

# **Novel Inference, Training and Decoding Methods over Translation Forests**

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Johns Hopkins University**

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**Co-advisor: Jason Eisner**

# Statistical Machine Translation Pipeline

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# Statistical Machine Translation Pipeline

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Bilingual  
Data

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# Statistical Machine Translation Pipeline

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# Statistical Machine Translation Pipeline

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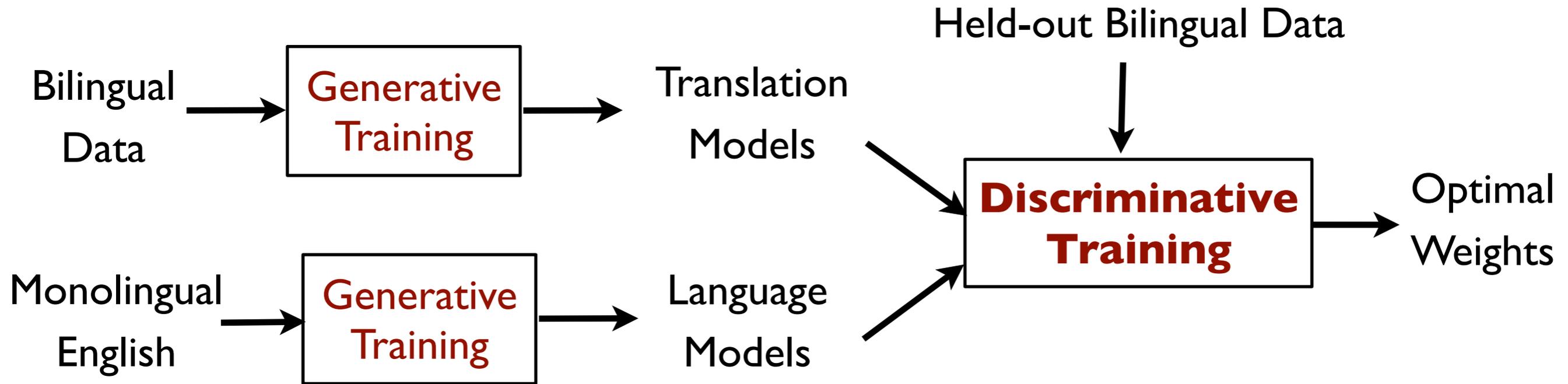
Monolingual  
English

# Statistical Machine Translation Pipeline

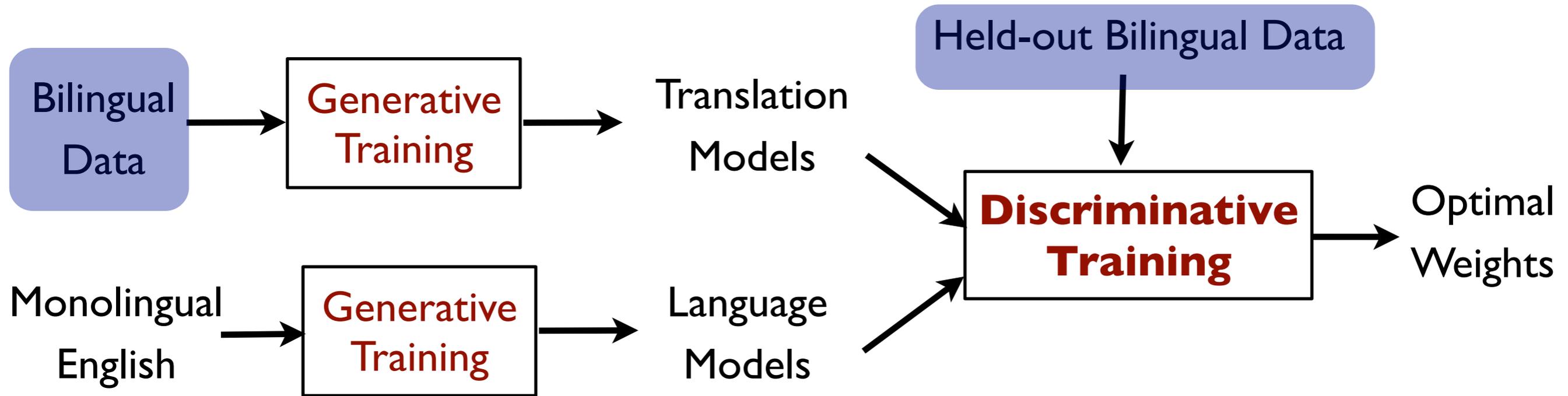
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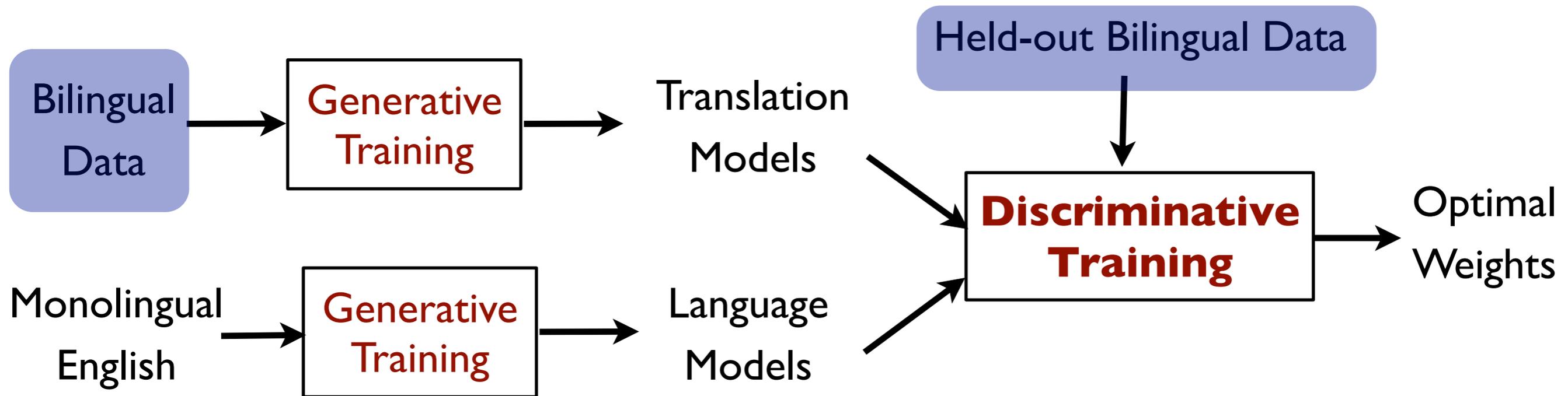
# Statistical Machine Translation Pipeline



# Statistical Machine Translation Pipeline

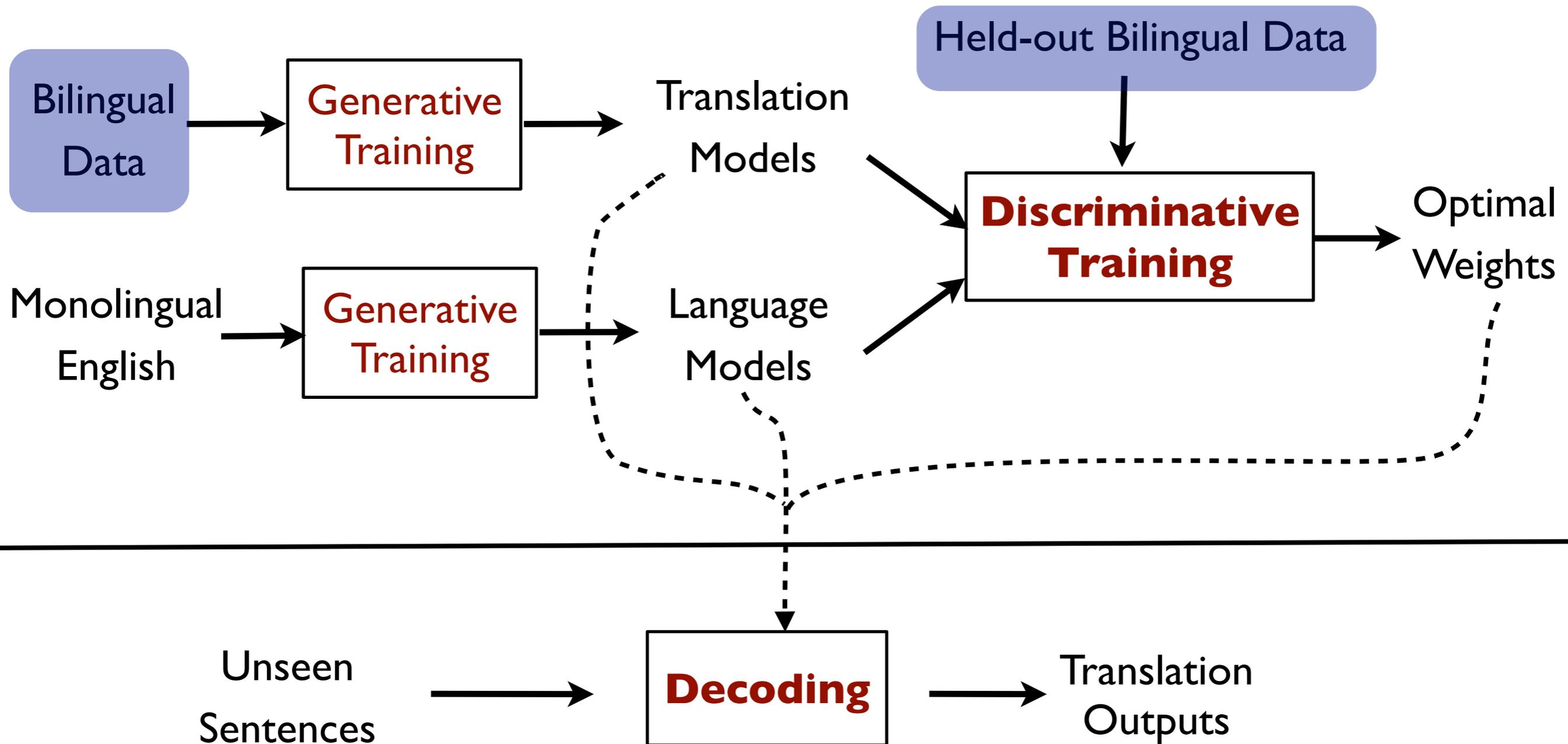


# Statistical Machine Translation Pipeline



Unseen  
Sentences

# Statistical Machine Translation Pipeline



# Training a Translation Model



# Training a Translation Model



垫子 上 的 猫  
dianzi shang de mao

# Training a Translation Model



垫子 上 的 猫  
dianzi shang de mao

a cat on the mat

# Training a Translation Model



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

# Training a Translation Model



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

# Training a Translation Model



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

# Training a Translation Model



垫子 上 的 猫

dianzi shang de  

  on the mat

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

# Training a Translation Model



$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

# Training a Translation Model



$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

$X \rightarrow \langle \text{dianzi shang de } X_0, X_0 \text{ on the mat} \rangle$

# Training a Translation Model



垫子 上 的 猫

$X_0$  de mao

a cat on  $X_0$

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

$X \rightarrow \langle \text{dianzi shang de } X_0, X_0 \text{ on the mat} \rangle$

# Training a Translation Model



垫子 上 的 猫

$X_0$  de mao

a cat on  $X_0$

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

$X \rightarrow \langle \text{dianzi shang de } X_0, X_0 \text{ on the mat} \rangle$

$X \rightarrow \langle X_0 \text{ de mao, a cat on } X_0 \rangle$

# Training a Translation Model



垫子 上 的 猫

$X_0$  de  $X_1$

$X_1$  on  $X_0$

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

$X \rightarrow \langle \text{dianzi shang de } X_0, X_0 \text{ on the mat} \rangle$

$X \rightarrow \langle X_0 \text{ de mao, a cat on } X_0 \rangle$

# Training a Translation Model



垫子 上 的 猫

$X_0$  de  $X_1$

$X_1$  on  $X_0$

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

$X \rightarrow \langle \text{dianzi shang de } X_0, X_0 \text{ on the mat} \rangle$

$X \rightarrow \langle X_0 \text{ de mao, a cat on } X_0 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

# Decoding a Test Sentence

# Decoding a Test Sentence



# Decoding a Test Sentence

垫子 上 的 狗



# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

---

# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

---

# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

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$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

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# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

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dianzi shang de gou

# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

|  
dianzi shang de gou

# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

|  
dianzi shang de

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

|  
gou

# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

---

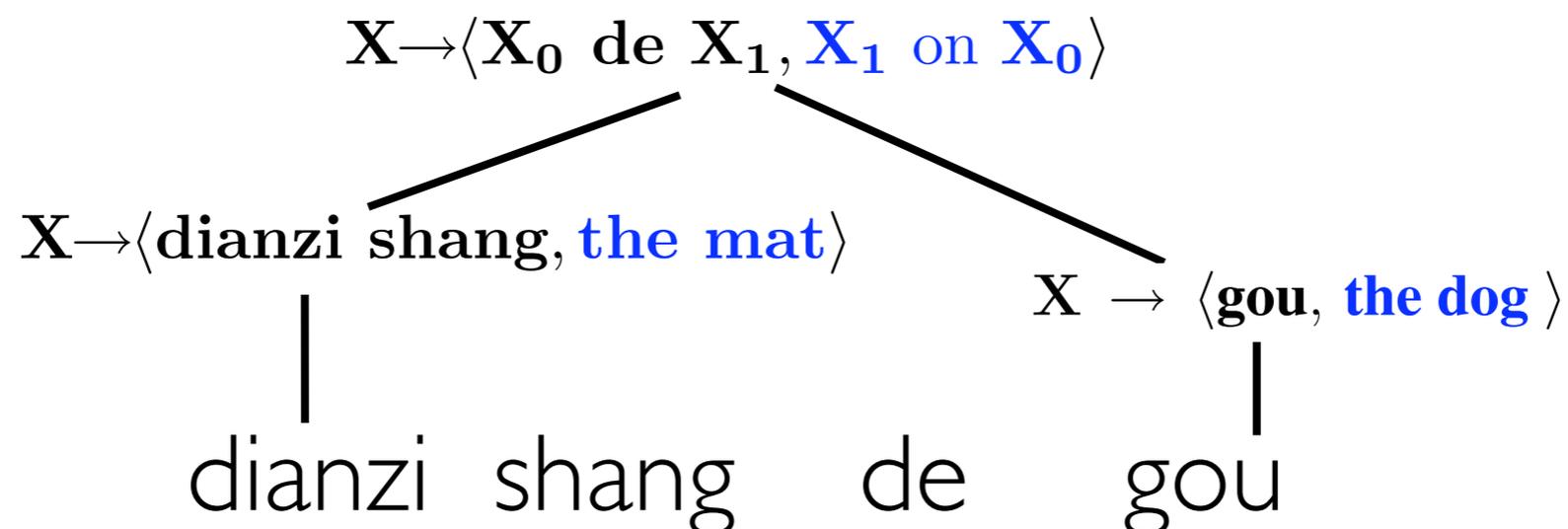
$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

---





# Decoding a Test Sentence



垫子 上 的 狗

dianzi shang de gou

the dog on the mat

---

$X \rightarrow \langle \text{dianzi shang}, \text{the mat} \rangle$

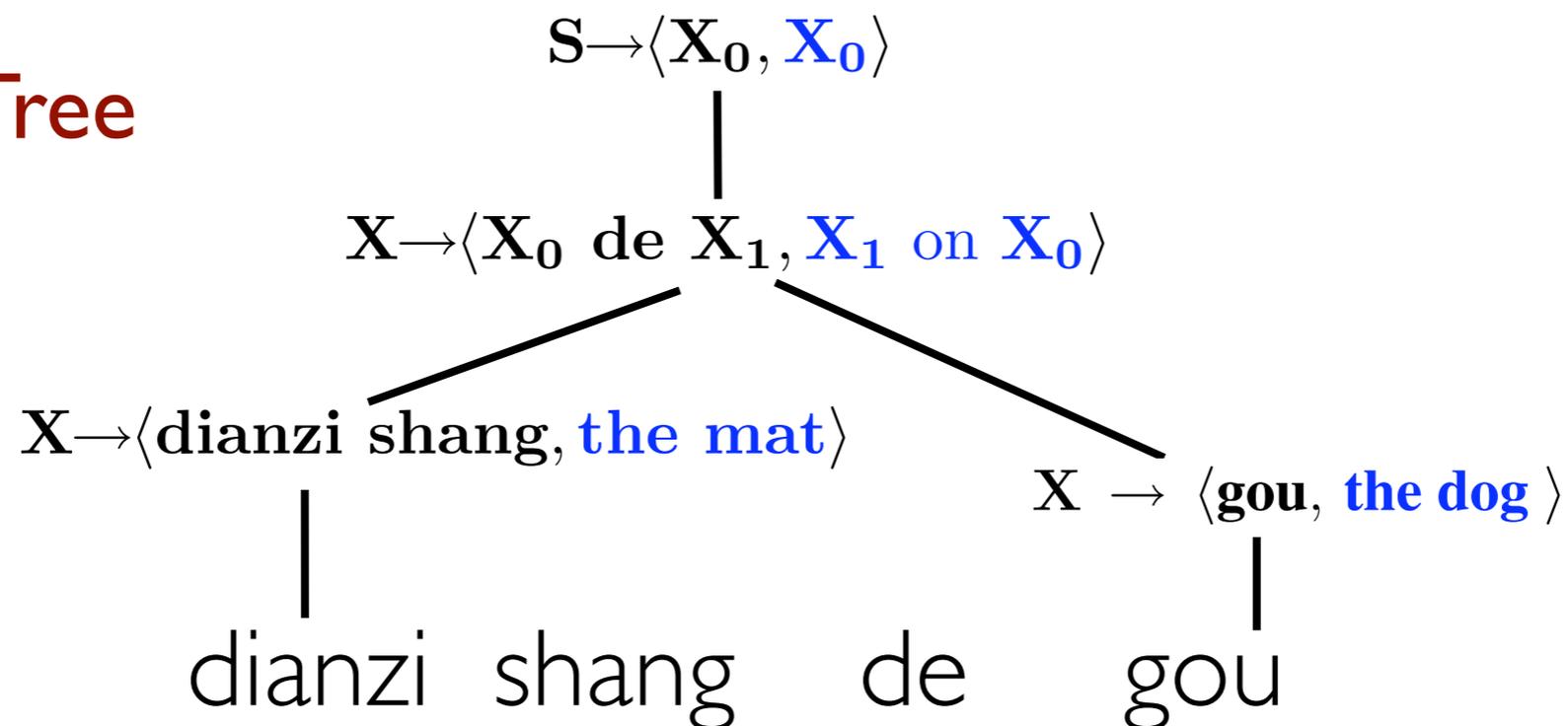
$X \rightarrow \langle \text{gou}, \text{the dog} \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$S \rightarrow \langle X_0, X_0 \rangle$

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## Derivation Tree





# Translation Ambiguity



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

# Translation Ambiguity



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

# Translation Ambiguity



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

zhongguo de shoudu  
capital of China

# Translation Ambiguity



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

zhongguo de shoudu

capital of China

# Translation Ambiguity



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

zhongguo de shoudu

capital of China

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

# Translation Ambiguity



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

zhongguo de shoudu

capital of China

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

wo de mao

my cat

# Translation Ambiguity



垫子 上 的 猫  
dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

zhongguo de shoudu  
capital of China

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

wo de mao  
my cat

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 X_1 \rangle$

# Translation Ambiguity



垫子 上 的 猫  
dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

zhongguo de shoudu  
capital of China

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

wo de mao  
my cat

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 X_1 \rangle$

zhifei de mao  
zhifei 's cat

# Translation Ambiguity



垫子 上 的 猫

dianzi shang de mao

a cat on the mat

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

zhongguo de shoudu

capital of China

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

wo de mao

my cat

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 X_1 \rangle$

zhifei de mao

zhifei 's cat

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 \text{ 's } X_1 \rangle$

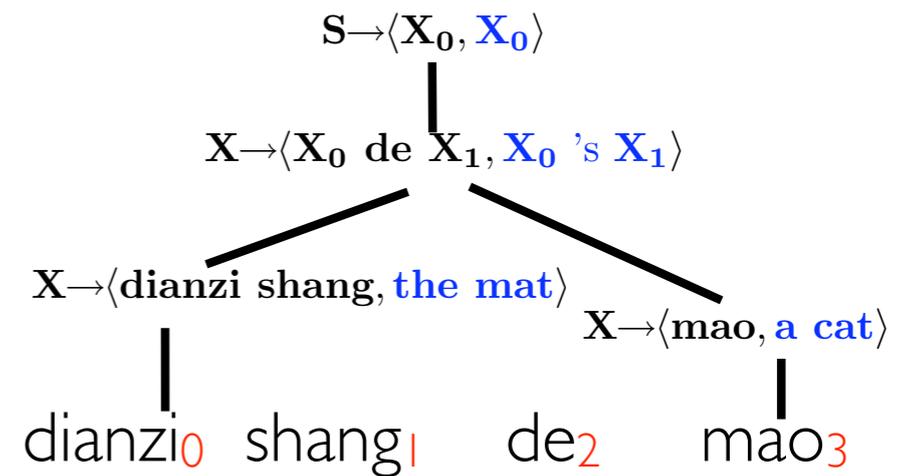
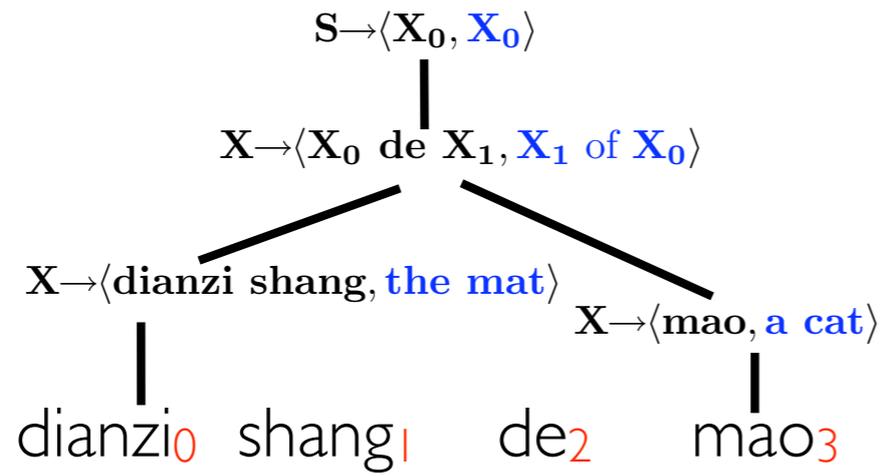
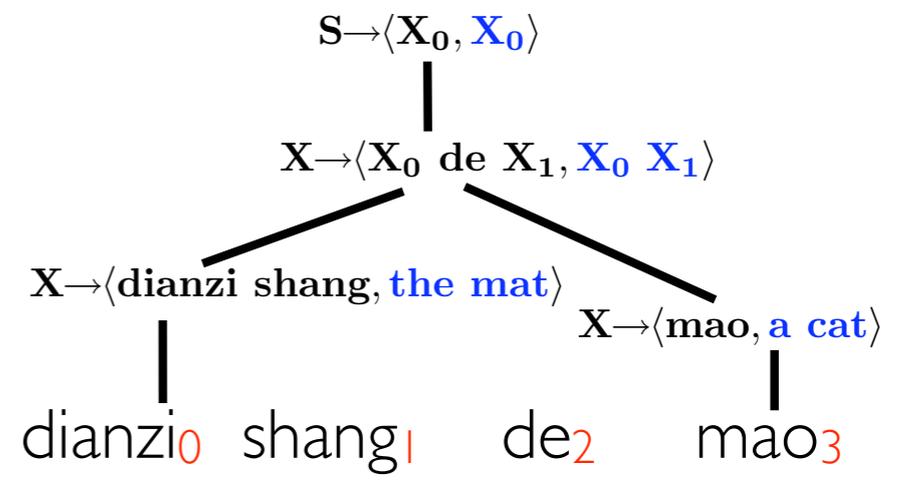
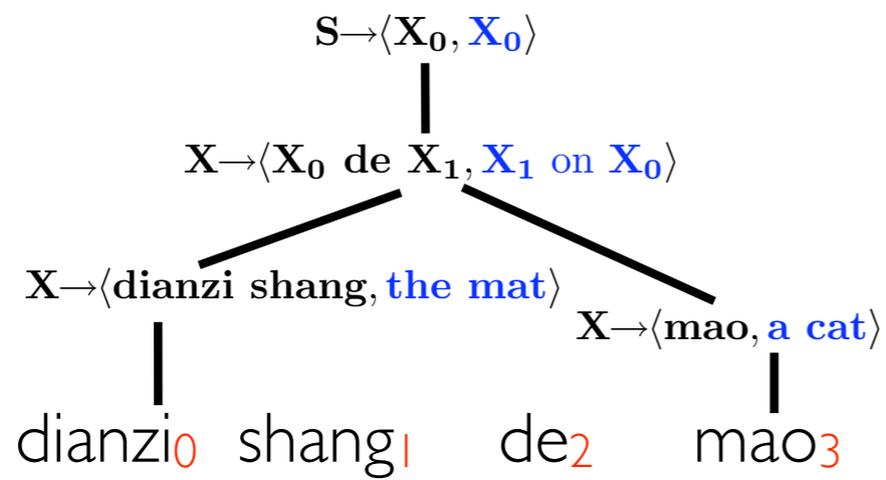


