Direct Plane Tracking in Stereo Images for Mobile Navigation

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The Problem

- **Input:**
  - Stream of rectified stereo images, known intrinsics.
- **Assumptions:**
  - Indoor scene with many well-textured planes.
- **Output:**
  - Compute relative motion for a mobile robot. (3 DOF)
- **How:**
  - Extract and track planes in view.
  - Compute motion relative to planes.
Typical Solution

• Fully calibrated system
• Solve with stereo (1 eg. algorithm)
  – Image scene and compute disparity.
  – Build list of significant features in view.
  – For each subsequent frame,
    • Match significant features
    • Compute motion against them.
• Pros - General Approach
• Cons - Expensive correspondence search.
  – Sparse, error prone results.
  – Pixel Accuracy, typical
Our Proposed Solution

- Exploit properties of the planar scene when viewed under rectified stereo cameras.

\[
\begin{align*}
ax + by + cz &= d \\
x/a + y/b + c &= d/z \\
D(u,v) &= \frac{Baseline}{z} \\
a'u + b'v + c' &= D(u,v)
\end{align*}
\]

\(D\) is a linear map
Tracking Algorithm

• Track 3 parameters per plane.
  – localize w.r.t. the tracked planes.

• Brightness Constancy Assumption
  \[ L(u,v) - R(u - D^*(u,v),v) = 0 \]

• Define Error Function
  \[ e(u,v) = L(u,v) - R(u - D(u,v),v) \]

• Least Squares Equation
  \[ \arg\min_{(a,b,c)} \sum_{u,v} e(u,v)^2 \]
Tracking Algorithm

- First order Taylor Expansion about (a,b,c)

\[
\arg\min_{\delta_a, \delta_b, \delta_c} \sum_{u, v} \left[ e(u, v) + e'(u, v) \begin{bmatrix} \delta_a & \delta_b & \delta_c \end{bmatrix}^T \right]^2
\]

where \( x^* = x + \delta_x, x \in \{a, b, c\} \)

- Stack up these equations and solve by SVD.

\[
\begin{bmatrix}
  e(u_1, v_1) \\
e(u_2, v_2) \\
  \vdots \\
e(u_n, v_n)
\end{bmatrix} =
\begin{bmatrix}
  u_1 R_x(u_1, v_1) & v_1 R_x(u_1, v_1) & R_x(u_1, v_1) \\
u_2 R_x(u_2, v_2) & v_2 R_x(u_2, v_2) & R_x(u_2, v_2) \\
  \vdots & \vdots & \vdots \\
u_n R_x(u_n, v_n) & v_n R_x(u_n, v_n) & R_x(u_n, v_n)
\end{bmatrix}\begin{bmatrix}
  \delta_a \\
  \delta_b \\
  \delta_c
\end{bmatrix}
\]
Binary Weighting Matrix

• Multiple planes are needed to localize.
• Include a mask for each plane.
• No need to maintain explicit boundary.

\( (\xi) \ \arg \min_{(a,b,c)} \sum_{u,v} \left[ \delta_{\{W(u,v) = 1\}} e(u,v) \right]^2 \)

• Fully recompute mask each frame.
  – Normalized Cross Correlation.
  – Horizontal Texture Variance.
Tracking Initialization

• Must compute a set of seed parameters for each plane.
• Use disparity calculation to detect significant planes (constant gradient).
• Fit a plane in 3D via PCA on points.
• If error in planar fit is small, keep plane.
• Else repeat search.
Tracking Algorithm Summarized

1. Compute an initial guess of (a,b,c)
2. Build binary weighting mask, W
3. For each subsequent frame
   1. Compute Derivative of Right Image (Rx)
   2. Solve (§) and update (a,b,c)
   3. Re-compute W
Plane Tracking Video
Results

• Two sets of experiments
  – Convergence Radius
  – Parameter Estimation Accuracy

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<th>Z Mean</th>
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<td>0.2947°</td>
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Error in Seed Depth Experiment
Error in Seed Angle Experiment
Depth Accuracy Experiment
Orientation Accuracy Experiment

![Graph showing orientation accuracy over time](image)

- **Estimation**
- **Odometry**

**Y-axis:** Robot Orientation Angle (radians)

**X-axis:** Time Step (frames)
Conclusions

• Novel plane tracking algorithm
• Direct Method (not feature-based)
  – Sub-pixel accuracy
• Super frame-rate
  – 2 iterations per frame at 30 Hz
• Extension to higher order surfaces
• Applications to augmented reality
  – 3D Registration
• Thank You.
• Questions/Comments?
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