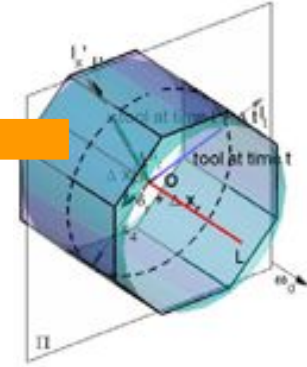
Optimization

$$\arg \min_{\Delta \vec{q}} C(\vec{x}(\vec{q} + \Delta \vec{q}), \vec{s}, \vec{x}^d)$$

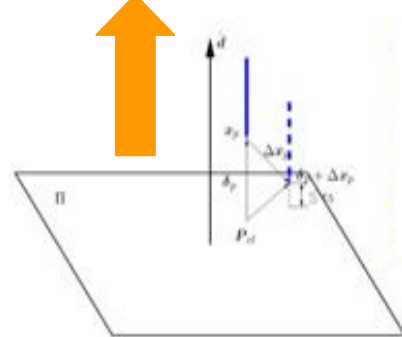
s. t. $A(\vec{x}(\vec{q} + \Delta \vec{q}), \vec{s}) \leq \vec{b},$

$$\vec{s}_{up} \geq s \geq \vec{s}_{low} \geq 0,$$

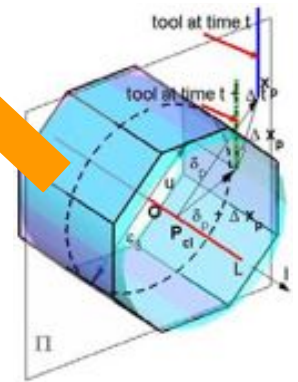
$$\Delta \vec{q}_{up} \geq \Delta \vec{q} \geq \Delta \vec{q}_{low}$$



Move along a line



Prevent plane penetrating



Rotate around a line



Example Constraint: Stay on a Point

Target Position: \vec{x}_0

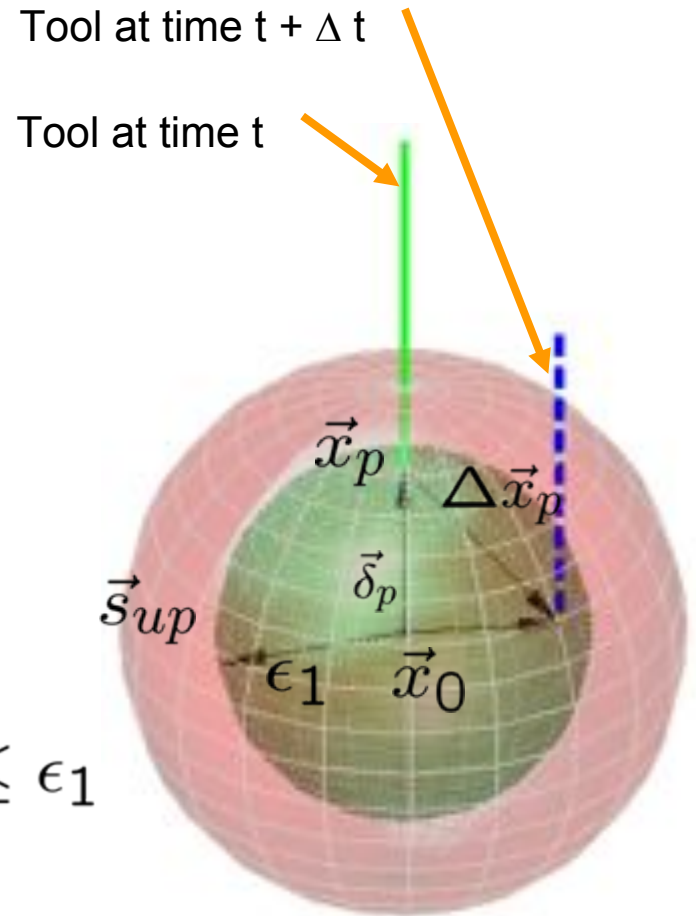
After incremental motion

$$\vec{x}_p + \Delta\vec{x}_p \text{ close to } \vec{x}_0$$

We want...