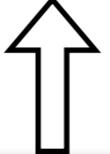
dianzi shang de mao



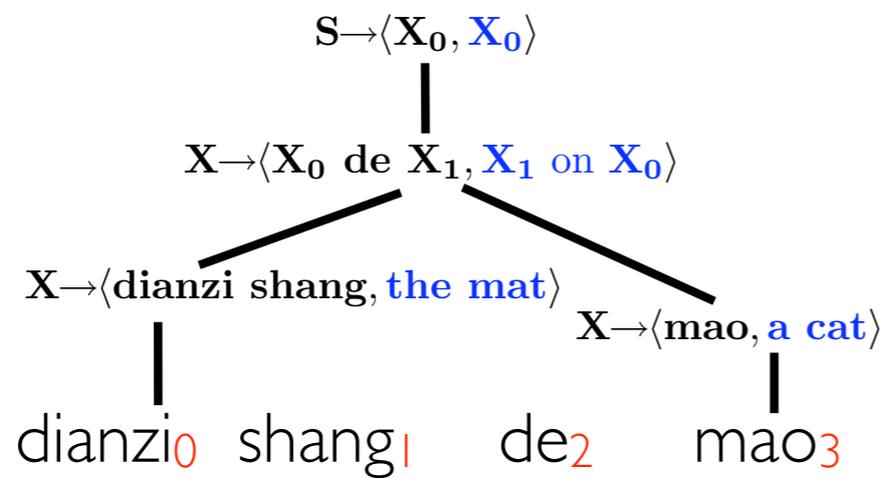
**Joshua**  
(chart parser)

dianzi shang de mao

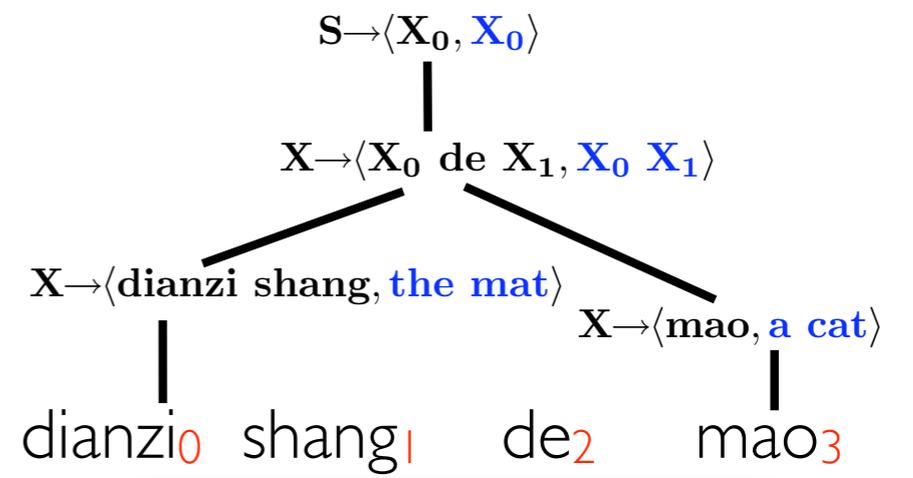


  
**Joshua**  
 (chart parser)

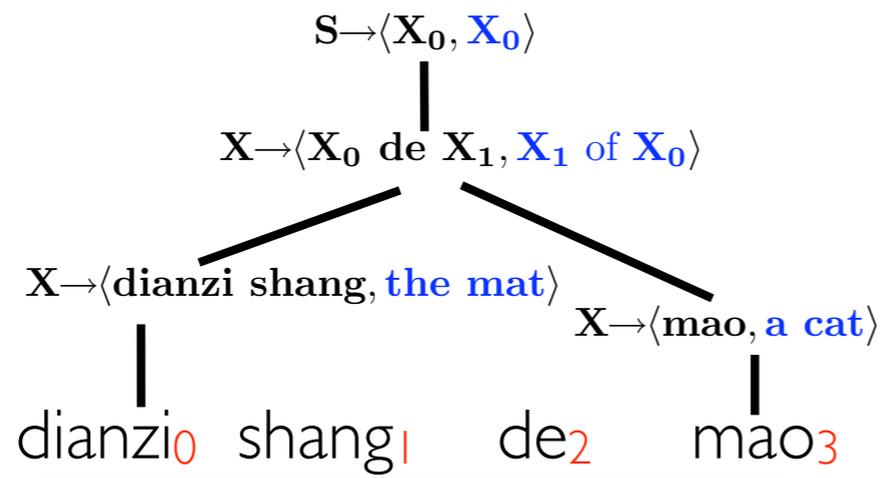
  
 dianzi shang de mao



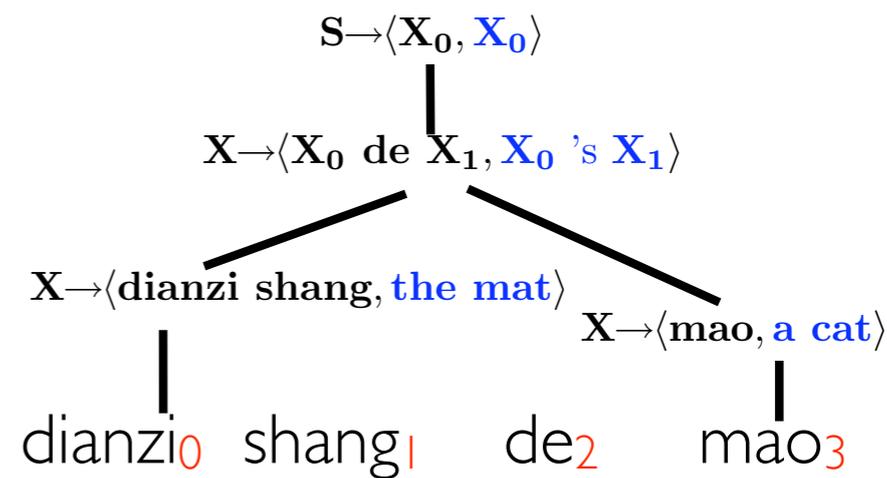
a cat on the mat



the mat a cat



a cat of the mat

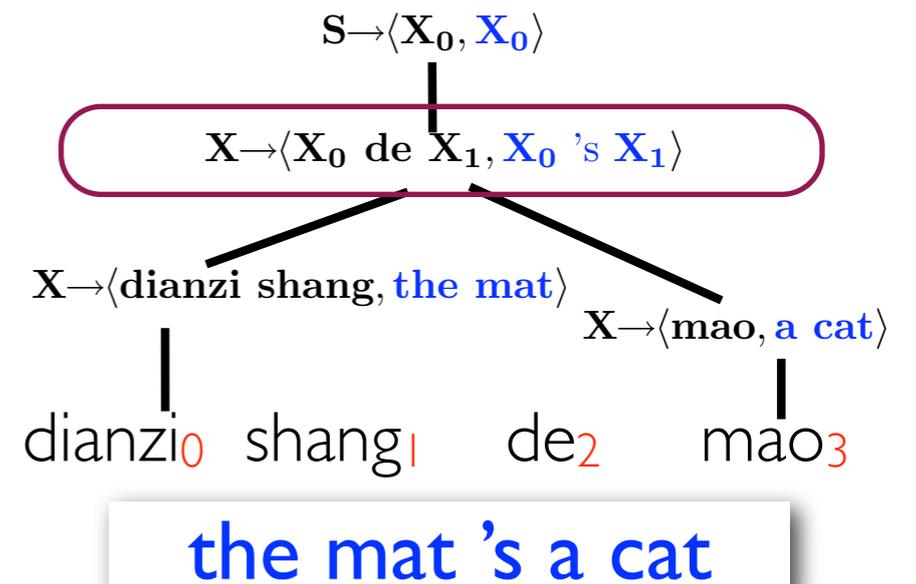
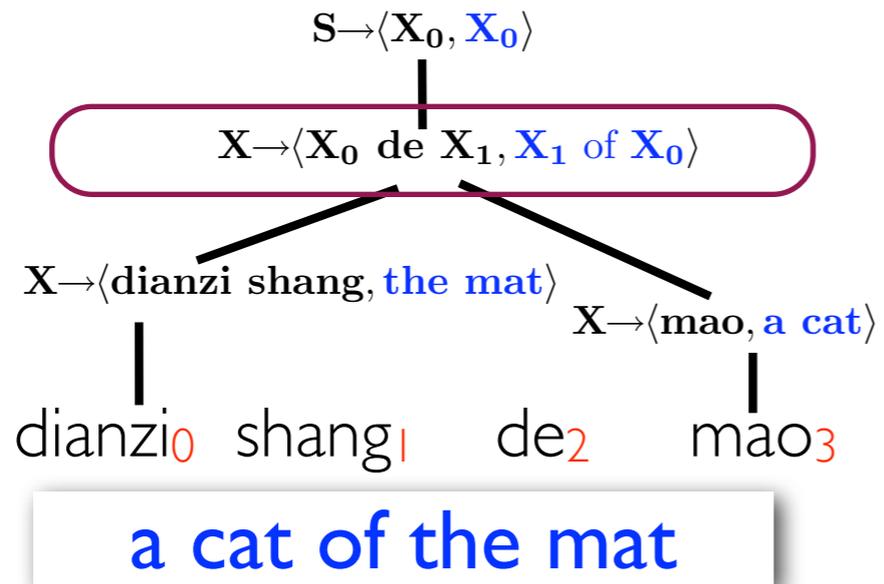
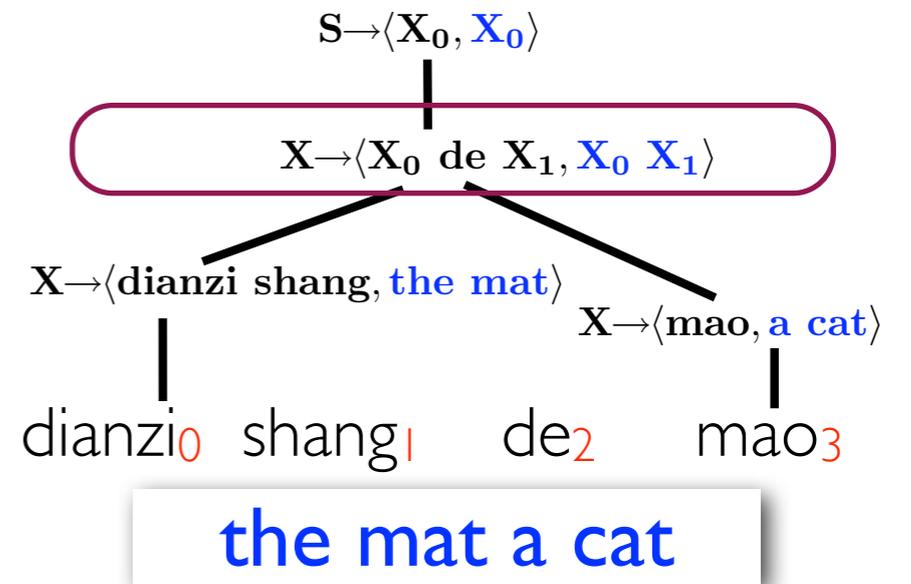
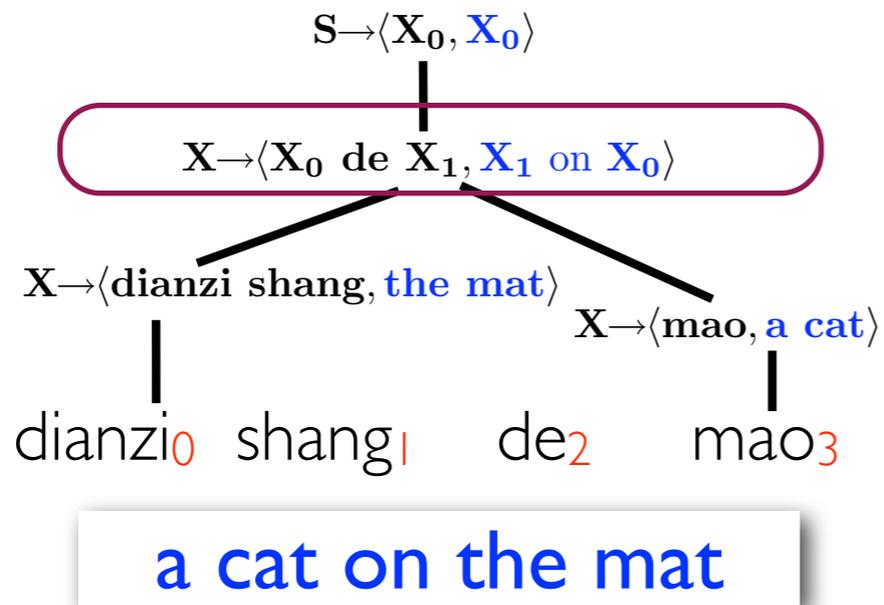


