600.464/664 Randomized Algorithms
Final Examination
May 11, 2008
In-class, Closed Book
Time: 2 hrs 30 mins.

I. Random variables A, B, and C are -1, 0, +1 valued. They are independent and uniformly distributed. Given that $P[A+B+C \leq -2] + P[A+C \leq 0] < 1$, fix a value for the r.v. A by the conditional probability method of derandomization.

II. Explain the 2 stage hashing scheme that achieves O(1) step response time to Member operations.

III. Extend the normal n-dimensional hypercube to 3^n size n-dimensional directed hypercube $\{0, 1, 2\}^n$ as follows. Each vertex is addressed by $\{i_1, i_2, \cdots, i_n\}$ in which each $i_j \in \{0, 1, 2\}$, and this vertex has edges to n neighbors, the j^{th} neighbor being $\{i_1, \cdots, i_{j-1}, i'_j, i_{j+1}, \cdots, i_n)$ in which $i'_j = i_j + 1 (\text{mod } 3)$. Consider a random walk on this graph in which at any vertex, the probability of walking to each of its n neighbors is $1/2n$, and probability of staying at the vertex is $1/2$. Apply the coupling method and derive a good upperbound for $\tau(\epsilon)$.

IV. Let $U = \{0, 1, \cdots, m-1\}$ where $m > n^3$. A class C of functions from U to $R = \{0, 1, \cdots, n^3 - 1\}$ is *cfgood* if for every size n subset S of U there exists a function f in C which maps elements of S into R without any collisions; i.e. $(\forall S \subseteq U,\ |S| = n)(\exists f \in C)(\forall x, y \in S,\ x \neq y)(f(x) \neq f(y))$. By applying the probabilistic method, find a suitable upper bound for the size of a cfgood class C. (Hint: Choose k independent random functions, and derive an inequality for insuring the probability of failure to be less than 1.)