$$A(\vec{x}, s) = \|\vec{\delta}_p + \Delta\vec{x}_p\|^2 - s \leq \epsilon_1$$

where $\vec{\delta}_p = \vec{x}_p - \vec{x}_0$





Using Linear Constrained Quadratic Optimization

Matrix representation

$$A \cdot \Delta \vec{x} - s \leq b$$

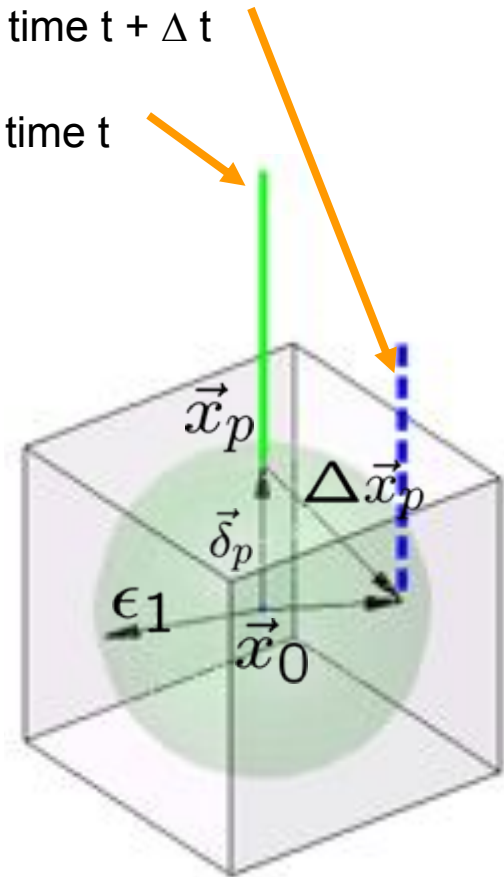
Use Constrained Least Squares to solve

$$\arg \min_{\Delta \vec{q}} \|\Delta \vec{x} - \Delta \vec{x}^d\|^2$$

$$s.t \quad A \cdot \Delta \vec{x} - s \leq b$$

Tool at time $t + \Delta t$

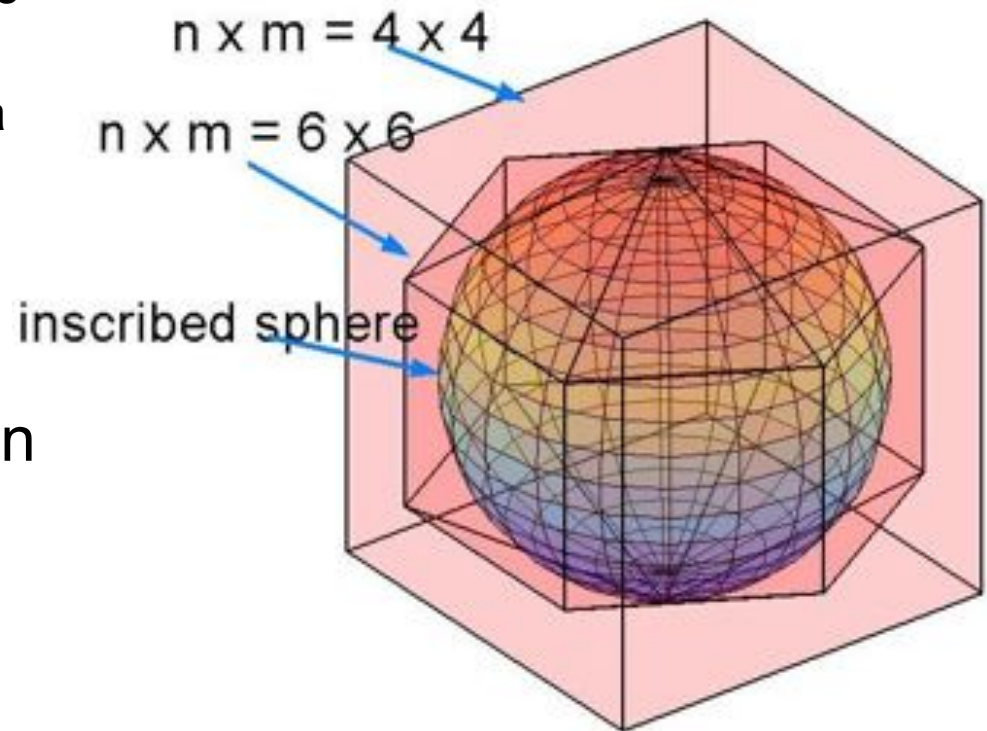
Tool at time t





Linear approximation for constraints

- $n \times m$ increase
 - Polyhedron approaches the inscribed sphere
 - Linearized conditions are a better approximation
 - More constraints require more time to solve the optimization problem
- Symmetrical polyhedron
 - $n \times m = 4 \times 4$
- Bounded polyhedron
 - $n \times m = 3 \times 3$