the mat 's a cat



**Joshua**  
(chart parser)

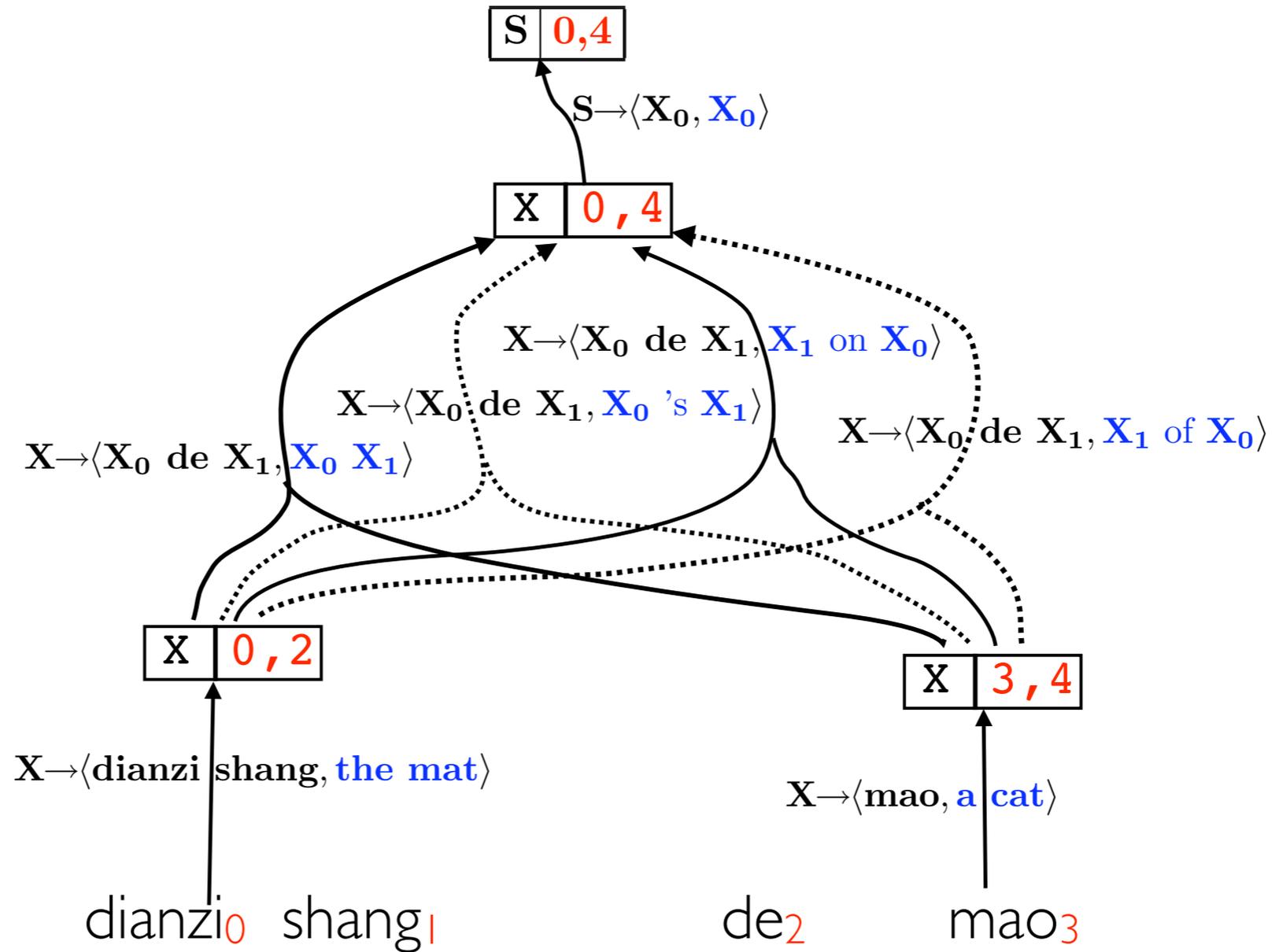
dianzi shang de mao



**Joshua**  
(chart parser)

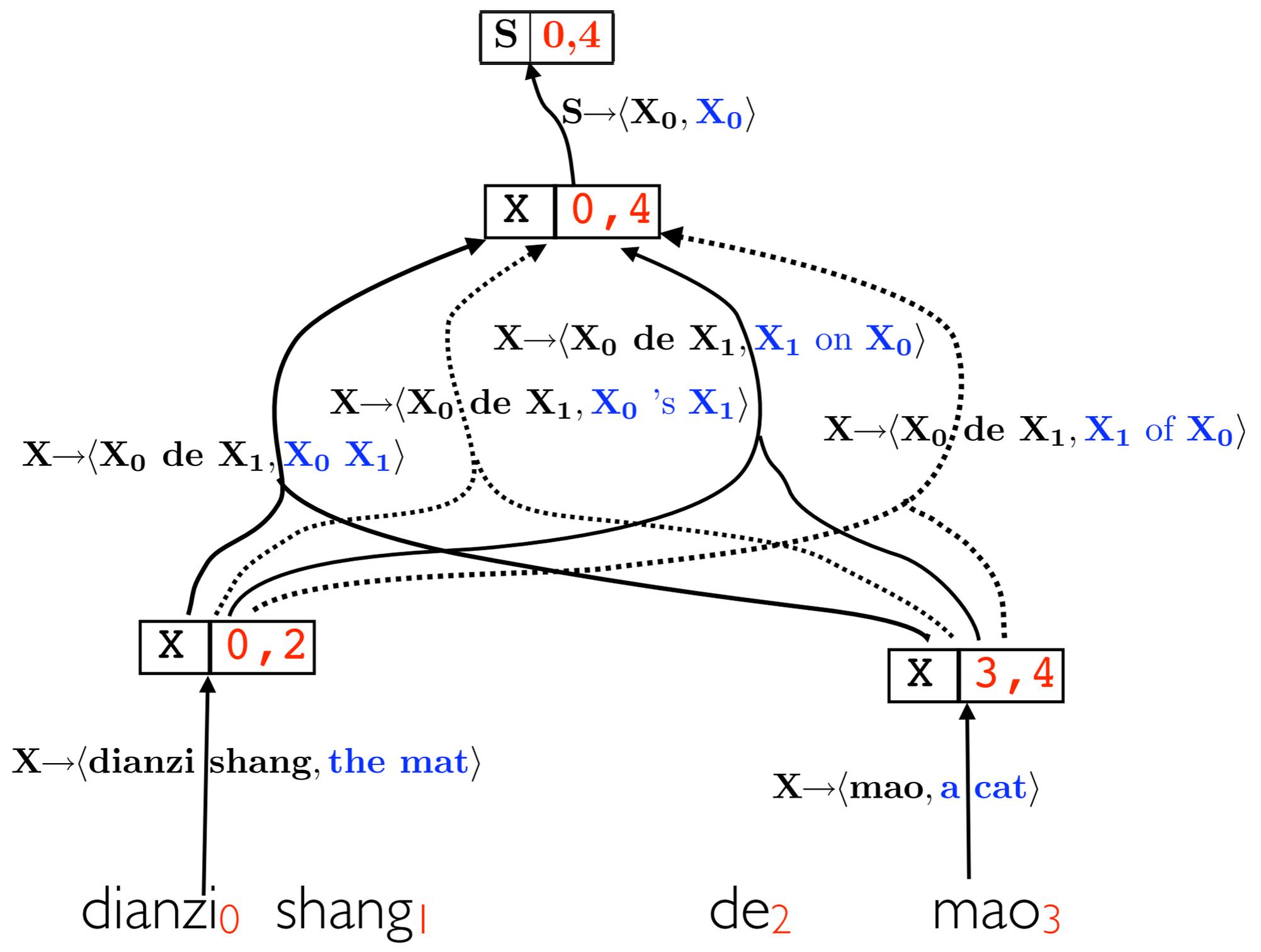
dianzi shang de mao

# hypergraph

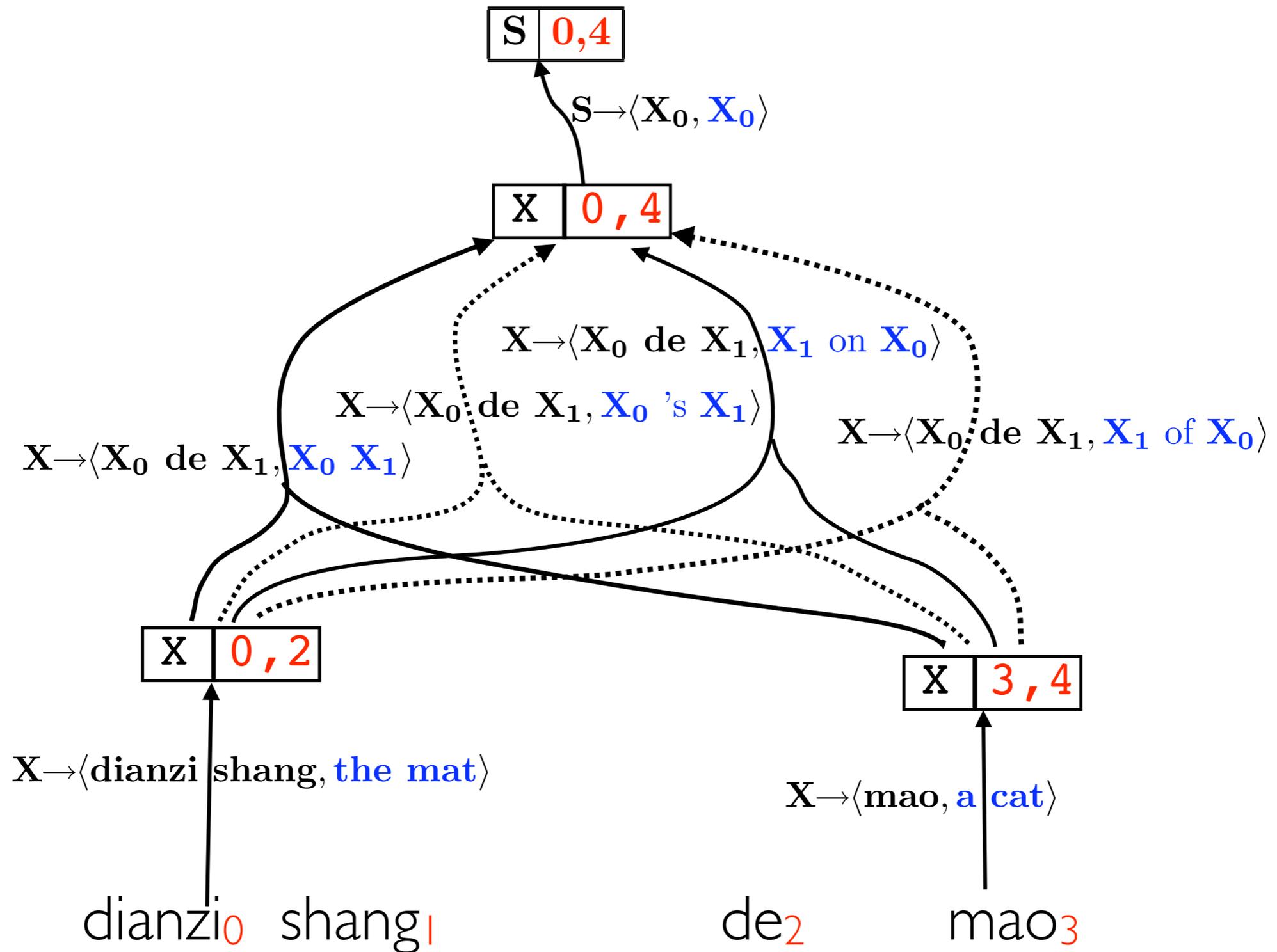


**Joshua**  
(chart parser)

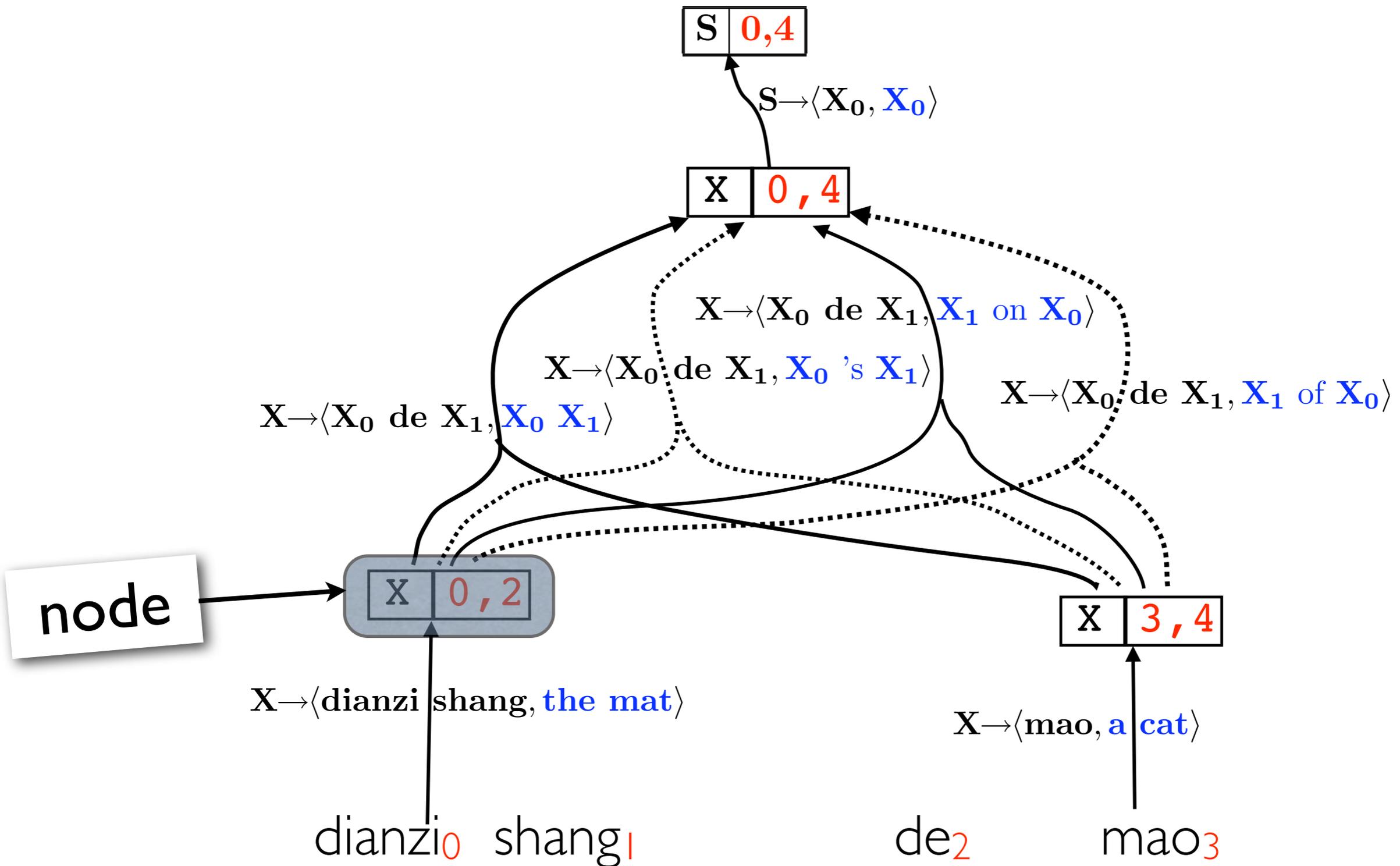
dianzi shang de mao



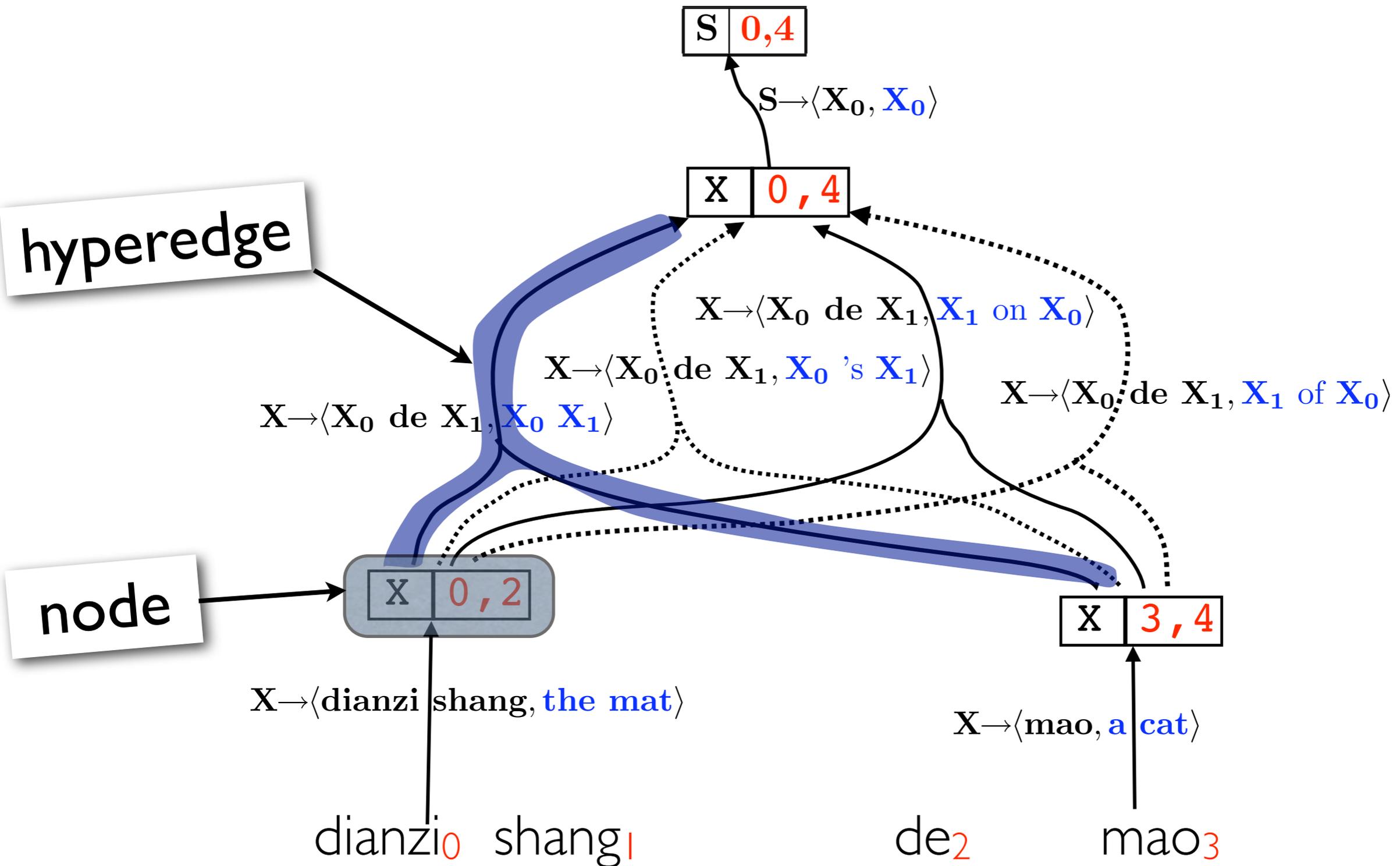
A hypergraph is a compact data structure to encode **exponentially many trees**.



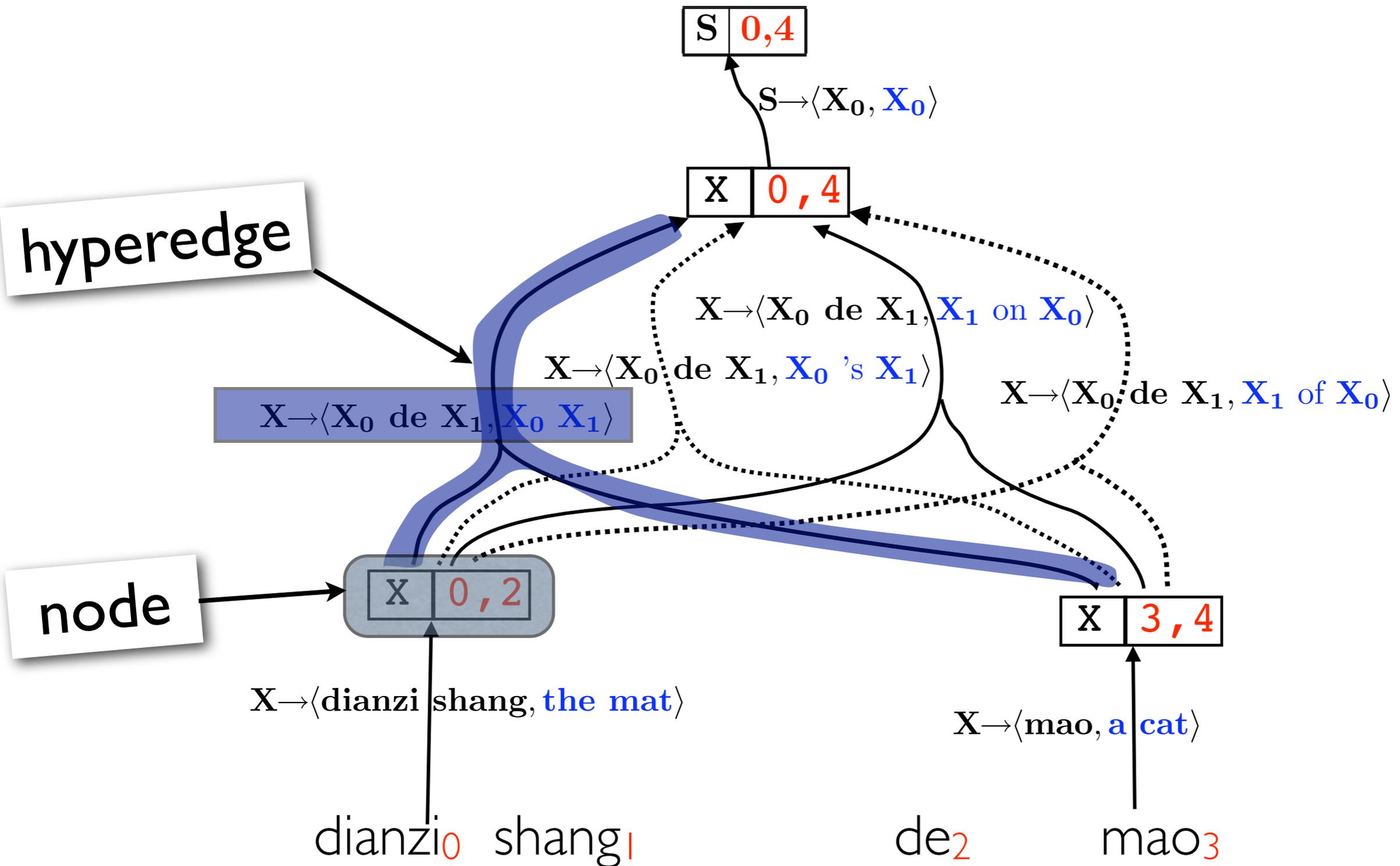
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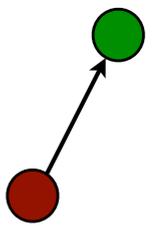


A hypergraph is a compact data structure to encode **exponentially many trees**.

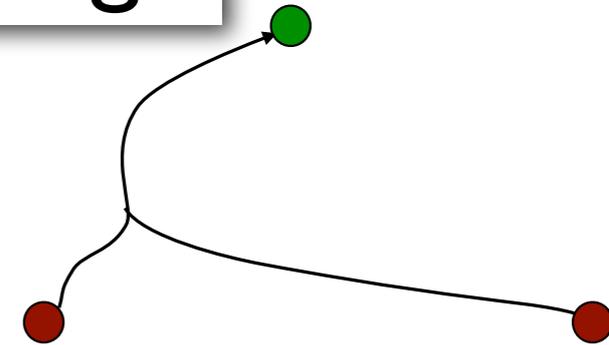


A hypergraph is a compact data structure to encode **exponentially many trees**.

edge



hyperedge



hyperedge

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 X_1 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 \text{'s } X_1 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

node

$X \quad 0, 2$

$X \quad 3, 4$

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

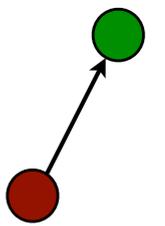
$X \rightarrow \langle \text{mao, a cat} \rangle$

dianzi<sub>0</sub> shang<sub>1</sub>

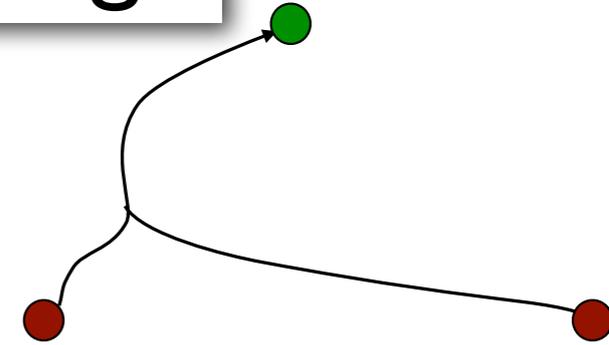
de<sub>2</sub> mao<sub>3</sub>

A hypergraph is a compact data structure to encode **exponentially many trees**.

edge



hyperedge



hyperedge

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 X_1 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 \text{ 's } X_1 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

FSA

node

$X \quad 0, 2$

$X \quad 3, 4$

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

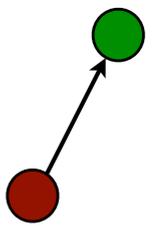
$X \rightarrow \langle \text{mao, a cat} \rangle$

dianzi<sub>0</sub> shang<sub>1</sub>

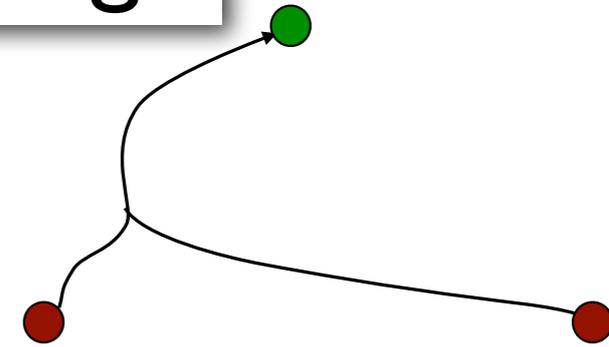
de<sub>2</sub> mao<sub>3</sub>

A hypergraph is a compact data structure to encode **exponentially many trees**.

edge



hyperedge



hyperedge

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 X_1 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_0 \text{ 's } X_1 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ on } X_0 \rangle$

