600.464/664 Randomized Algorithms
Mid Semester Examination
March 27, 2008
In-class, Closed Book
Time: 50 mins

Some useful formulas

- Chernoff bounds for 0,1 valued r.v.s:

  $$\Pr[X \geq (1+\delta)\mu] \leq \begin{cases} e^{-\frac{1}{3}\mu\delta^2} & \text{if } \delta \leq 1 \\ e^{-\frac{1}{2}\mu\delta\ln\delta} & \text{if } \delta > 1 \end{cases}$$

  $\Pr[X \leq (1-\delta)\mu] \leq e^{-\frac{1}{2}\mu\delta^2}$ for any $\delta \leq 1$

- $\binom{n}{x} \leq (\frac{ne}{x})^x$

I. n balls are thrown into n bins independently and u.a.r. For every $1 \leq i \leq n$, let $f(i)$ be the total number of balls that fell into the i bins with the most number of balls. Prove that with high probability $f(n^{2/3}) \leq c\, n^{2/3}\frac{\ln n}{\ln\ln n}$ for a suitably large constant c.

II. Random variables A and B are 0,+1 valued and random variables C, D, and E are +1,+2 valued. The five random variables are uniformly distributed and independent. Given that

$$\mathrm{P}[A+B+C+D \geq 5] + \mathrm{P}[A+C+D+E \geq 6] + \mathrm{P}[B+C+D+E \geq 6] < 1,$$

fix a value of A by the conditional probability method of derandomization.

III. Let an n vertex undirected graph $G=(V,E)$ be of uniform degree d. Prove by the probabilistic method that there exists a subset S of V such that ($|S|-$number of edges within S) is at least $\frac{n}{2d}$. Hint: Choose each vertex with probability $1/d$.