Using Non-Linear Constrained Optimization

- Use Sequential Quadratic Program* method
- SQP solves the following problem iteratively

$$\arg \min_{d^k} \nabla C(\vec{x}(\vec{q} + \Delta \vec{q}^k), \vec{s}^k, \vec{x}^d)^t \vec{d} + \frac{1}{2} \vec{d}^t B_k \vec{d}$$
$$s.t. \quad \nabla A_{\mathcal{A}_k}(\vec{x}_i(\vec{q} + \Delta \vec{q}^k), \vec{s}^k)^t \vec{d} \leq \vec{b}_{\mathcal{A}_k},$$

- Start with a solution $[\Delta \vec{q}^k, \vec{s}^k]^t$
- Descent direction along with step size determine next solution $[\Delta \vec{q}^{k+1}, \vec{s}^{k+1}]^t$

*P. Spellucci, *Math. Prog.*, '98

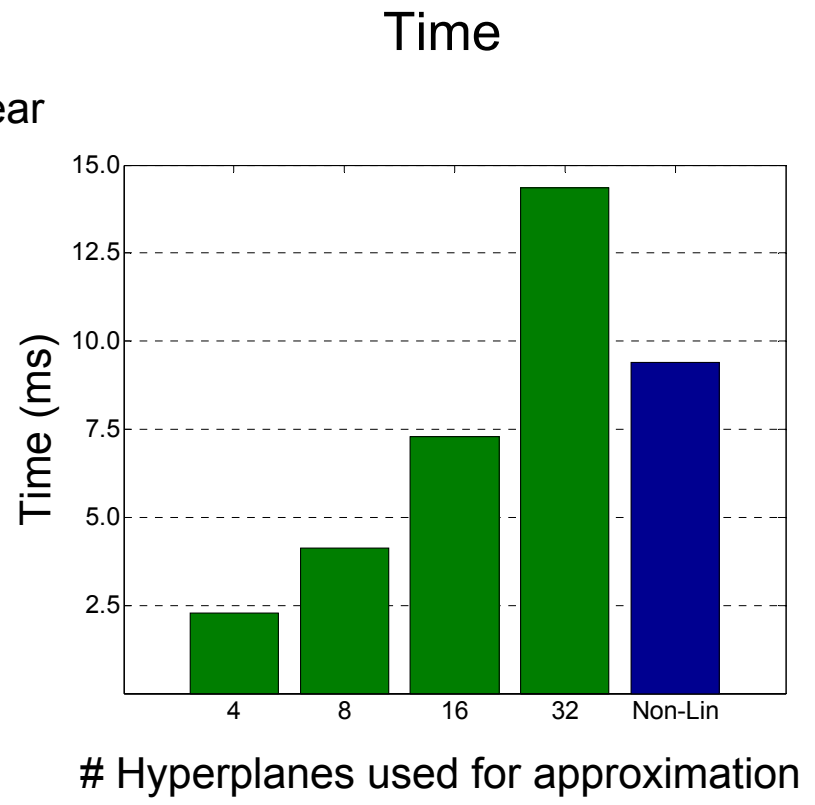
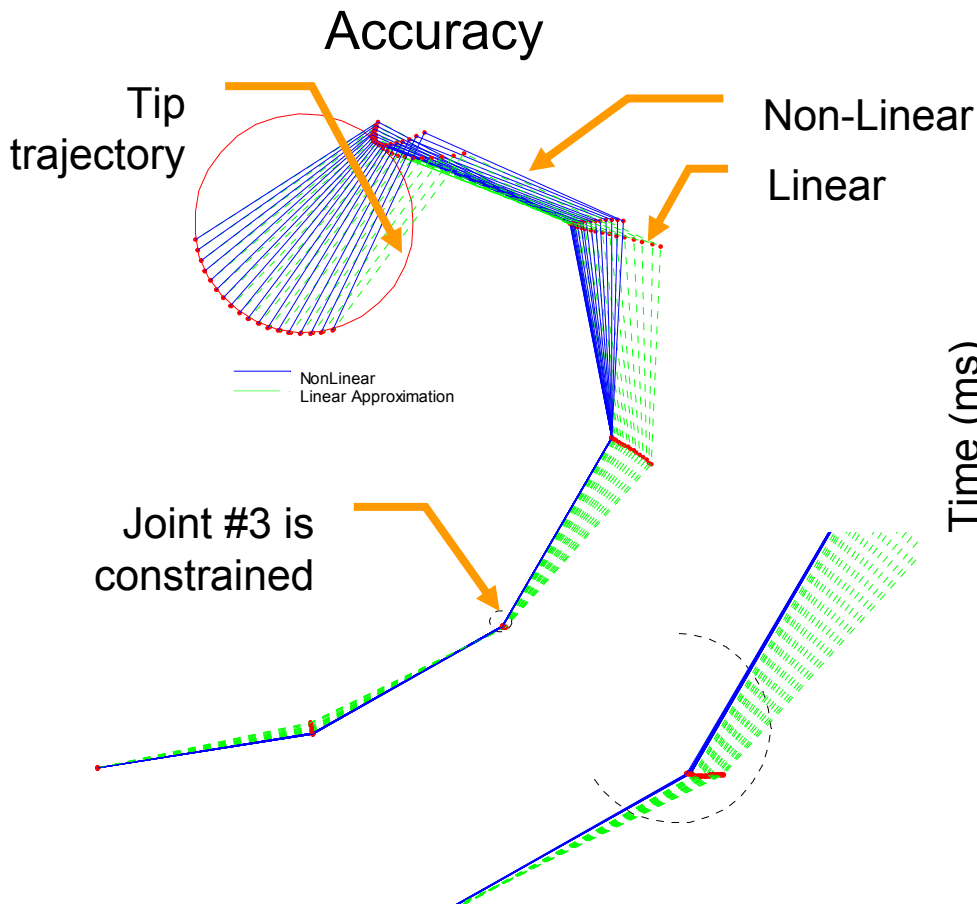