$X \rightarrow \langle X_0 \text{ de } X_1, X_1 \text{ of } X_0 \rangle$

node

$X \quad 0, 2$

$X \quad 3, 4$

$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

$X \rightarrow \langle \text{mao, a cat} \rangle$

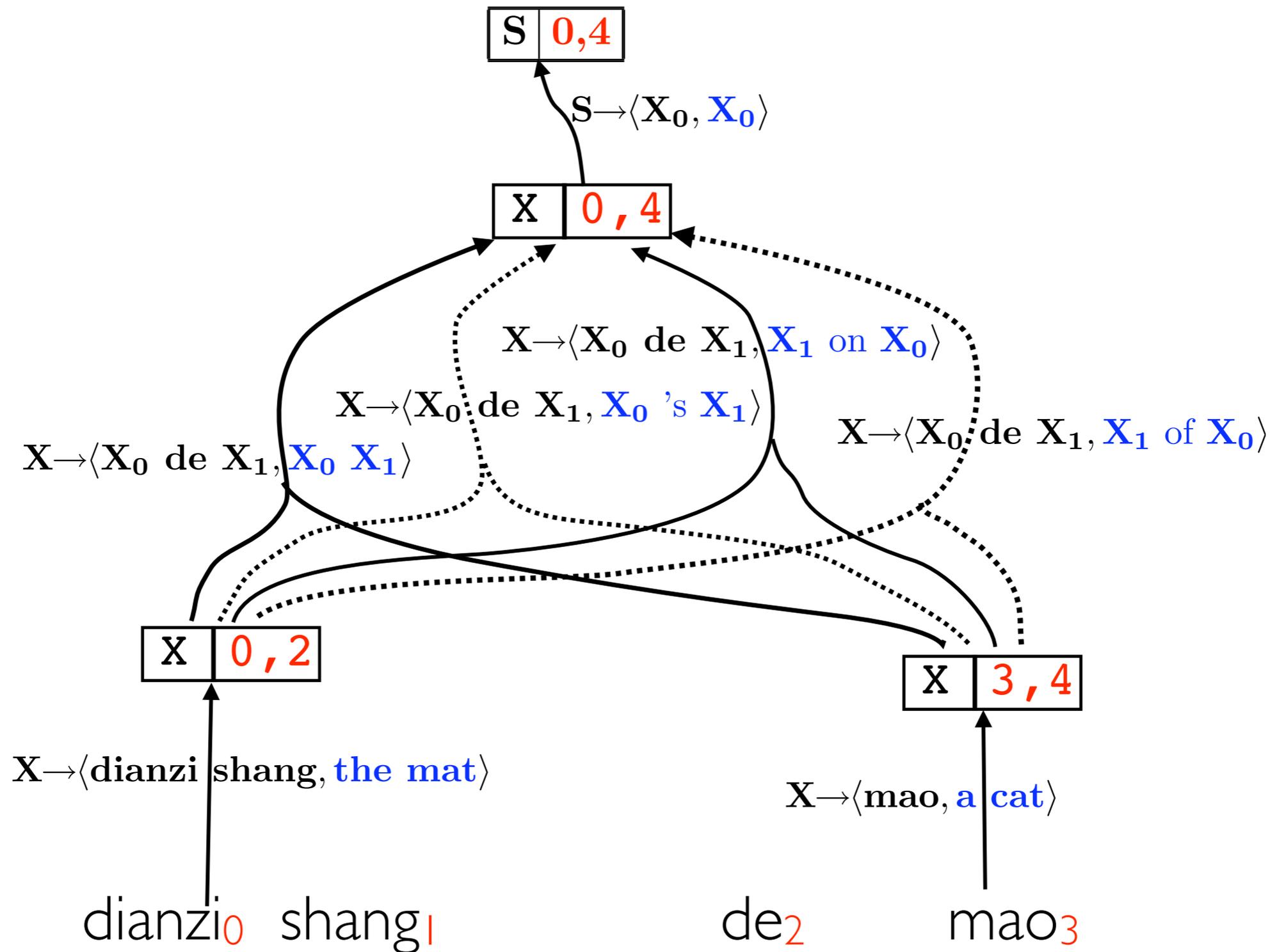
dianzi<sub>0</sub> shang<sub>1</sub>

de<sub>2</sub> mao<sub>3</sub>

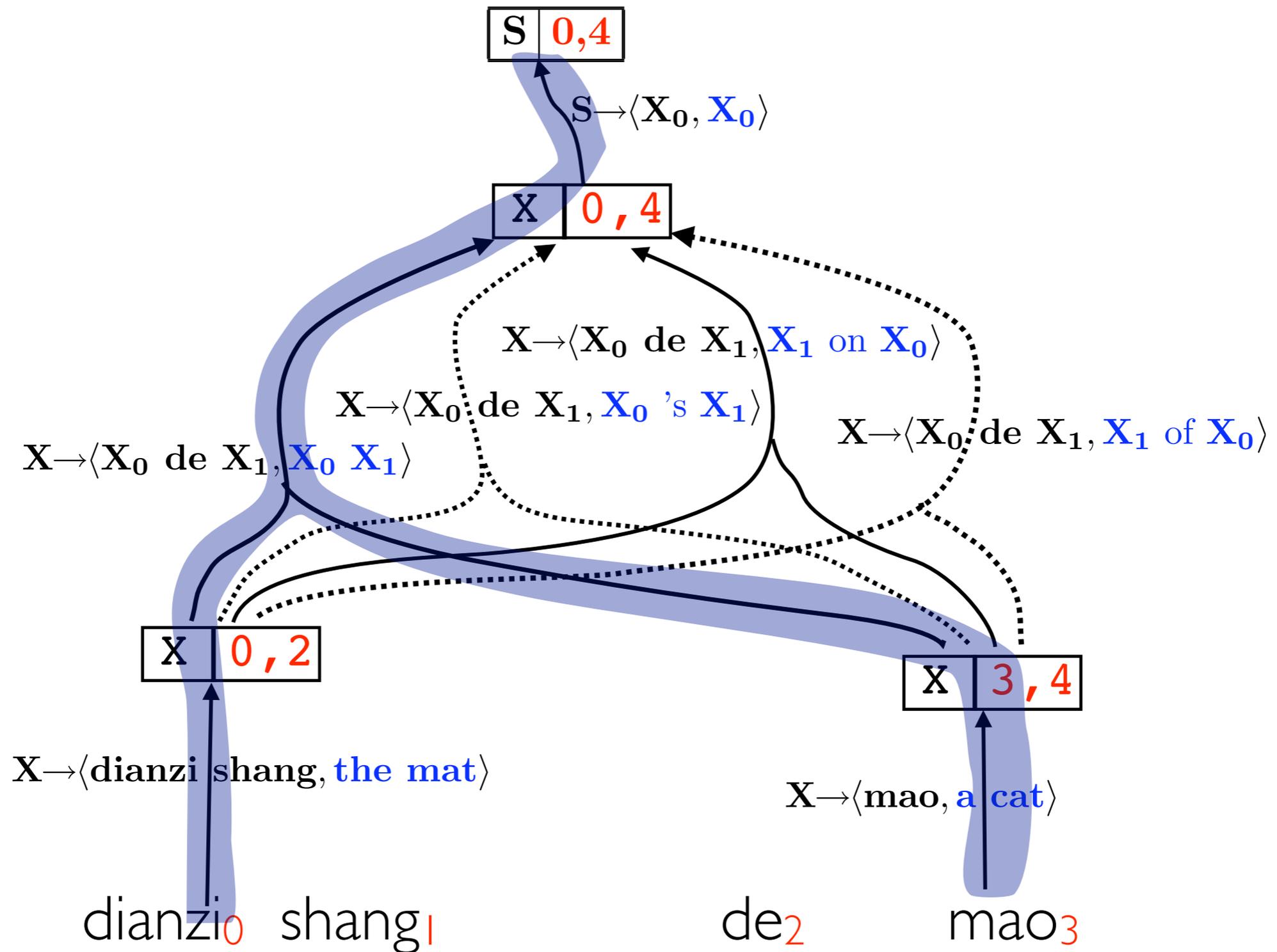
FSA

Packed Forest

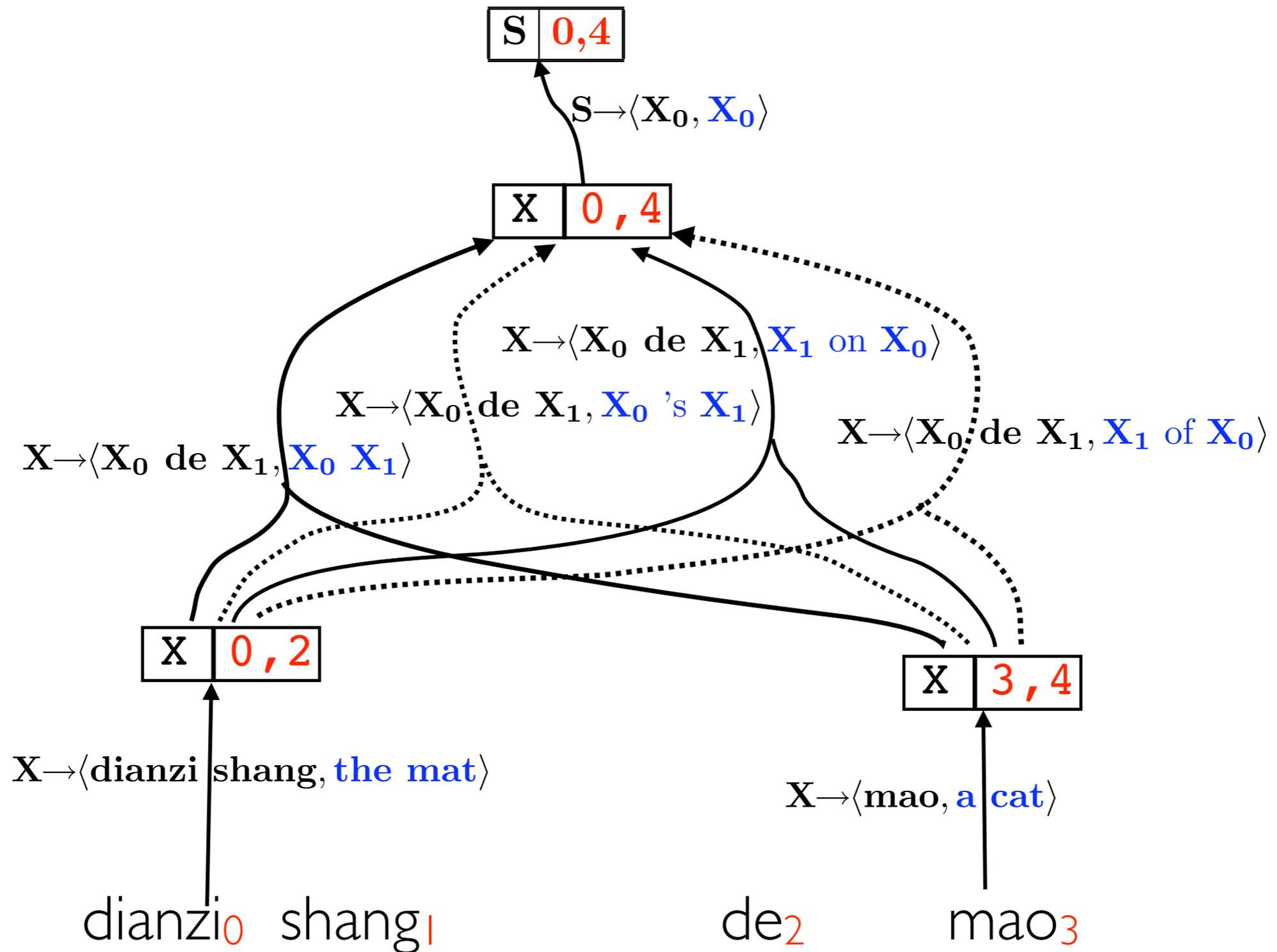
A hypergraph is a compact data structure to encode **exponentially many trees**.



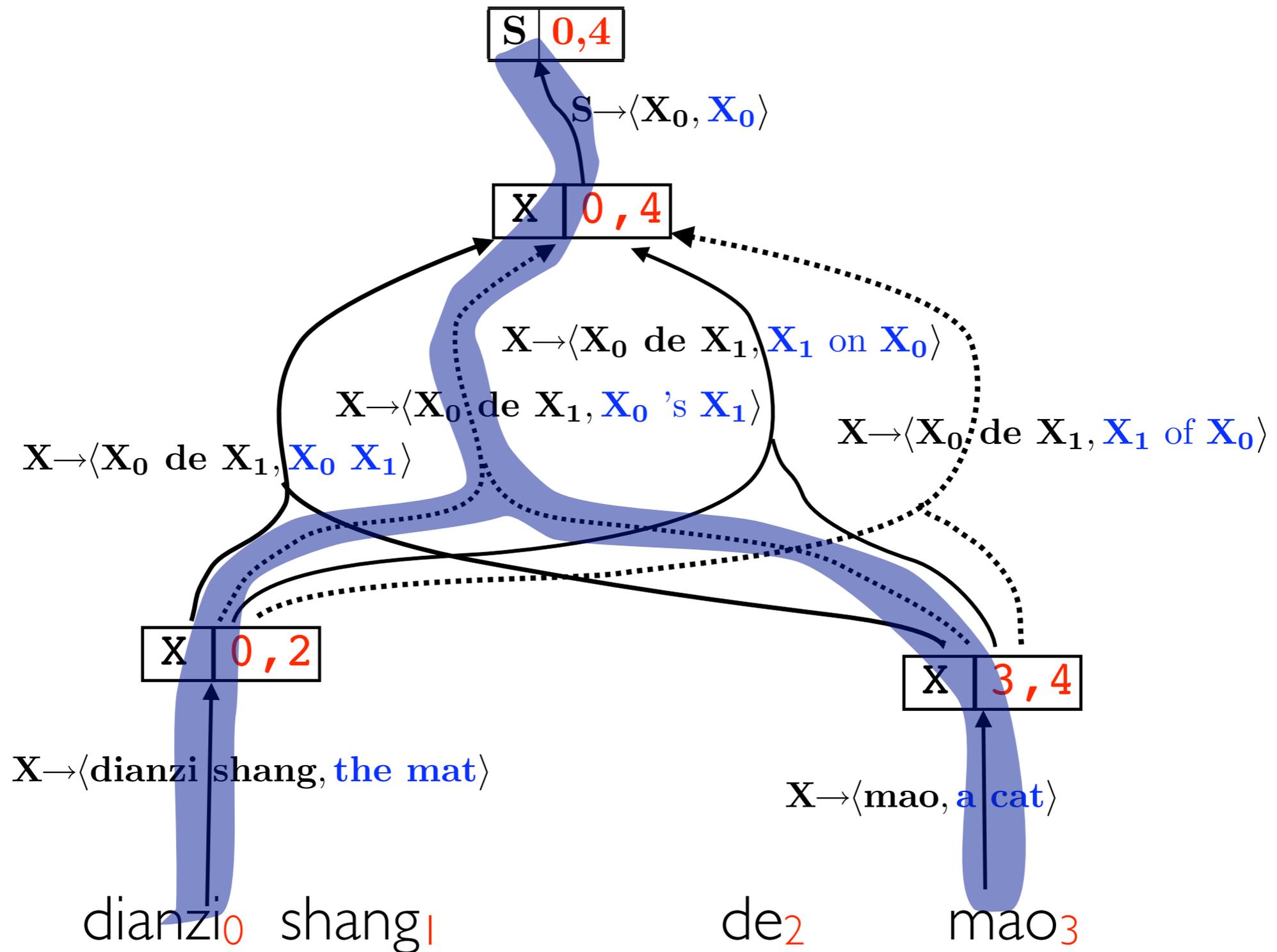
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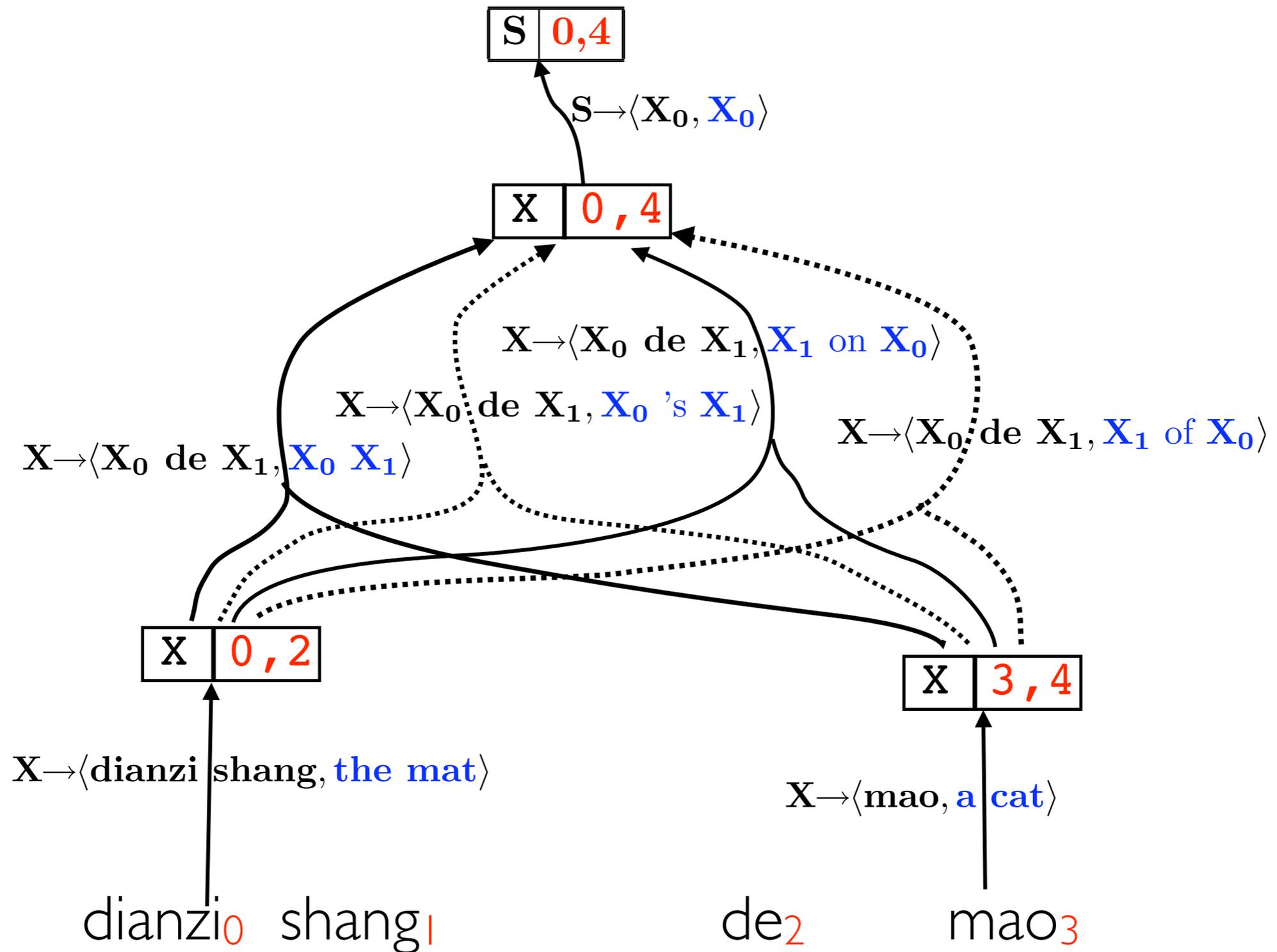
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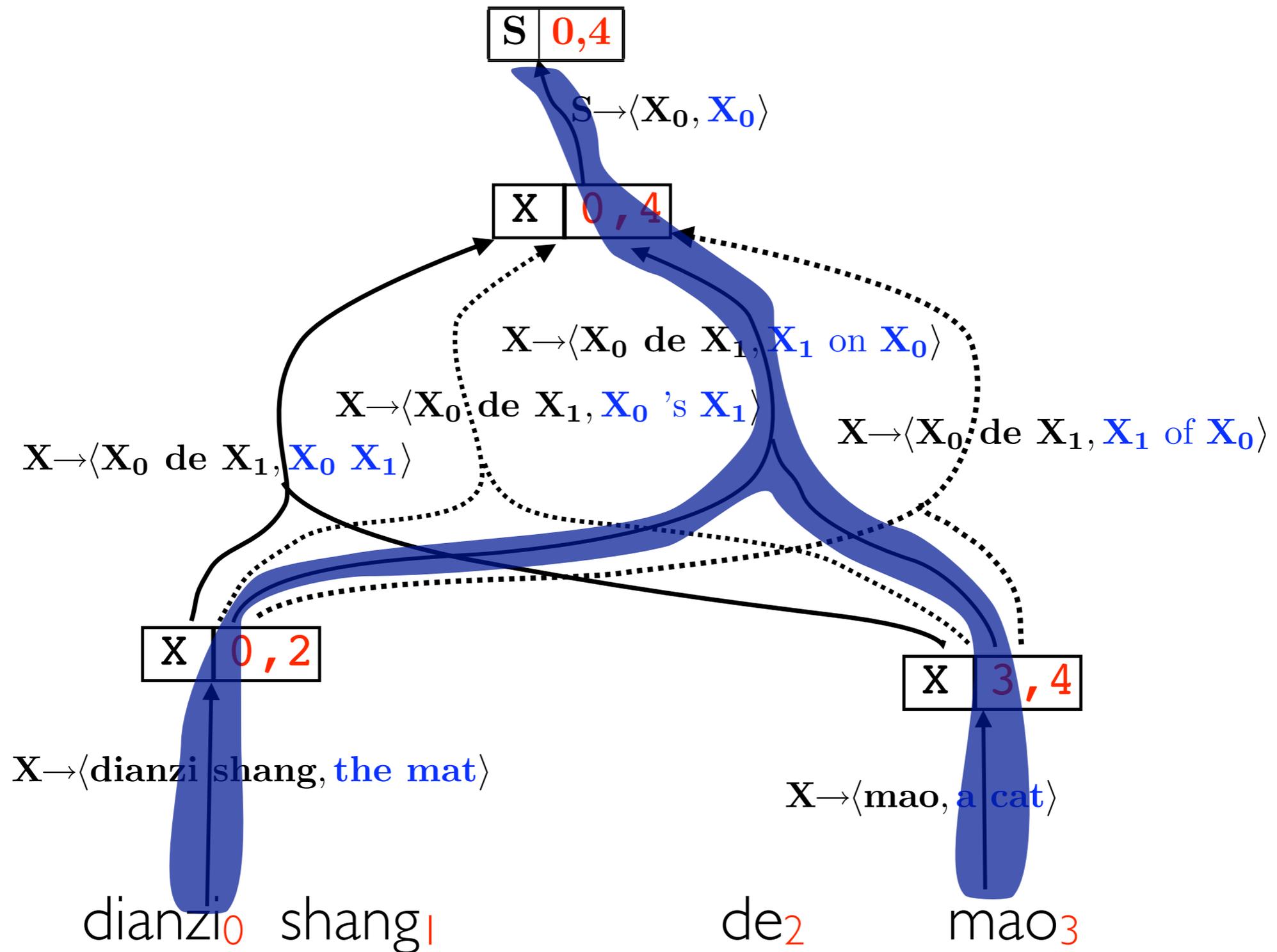
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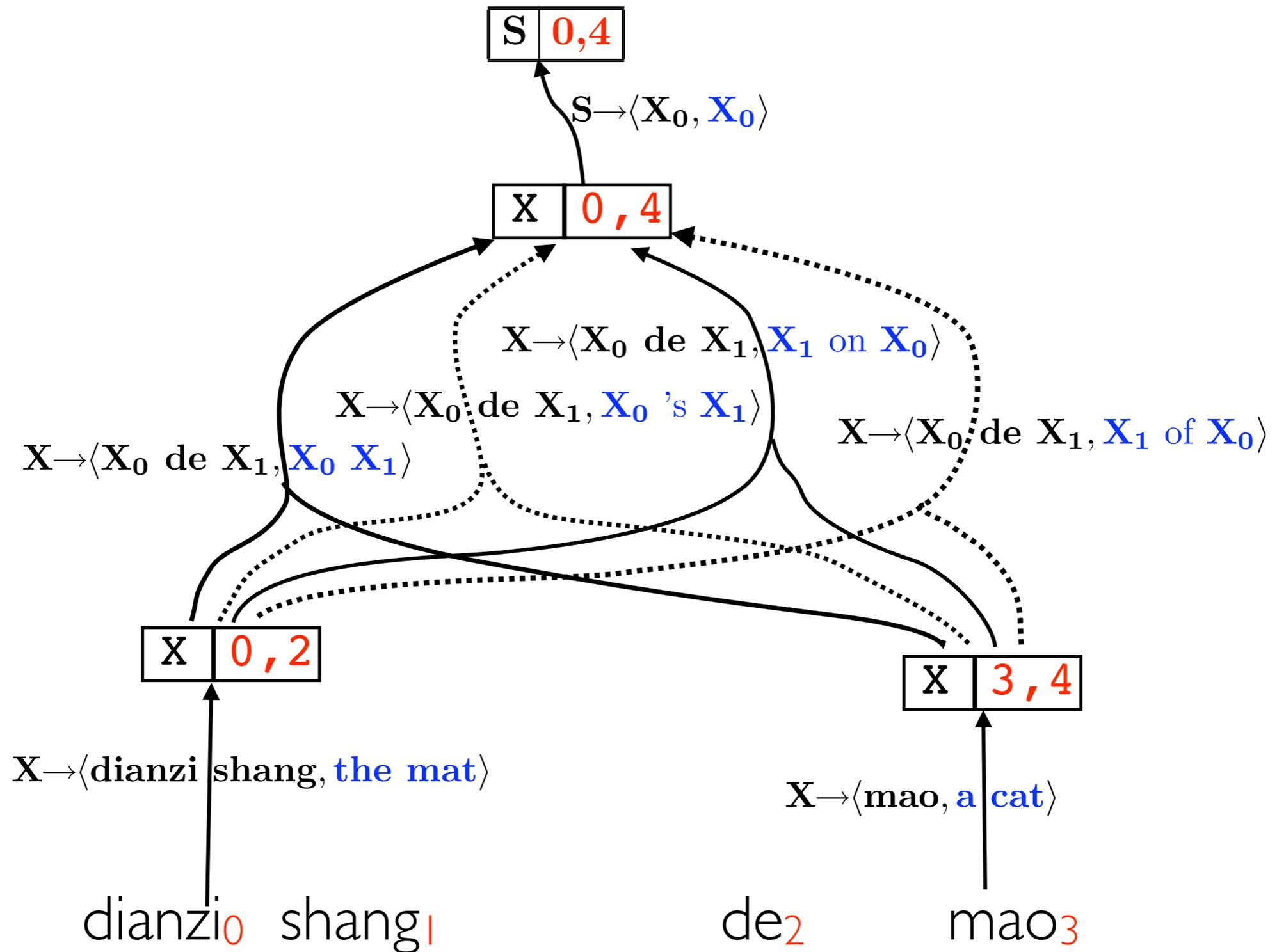
A hypergraph is a compact data structure to encode **exponentially many trees**.



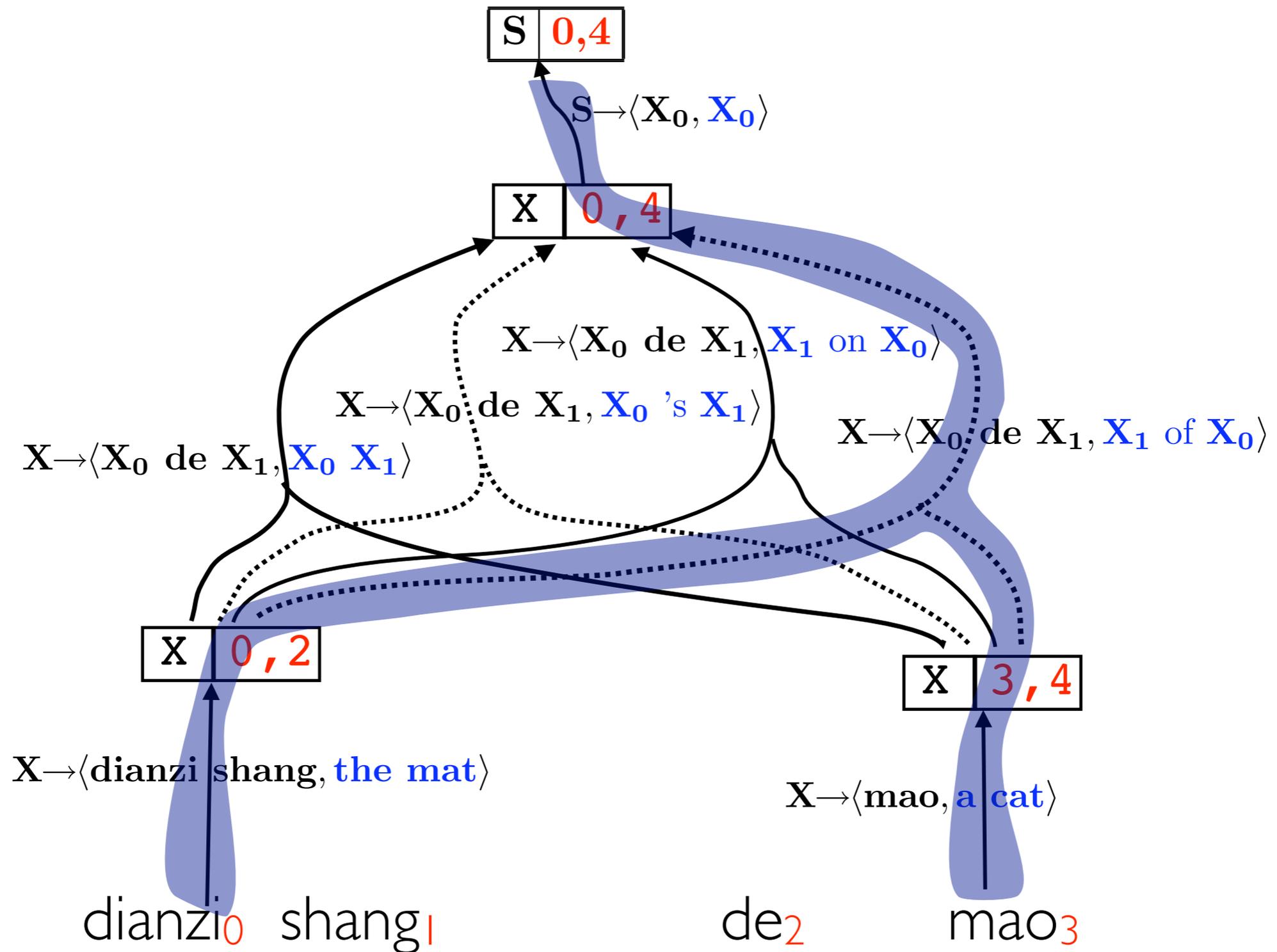
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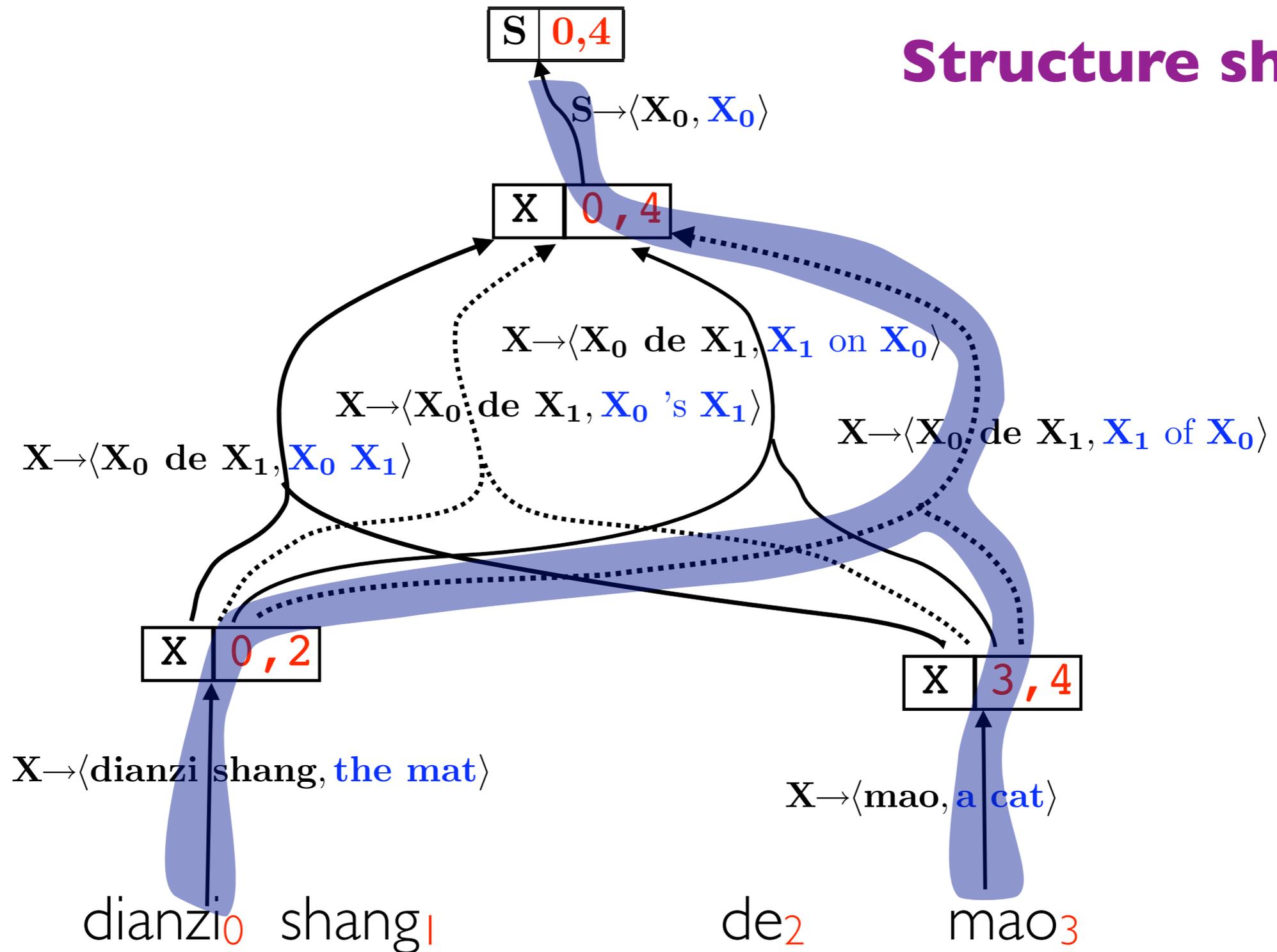


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A hypergraph is a compact data structure to encode **exponentially many trees**.

## Structure sharing

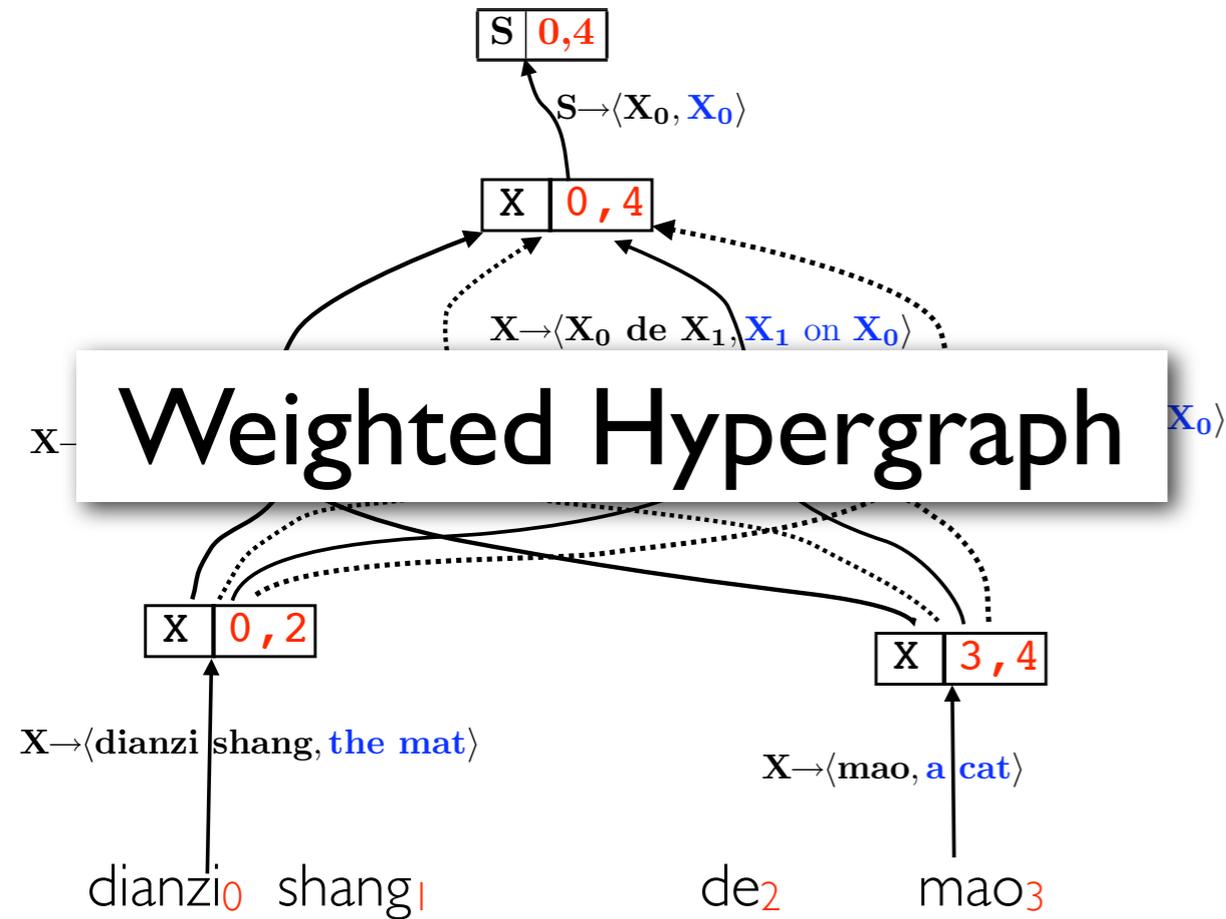


# Why Hypergraphs?

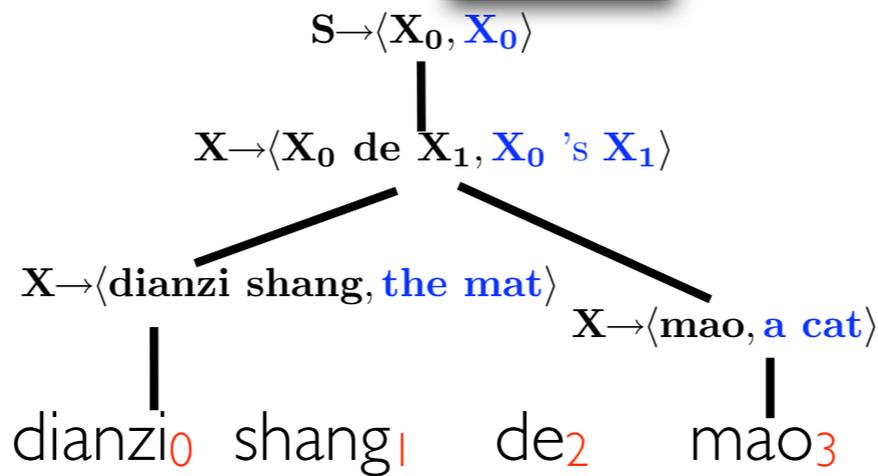
- Contains a much larger hypothesis space than a **k**-best list
- General compact data structure
  - special cases include
    - finite state machine (e.g., lattice),
    - and/or graph
    - packed forest
  - can be used for speech, parsing, tree-based MT systems, and many more



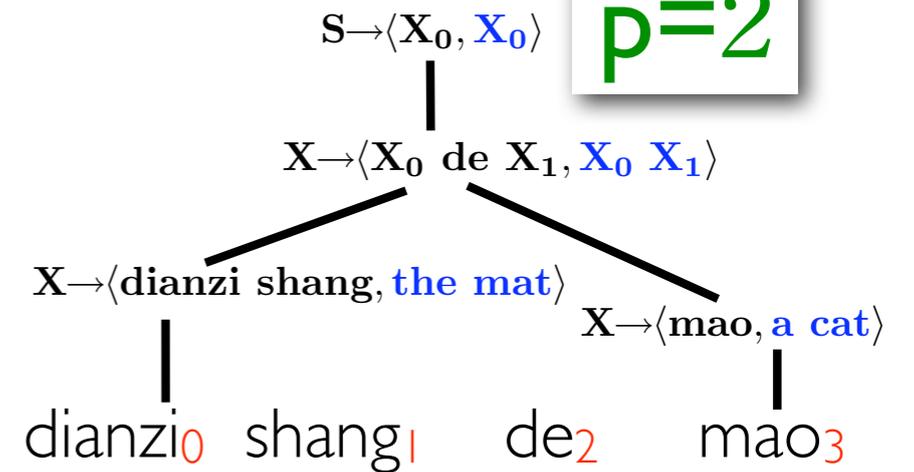
# Weighted Hypergraph



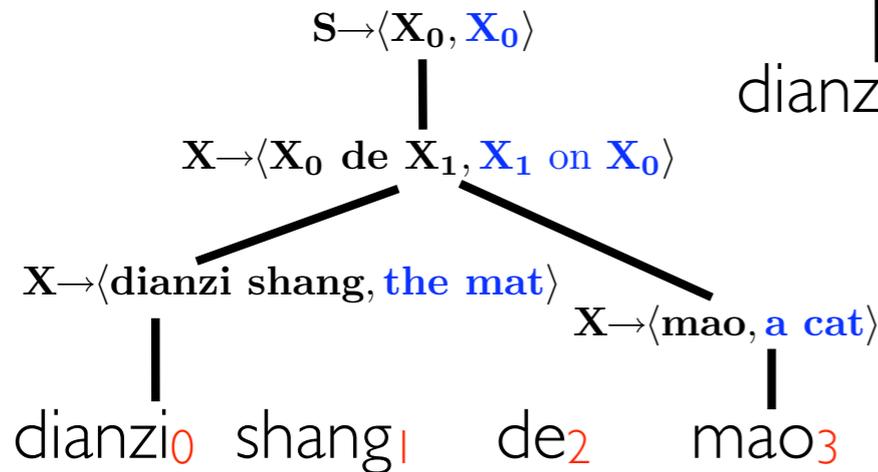
$p=3$



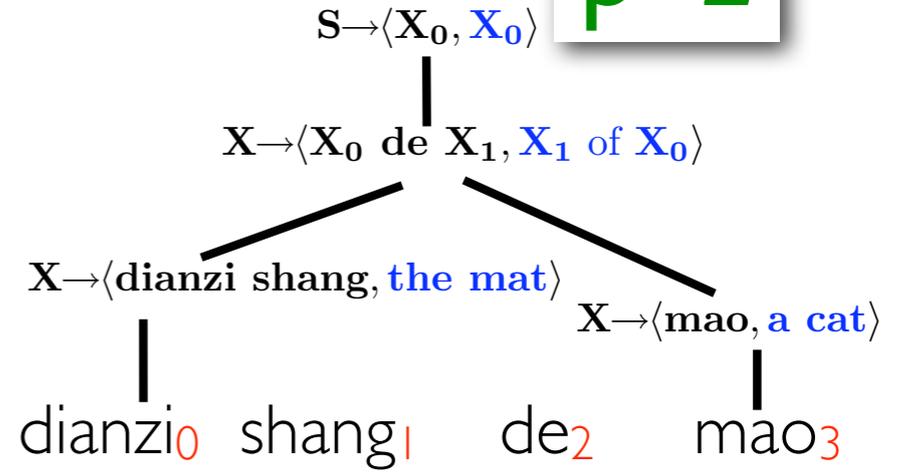
$p=2$



$p=2$



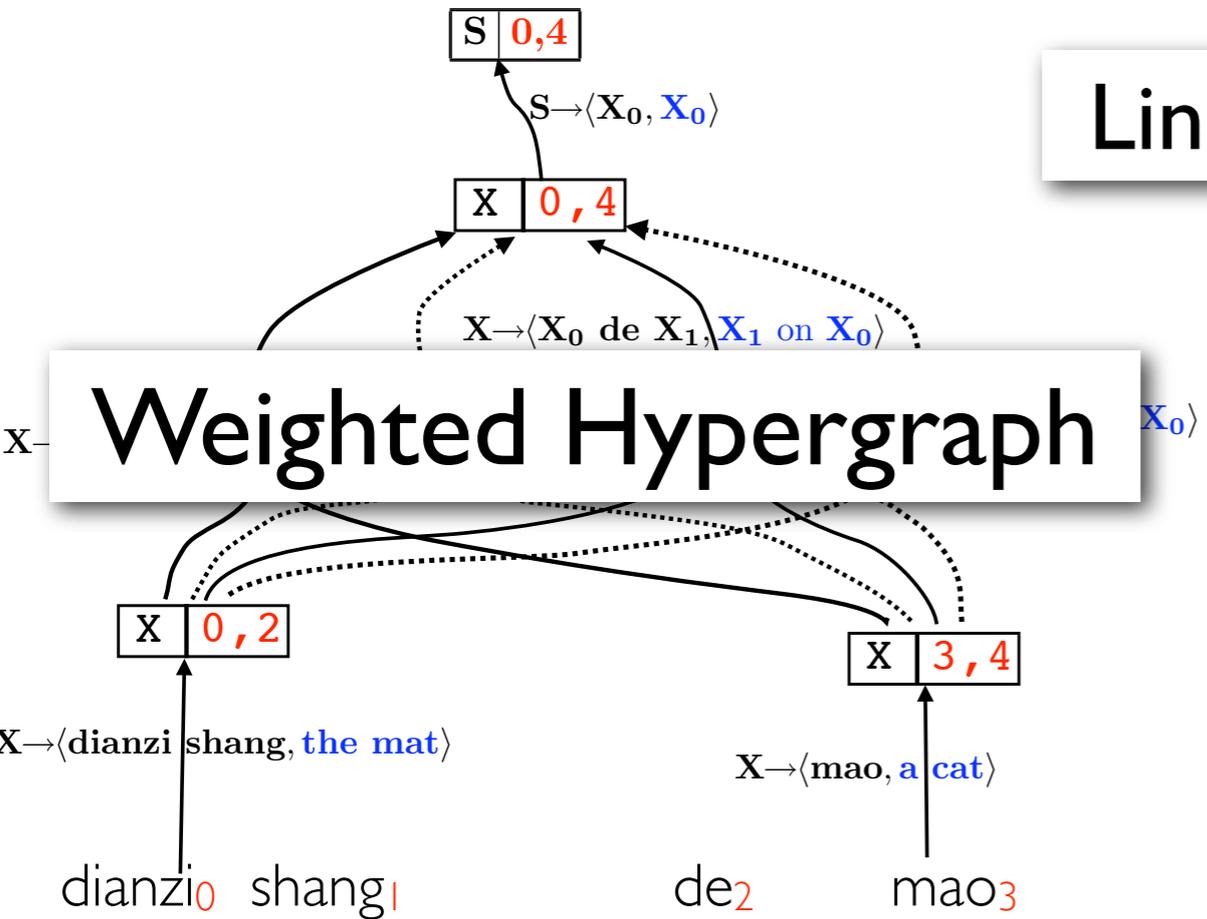
$p=1$



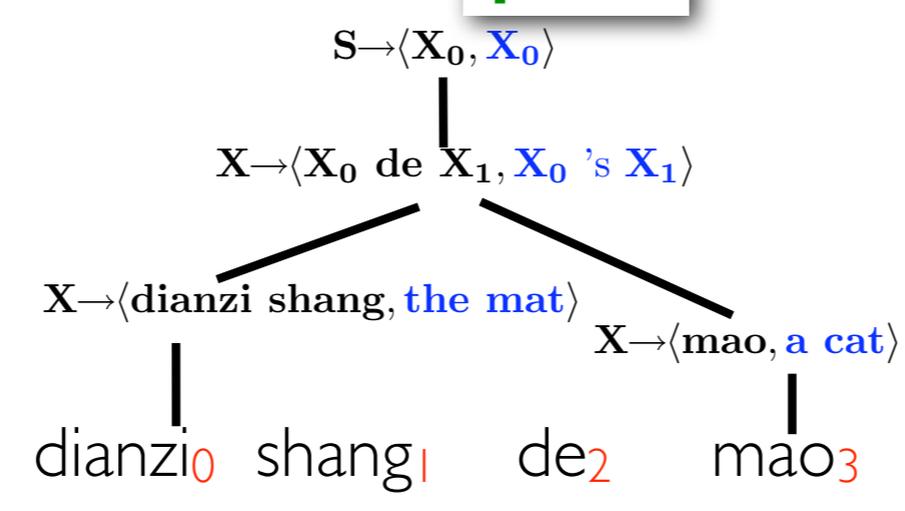
# Linear model:

$$p(d | x) = \theta \cdot \Phi(d, x)$$

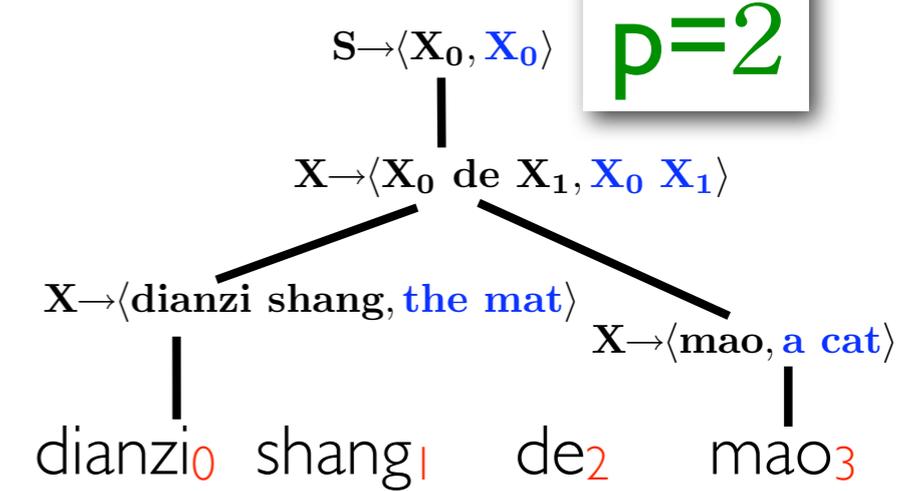
# Weighted Hypergraph



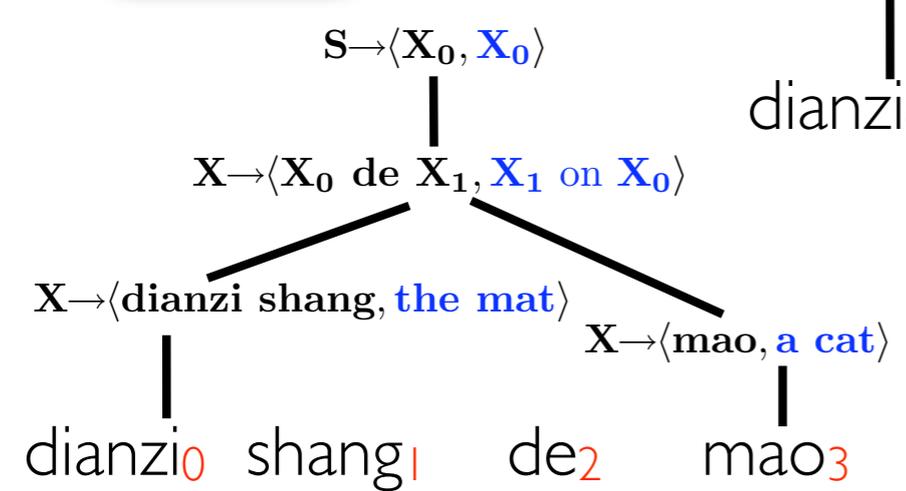
**p=3**



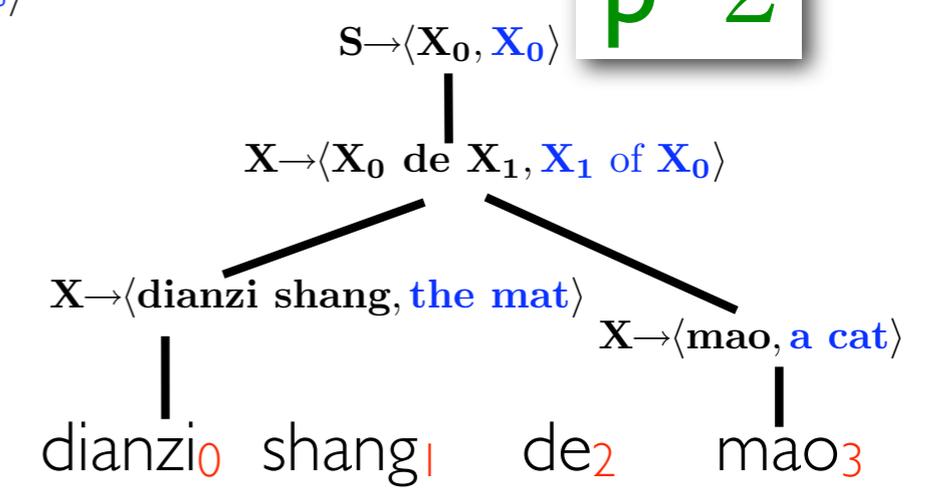
**p=2**



**p=1**



**p=2**

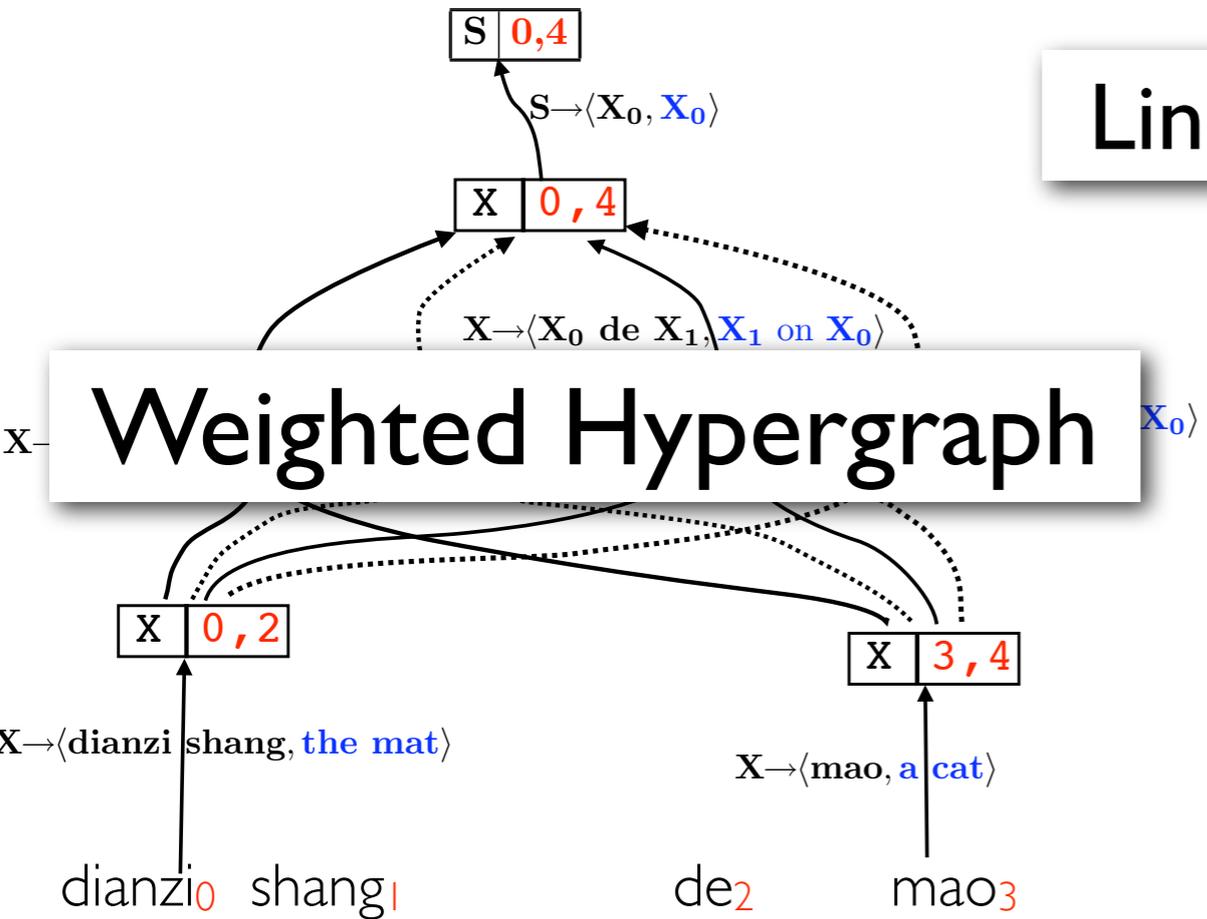


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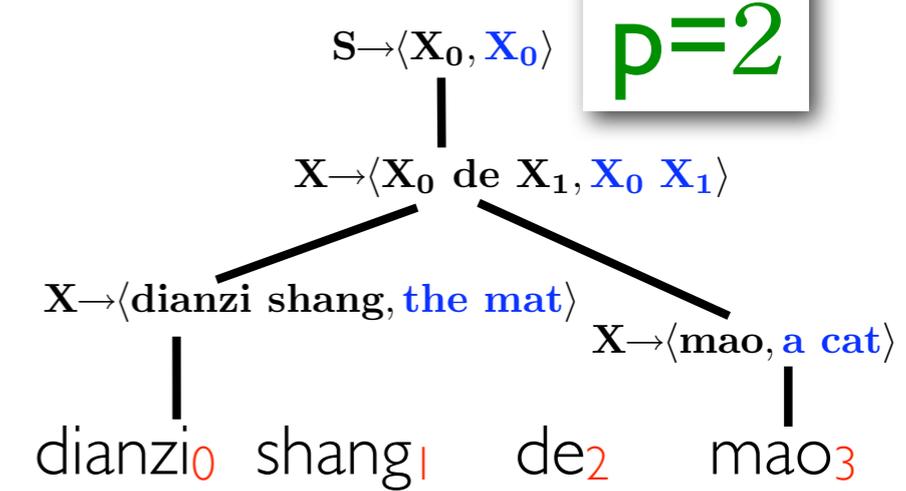
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foreign input