Remarks: Non-Linear Constraints

- Current incremental motion can be used as starting guess for next motion
- Worst case number of hyperplanes used by SQP is n times m , $n = \#$ variables, $m = \#$ nonlinear constraints
- Analytical gradient increases speed

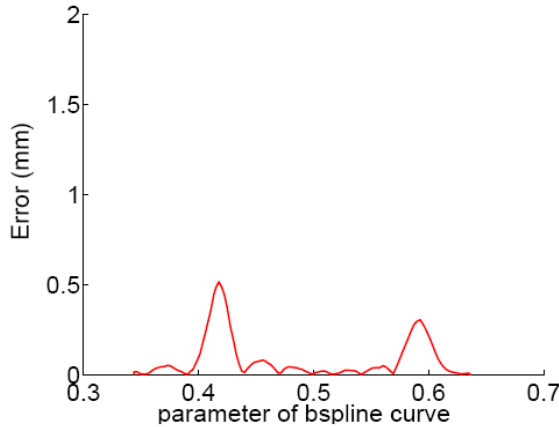


Linear v. Non-Linear Constraints

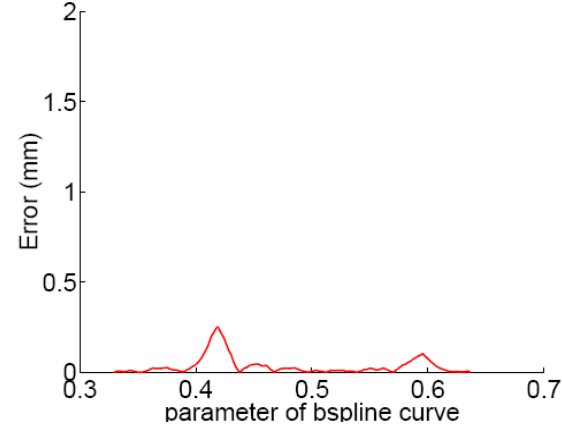




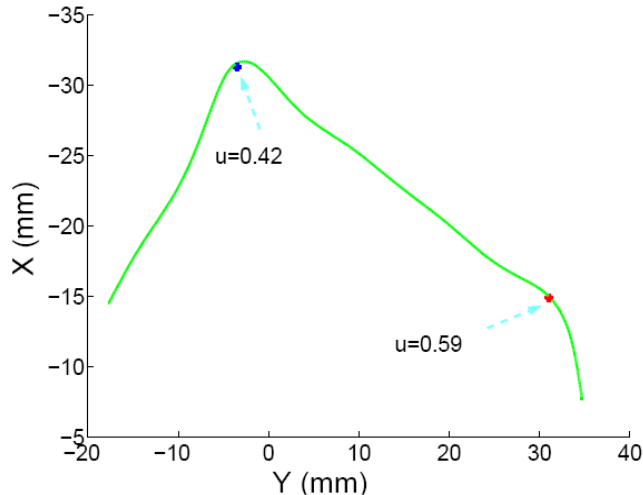
Effect of increasing control-loop time



Interval: 150ms



Interval: 40ms