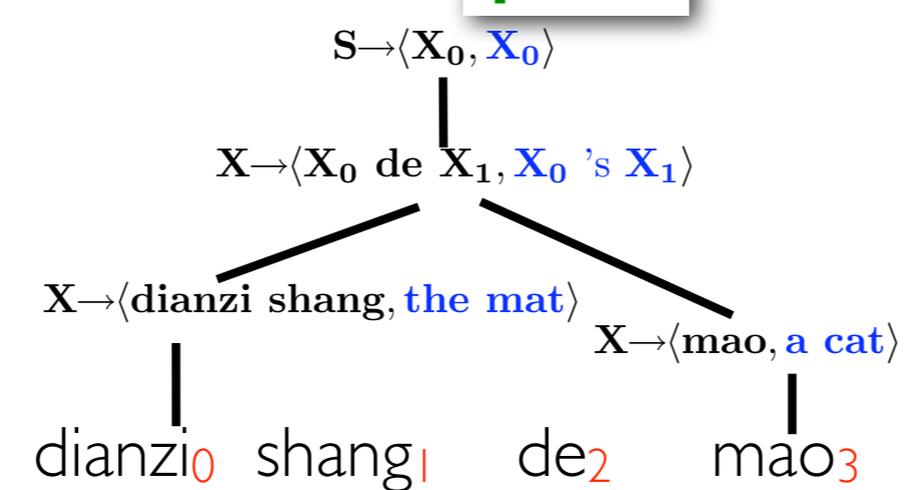
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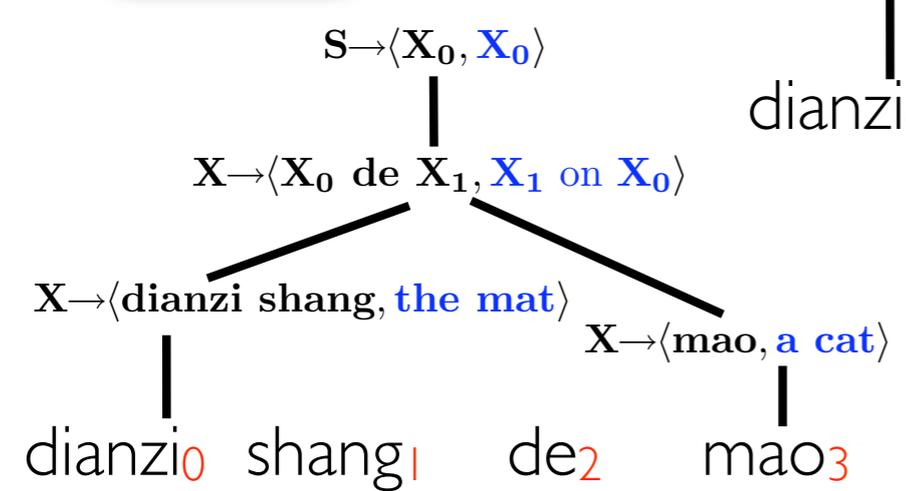
p=2



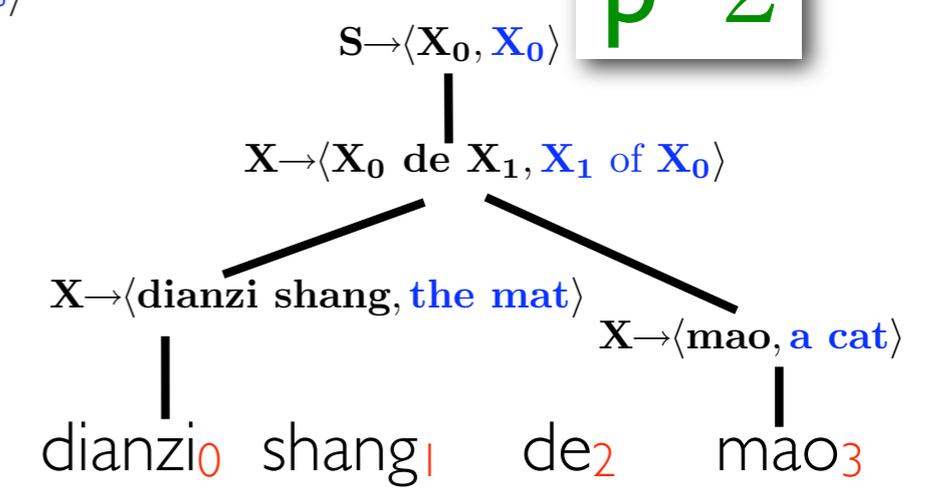
p=3



p=1



p=2

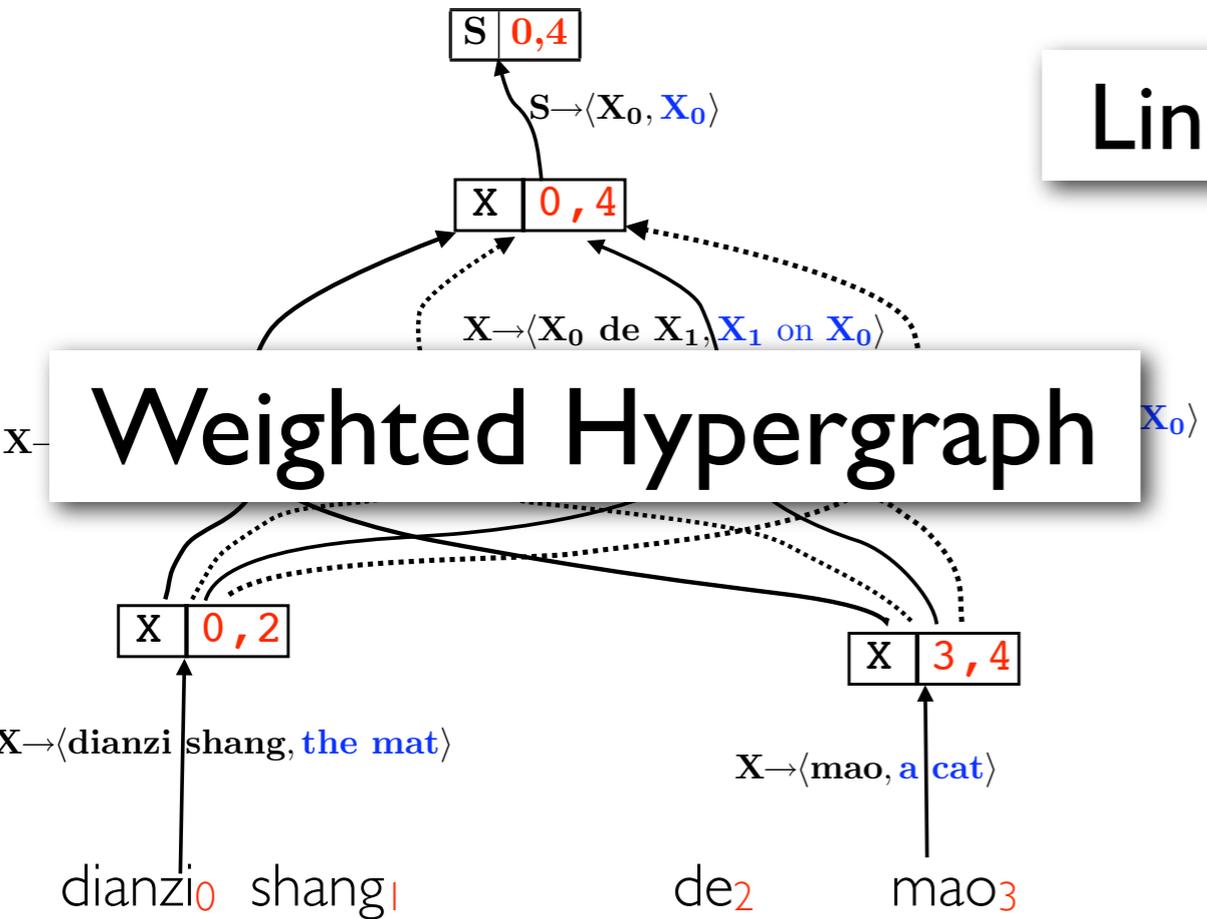


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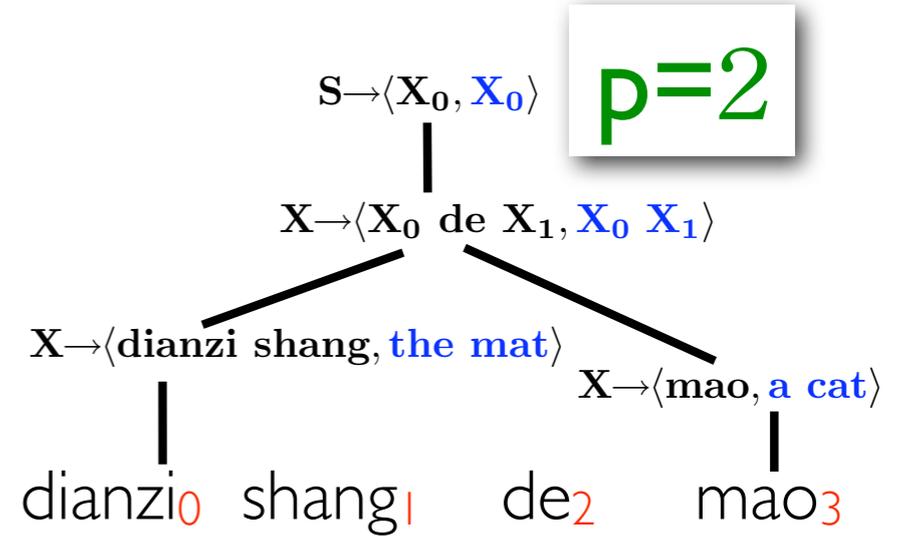
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derivation  
foreign input

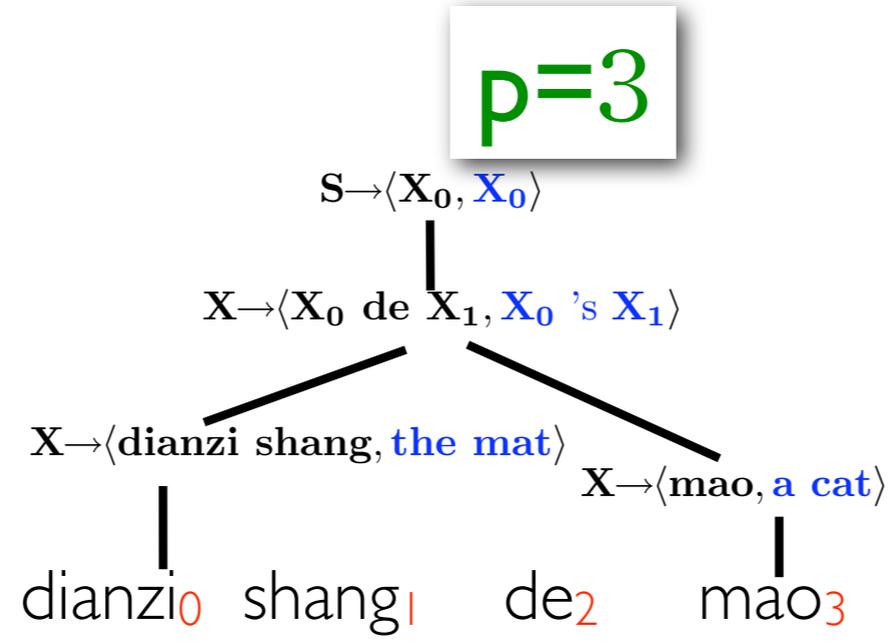
# Weighted Hypergraph



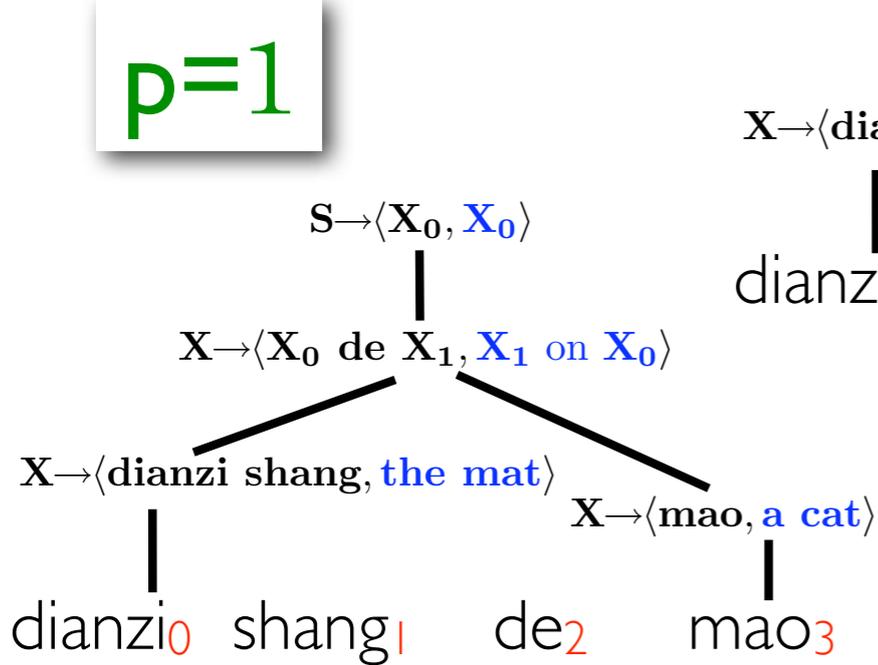
p=2



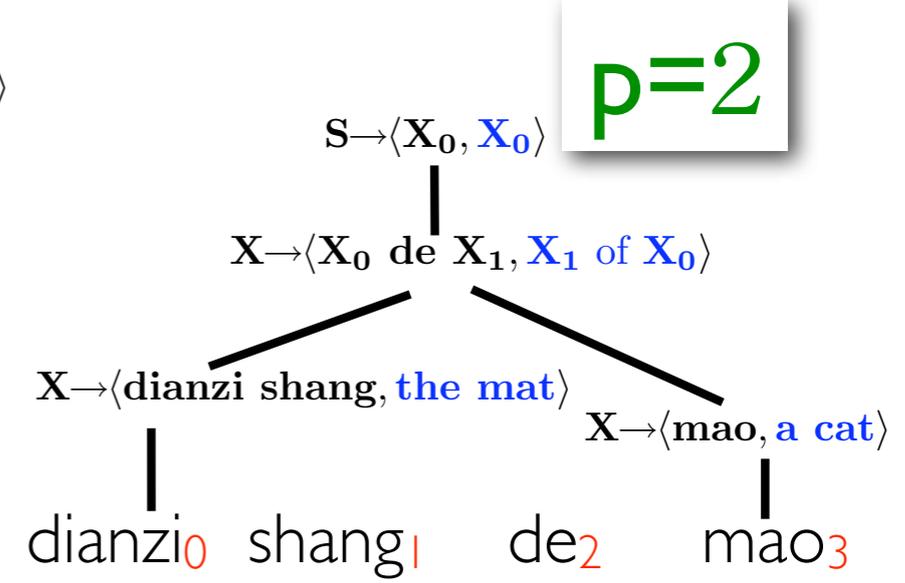
p=3



p=1

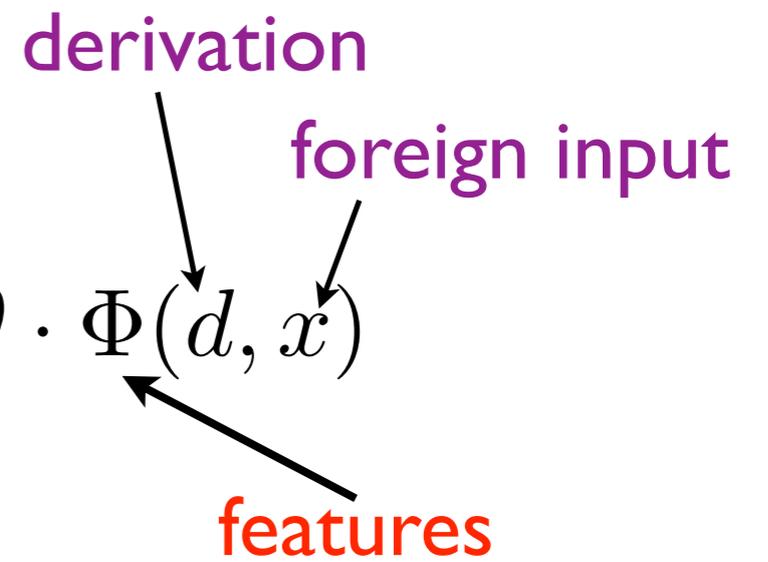


p=2

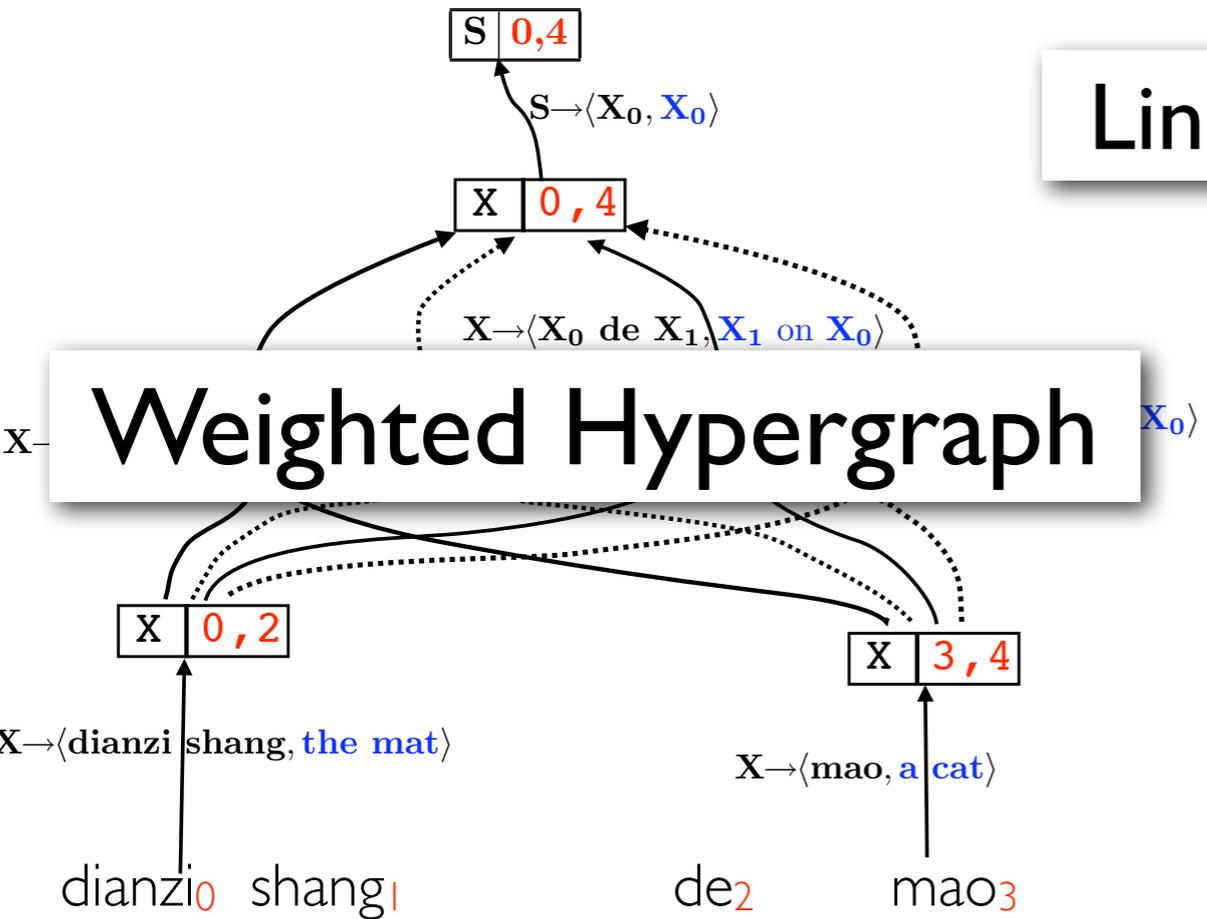


Linear model:

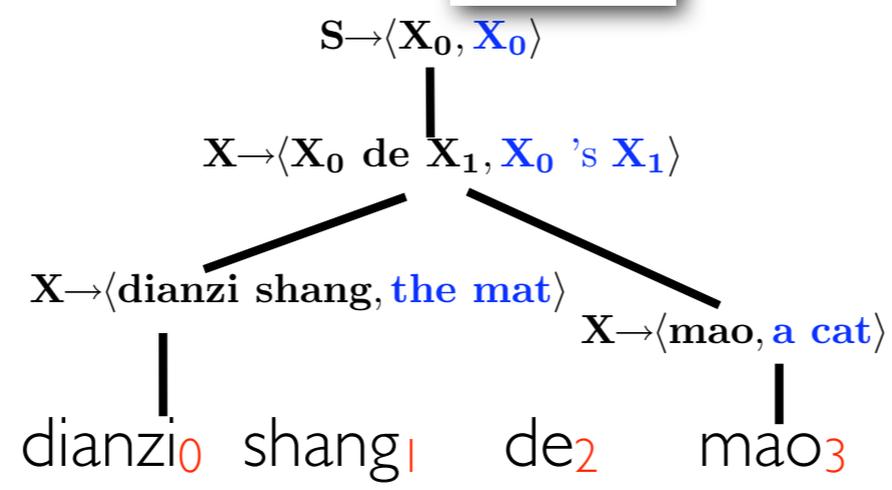
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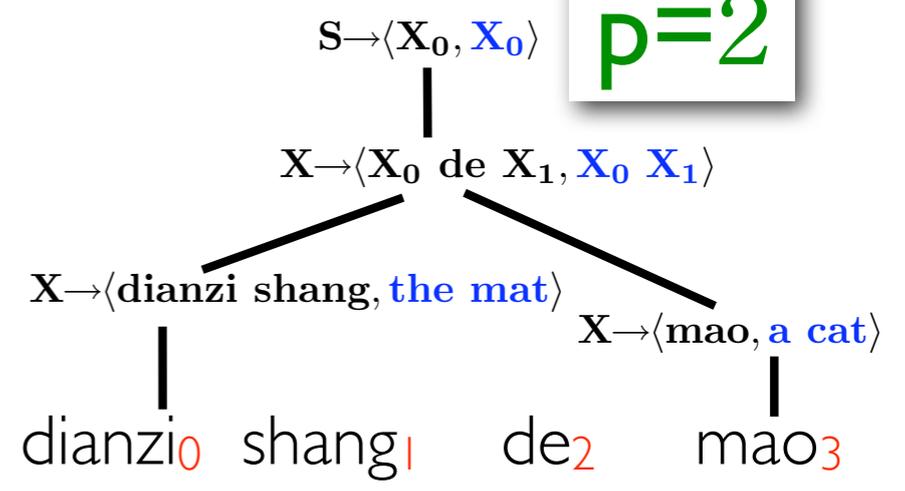
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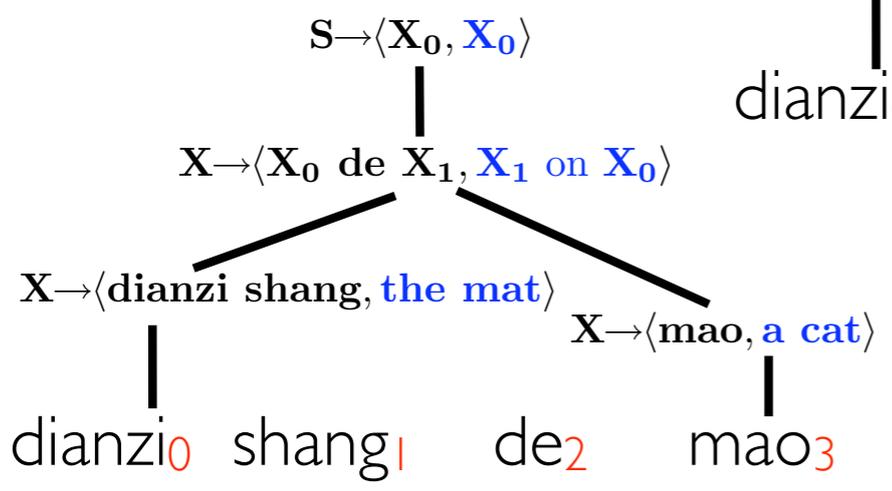
$p=3$



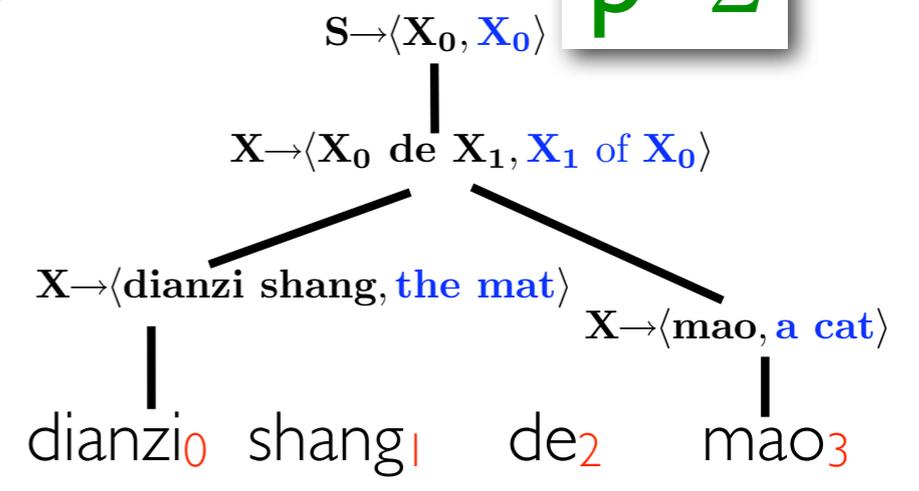
$p=2$



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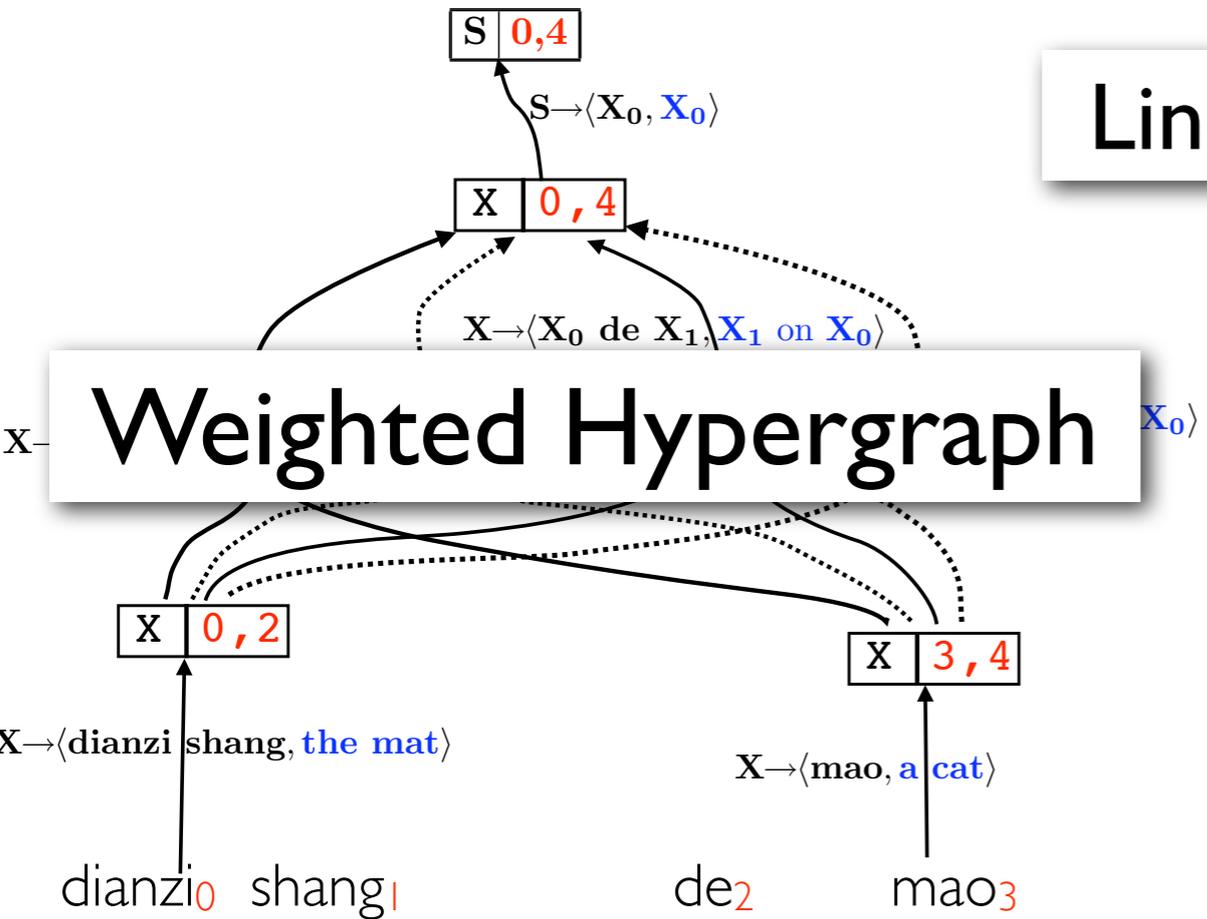
weights

features

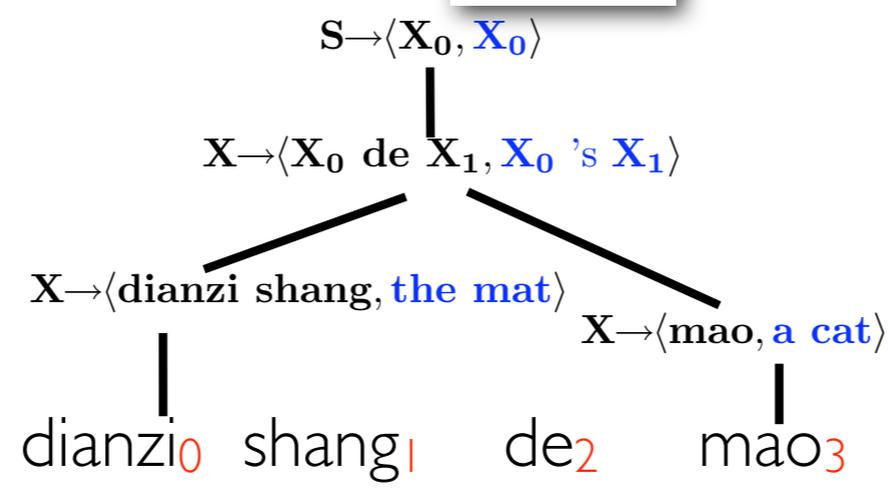
derivation

foreign input

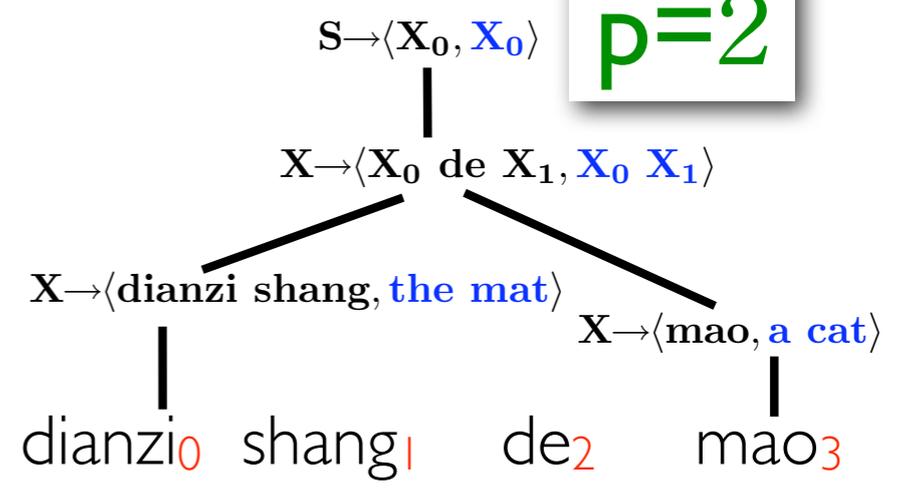
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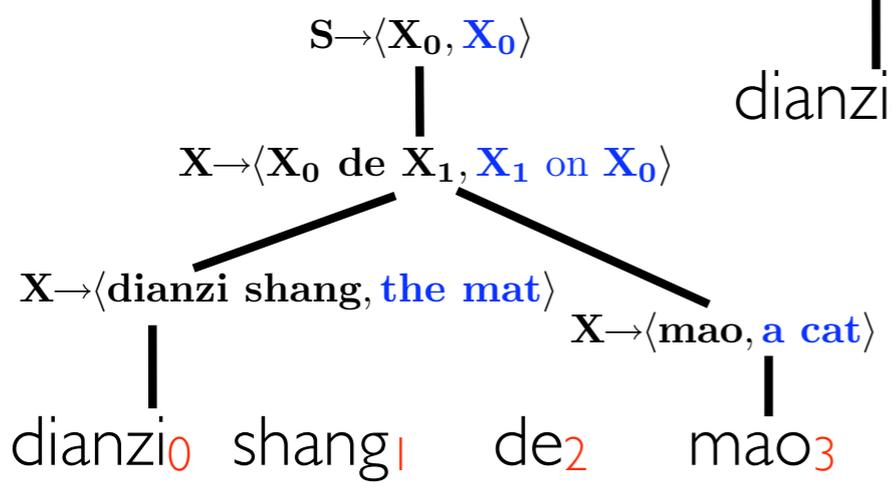
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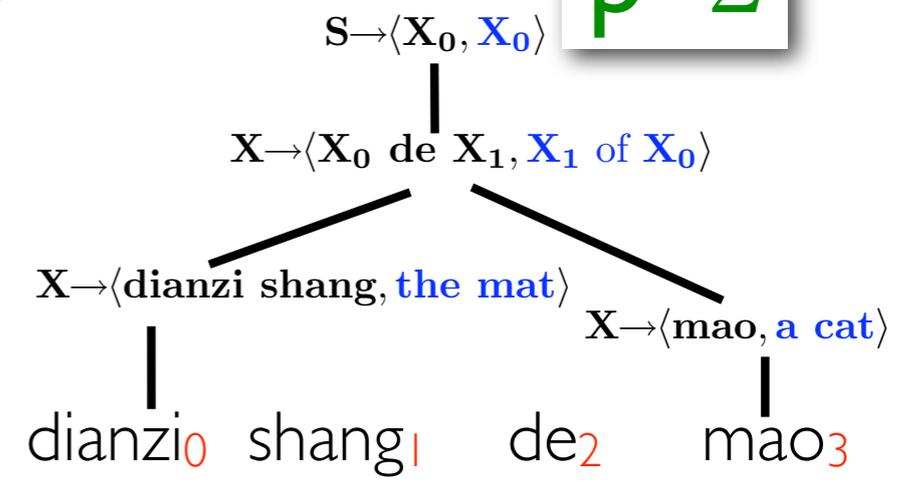
p=2



p=1



p=2

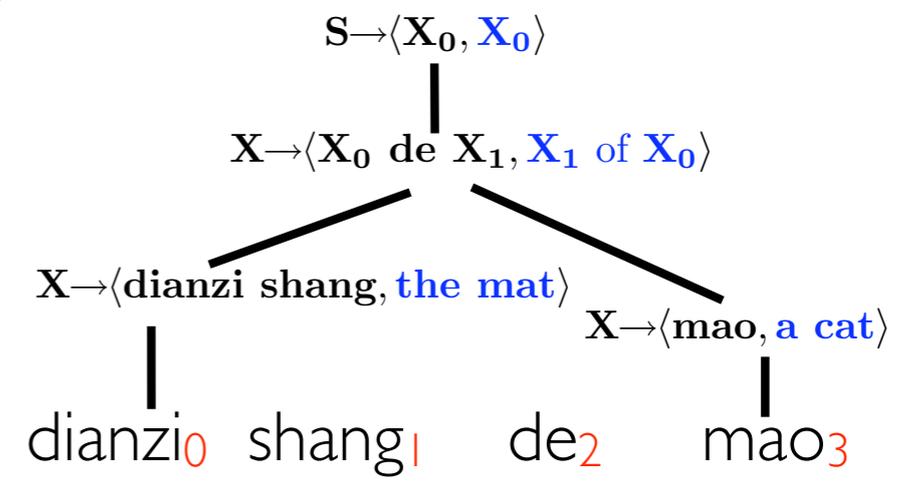
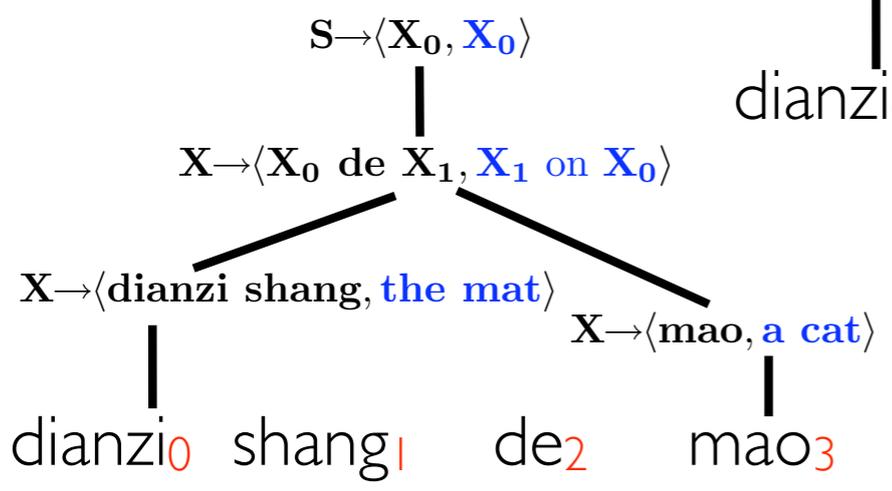
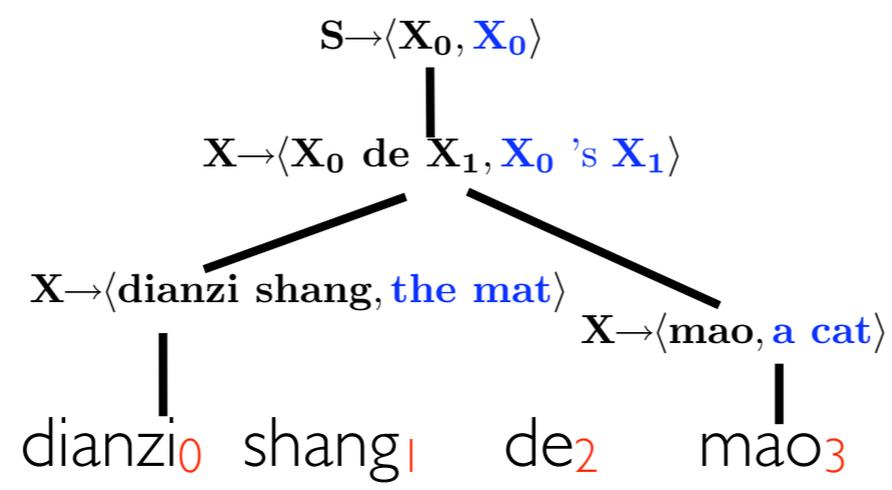
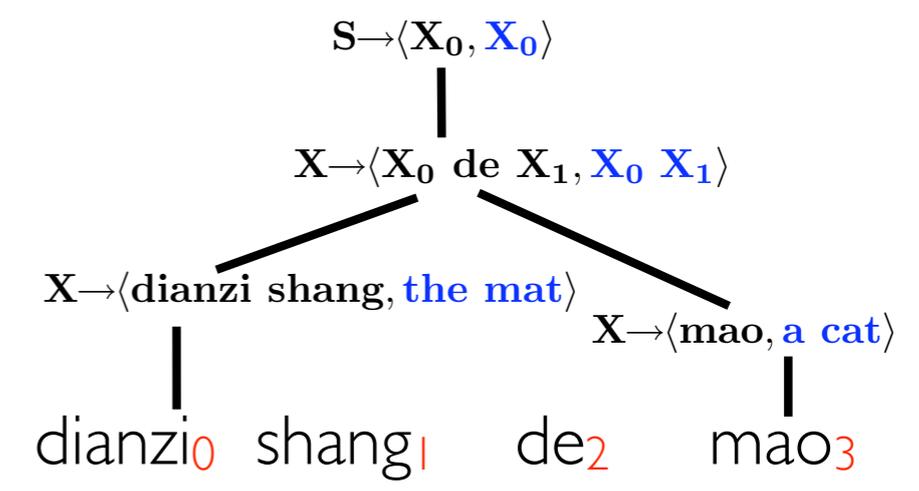
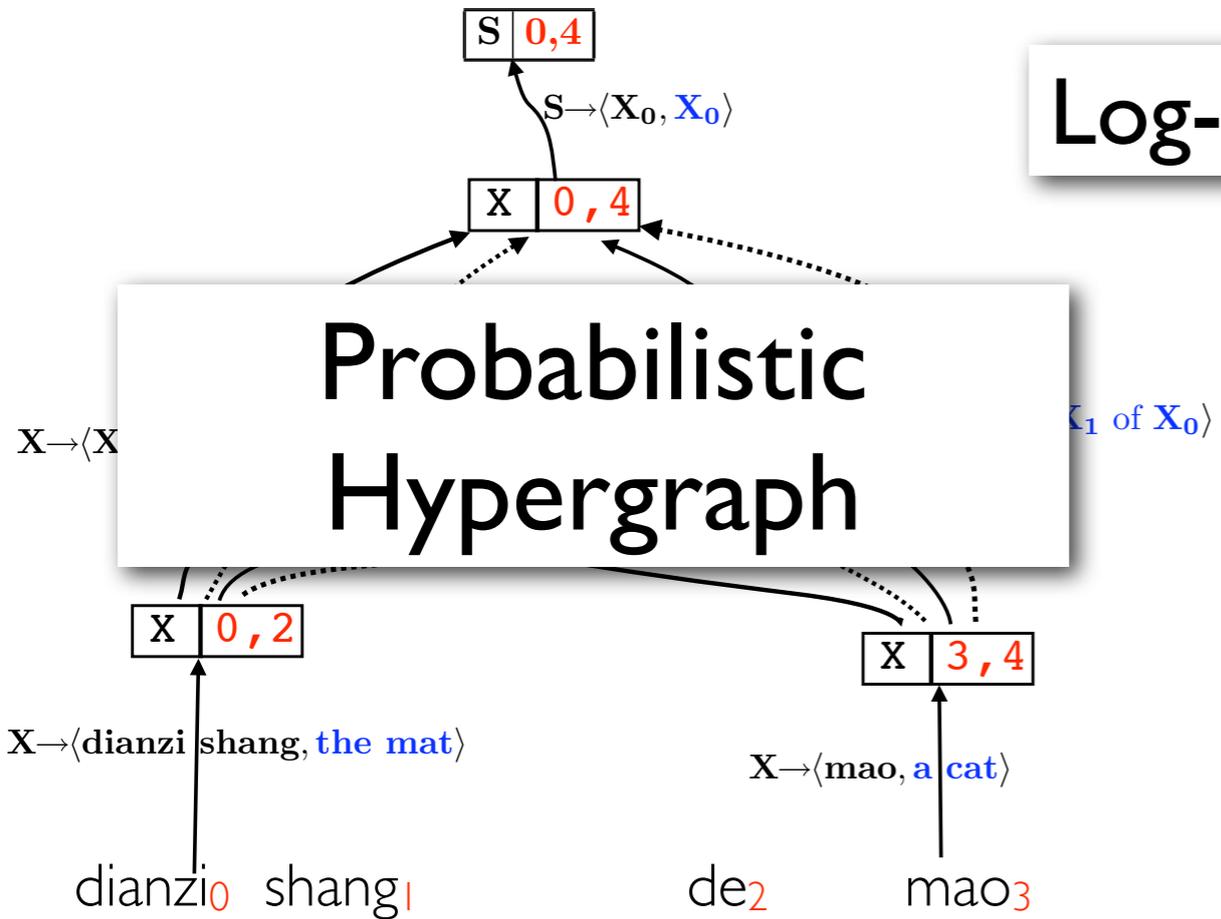