- Large error at sharp turning
- Small interval reduces error

Ming Li et al., IROS '05



“Soft” Constraints

- The i th constraint

$$A(\Delta \vec{x}_i) - s_i \leq b_i$$

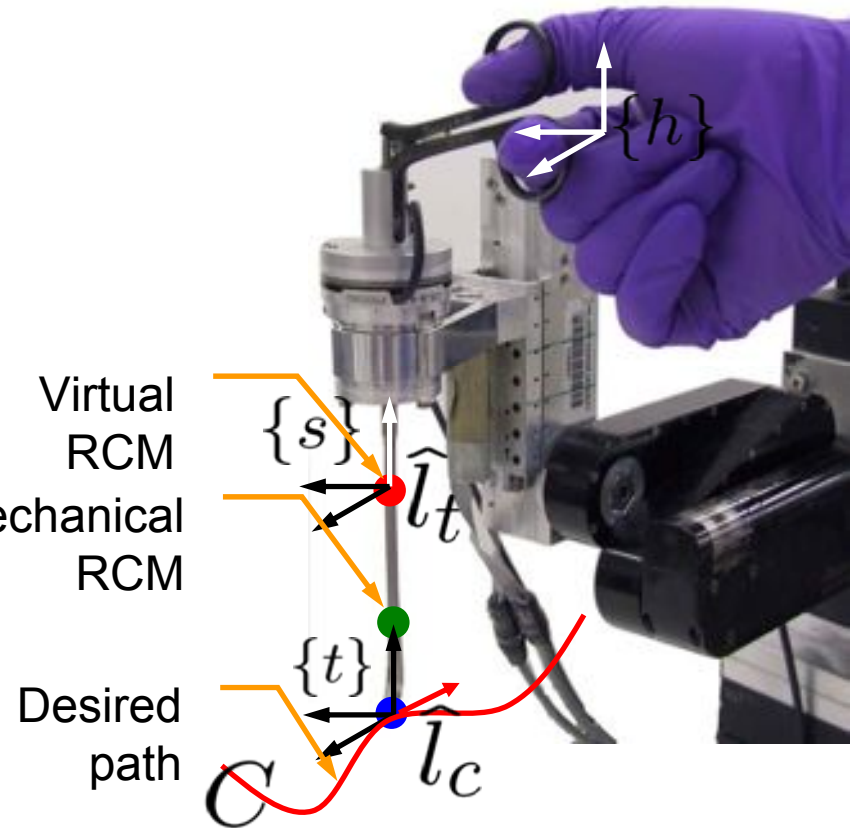
- Penalty on “slack” variable: $w_{s,i} \cdot s_i$
- Constraints on “slack”: $\vec{s}_{up} \geq \vec{s} \geq \vec{s}_{low} \geq 0$
- The complete objective function

$$C(\vec{x}(\vec{q} + \Delta \vec{q}), \vec{s}, \vec{x}^d) = \|J(\vec{q}) \cdot \Delta \vec{q} - \Delta \vec{x}^d\|_2^2 + \|W_s \vec{s}\|_2^2$$



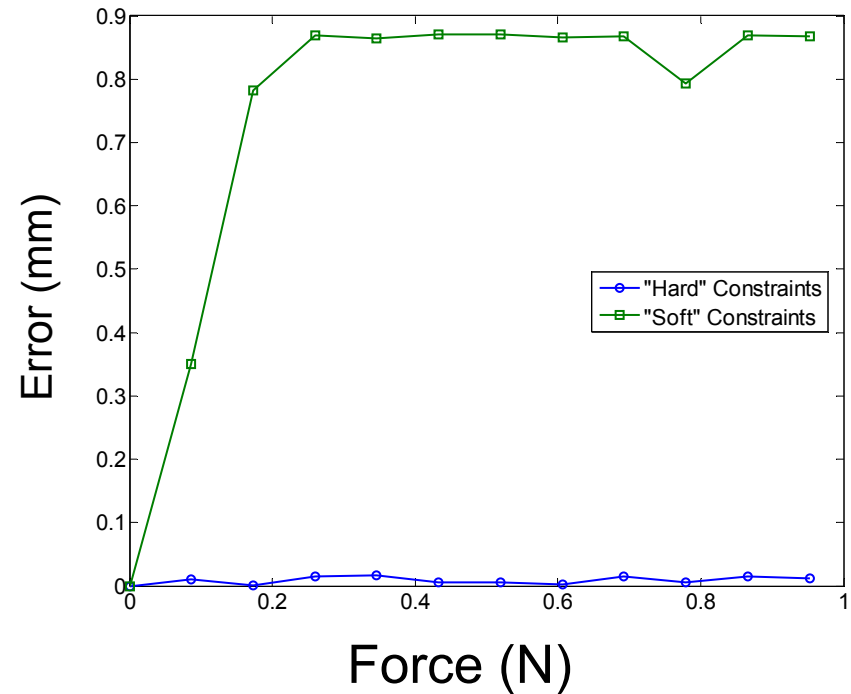
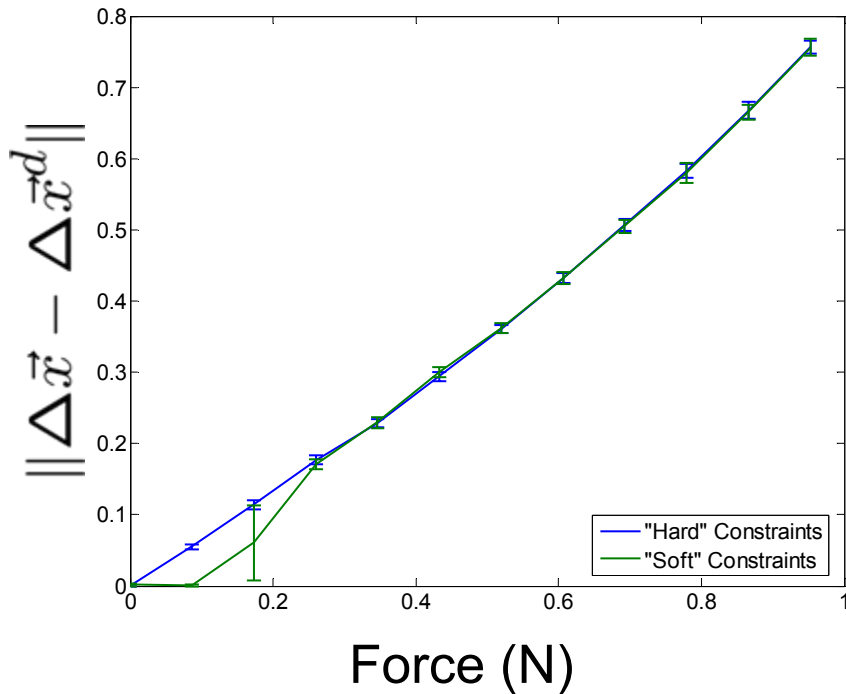
Example Task

- Constraint 1: Tip to move along curve C
- Constraint 2: Origin of $\{s\}$ to move along \hat{l}_t
- Objective: Handle to follow user input





Results for Example Task



"Hard" when $w_{s,i} = 1 \times 10^{+3}$

"Soft" when $w_{s,i} = 1 \times 10^{-3}$



Summary & Significance

- Constrained optimization approach to virtual fixtures
 - Linear and Non-Linear formulation
- Provides the link between surgeon-understandable task primitives and mathematical robot control
 - Specialized libraries of “macro” primitives
- Allows easy incorporation of uncertainty in surgical tasks
 - “Hard” and “Soft” constraints
- Permits surgical tasks to be described with reference to geometric information from patient-specific anatomic models (e.g., from CT or MRI scans)

Acknowledgements

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References

- **Hanson, R.J. and K.H. Haskell**, *{Algorithm 587}: Two Algorithms for the Linearly Constrained Least Squares Problem*. ACM Transactions on Mathematical Software, 1982. **8**(3): p. 323-333.
- **P. Spellucci**, “An SQP method for general nonlinear programs using only equality constrained subproblems,” *Math. Prog.*, vol. 82, pp. 413–448, 1998.