# Log-linear model:

$$p(d | x) = \frac{e^{\theta \cdot \Phi(d, x)}}{Z(x)}$$

$$Z = 2 + 1 + 3 + 2 = 8$$

## Probabilistic Hypergraph

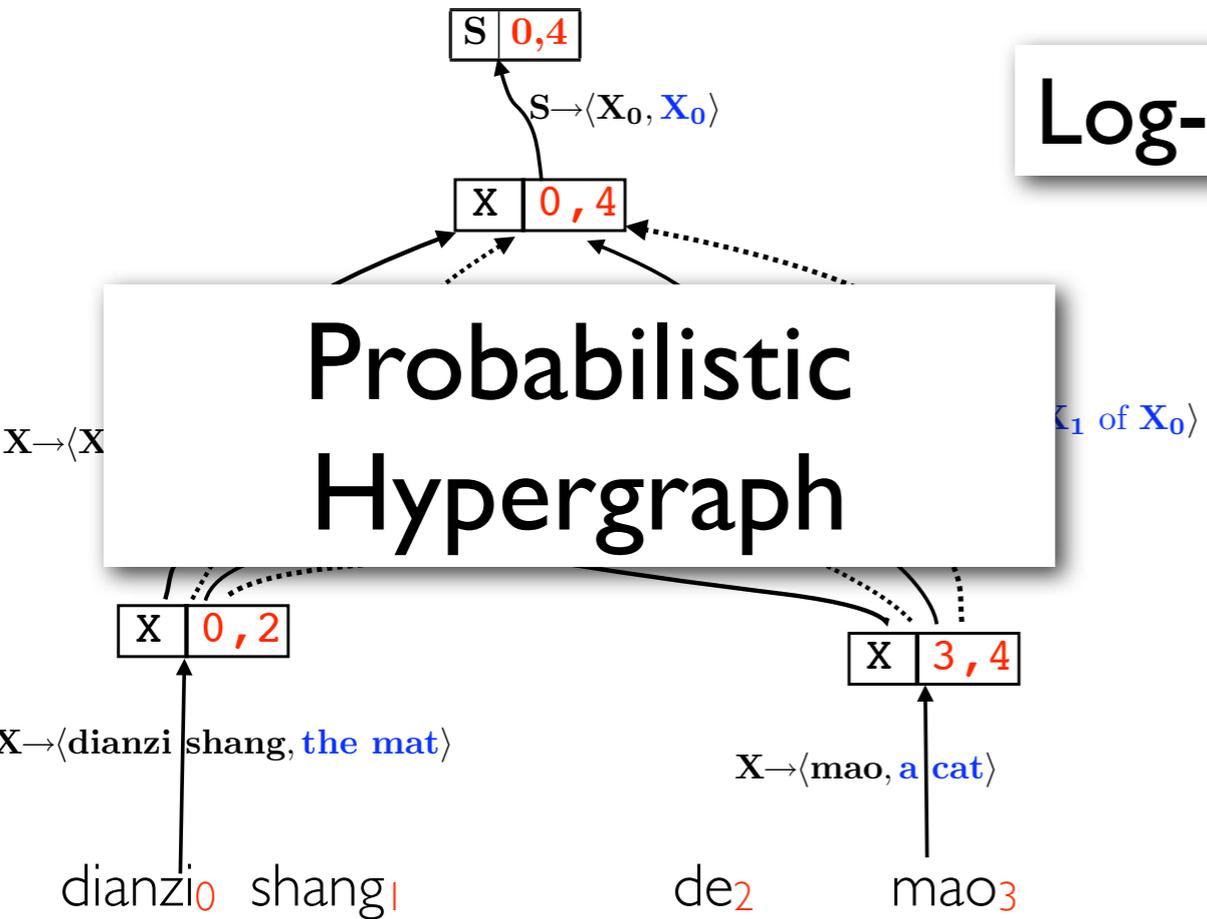


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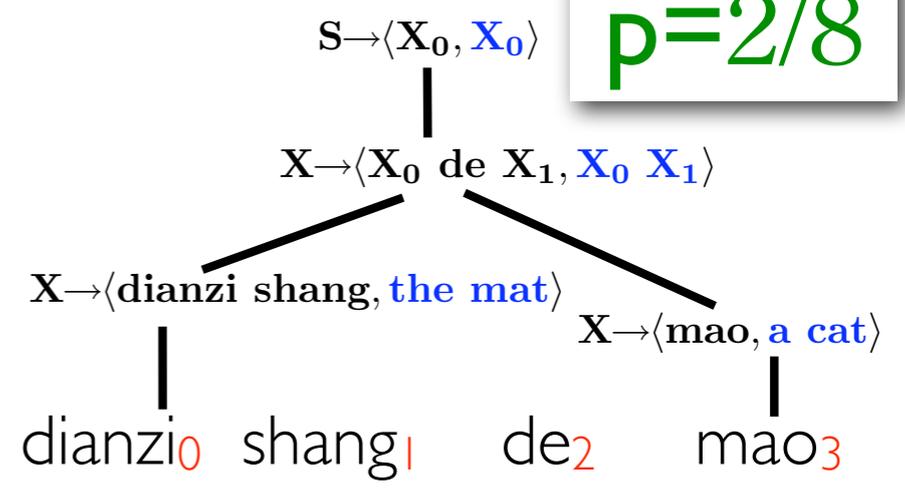
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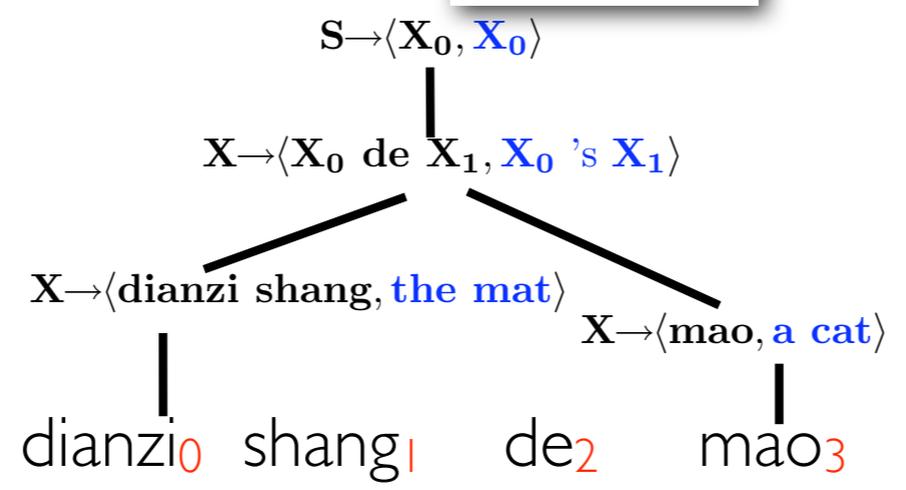
## Probabilistic Hypergraph



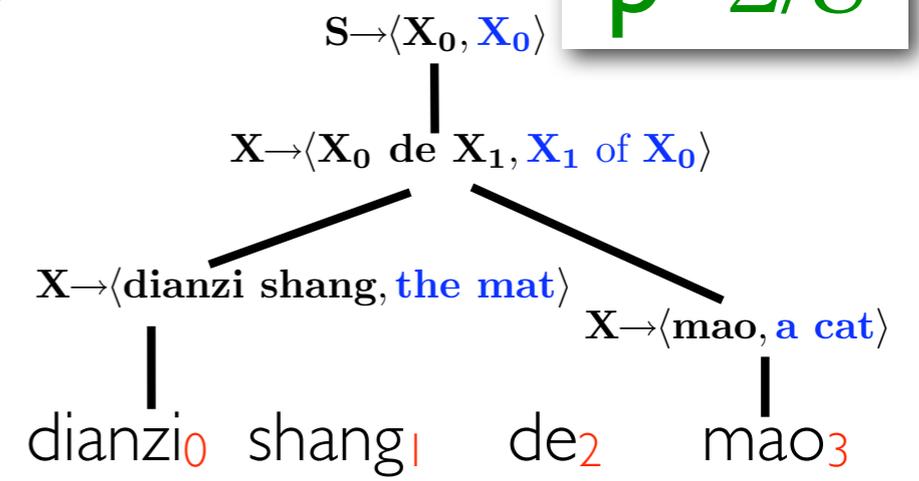
$$p = 2/8$$



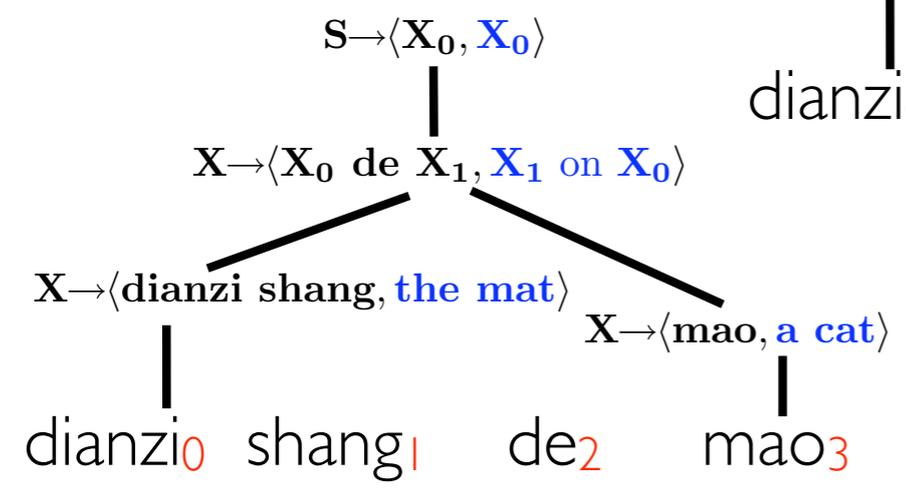
$$p = 3/8$$



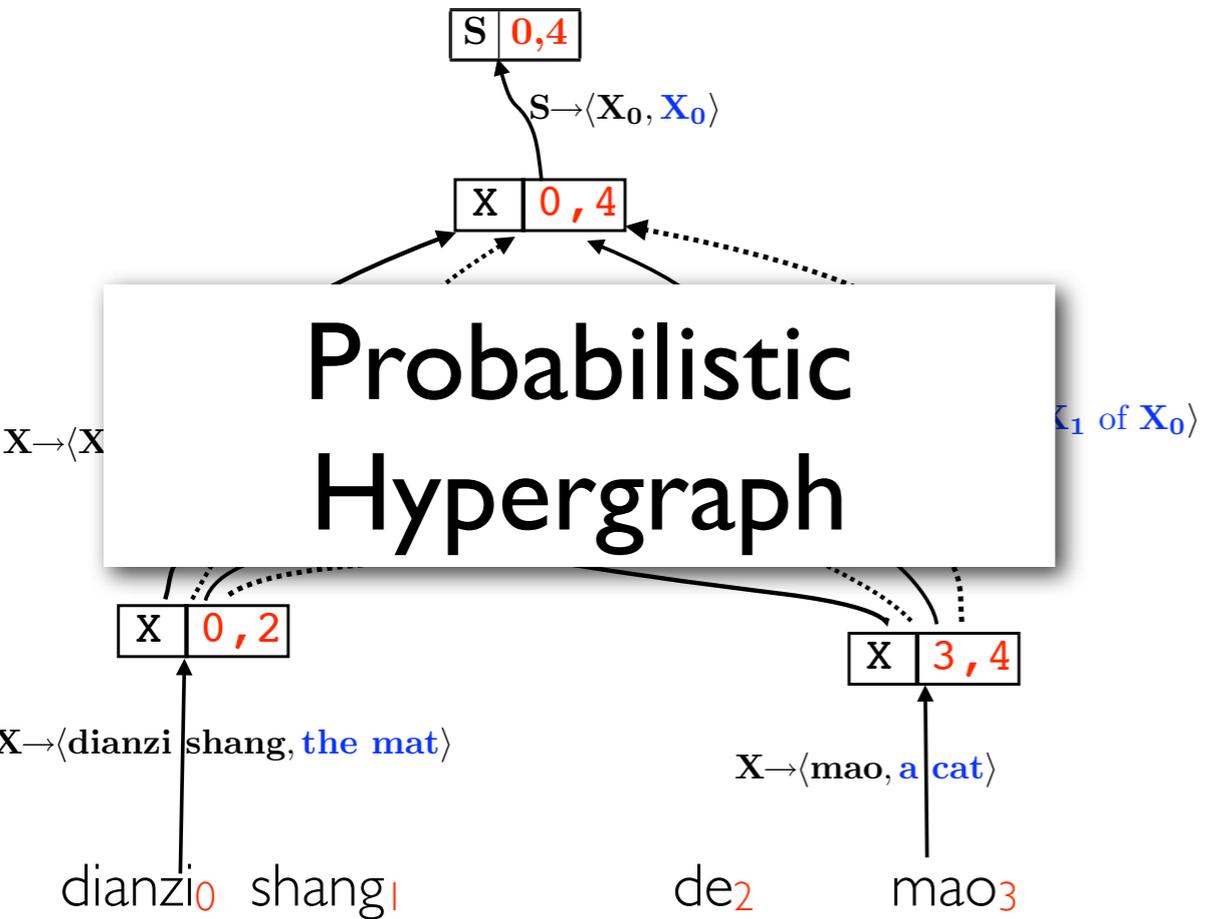
$$p = 2/8$$



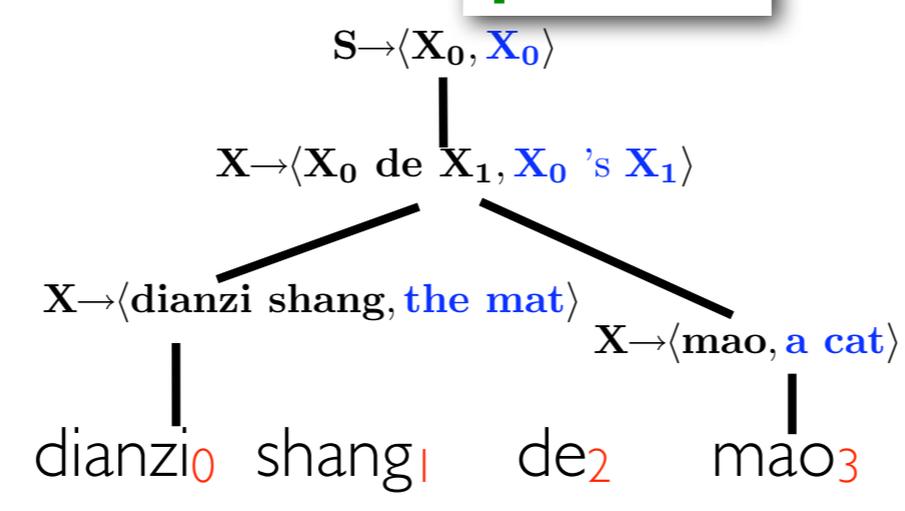
$$p = 1/8$$



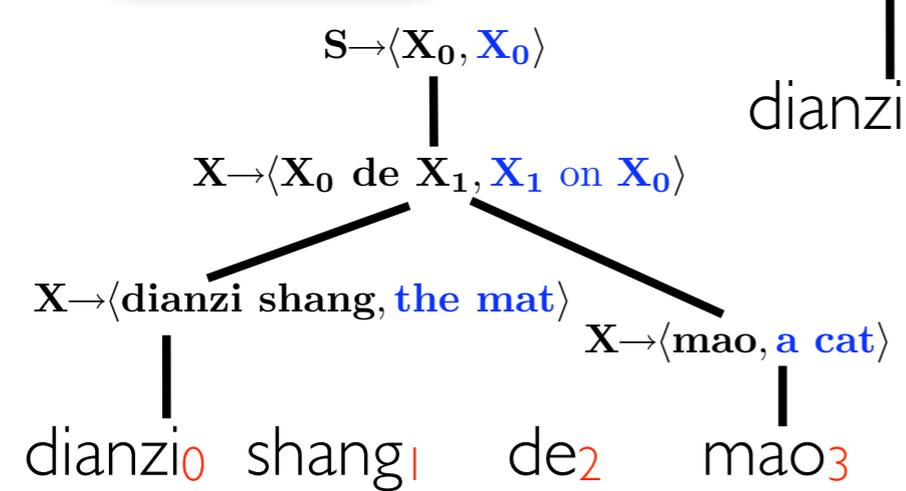
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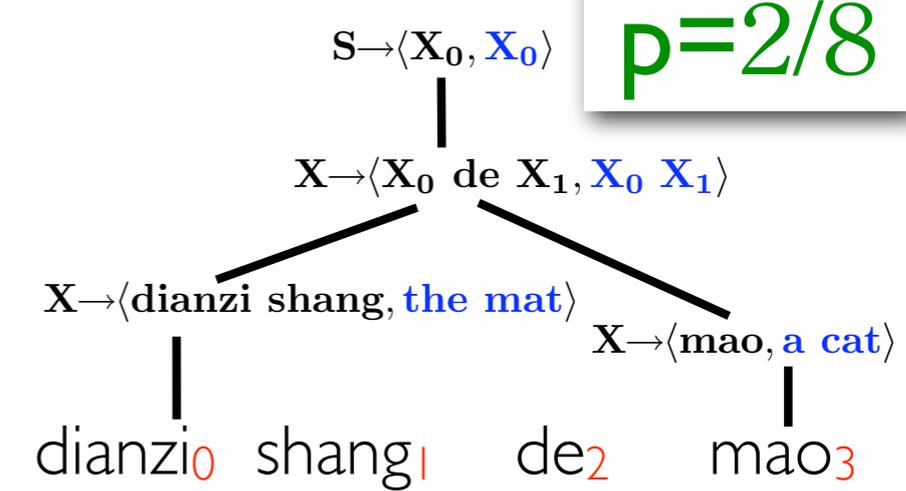
$p=3/8$



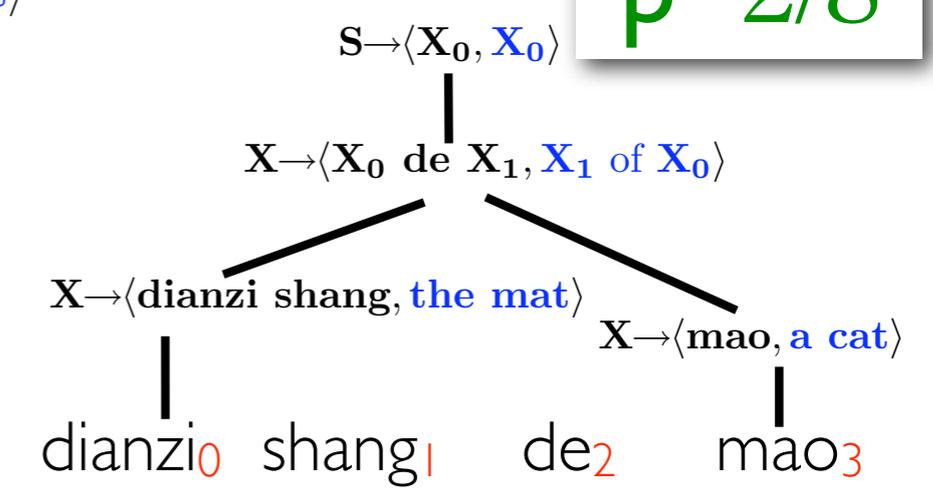
$p=1/8$



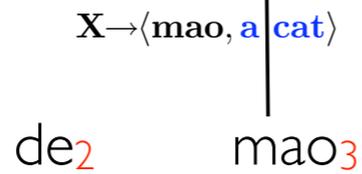
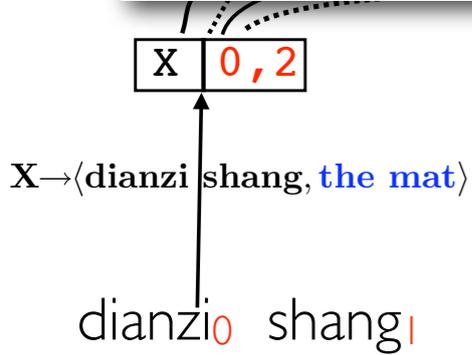
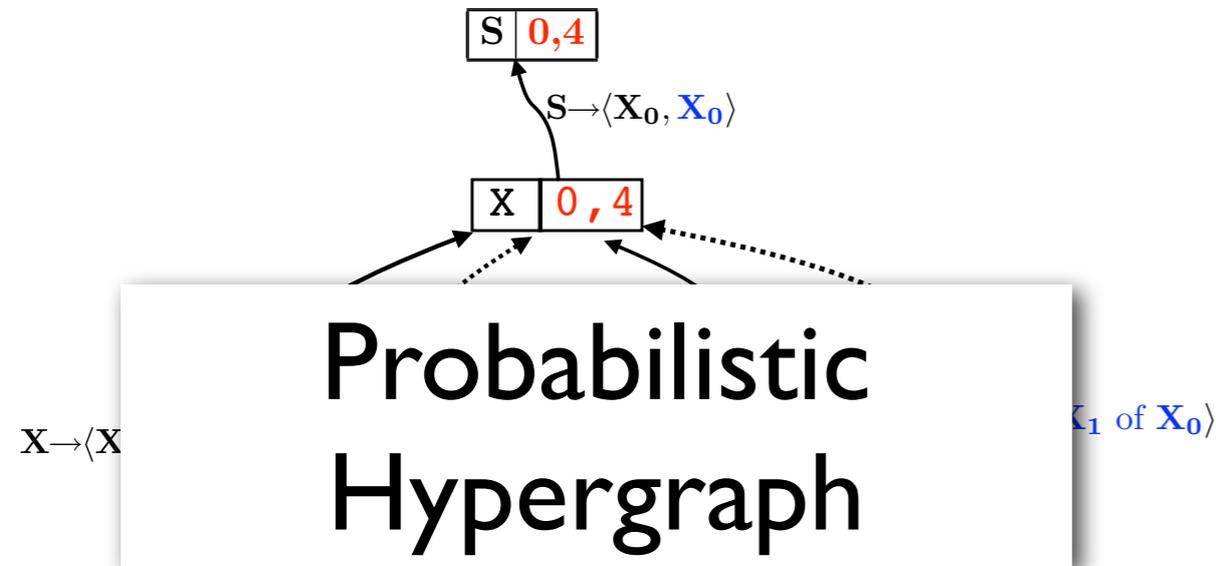
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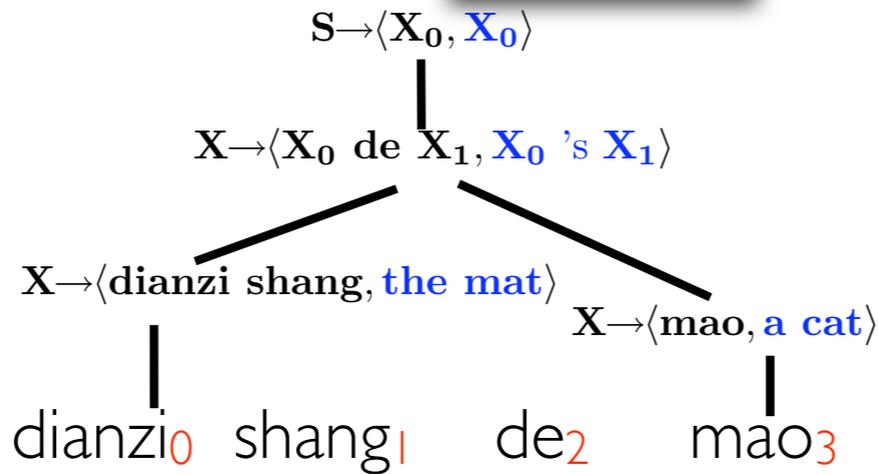
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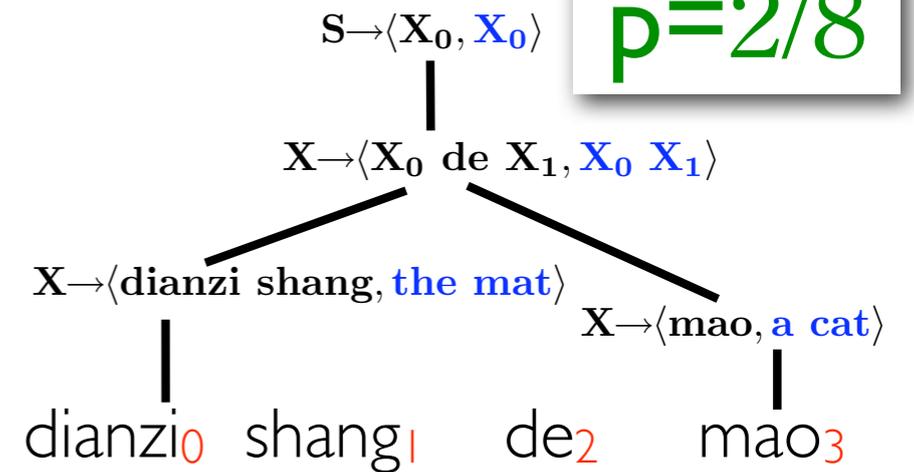
The hypergraph defines a probability distribution over **trees**!



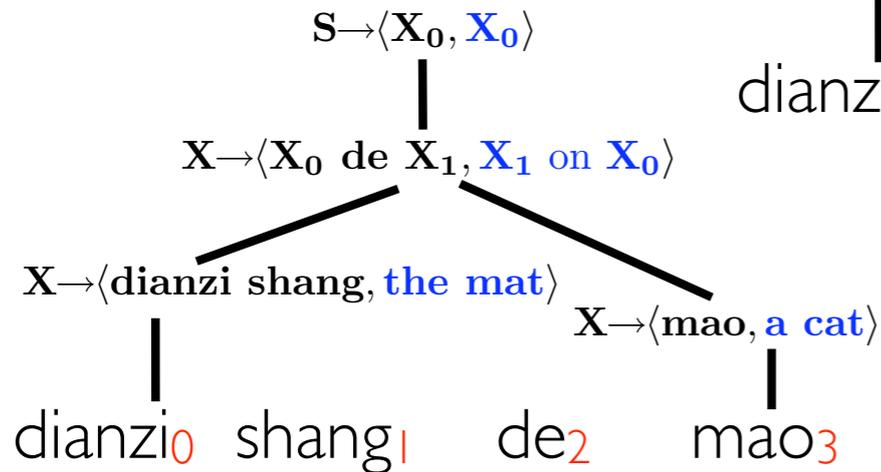
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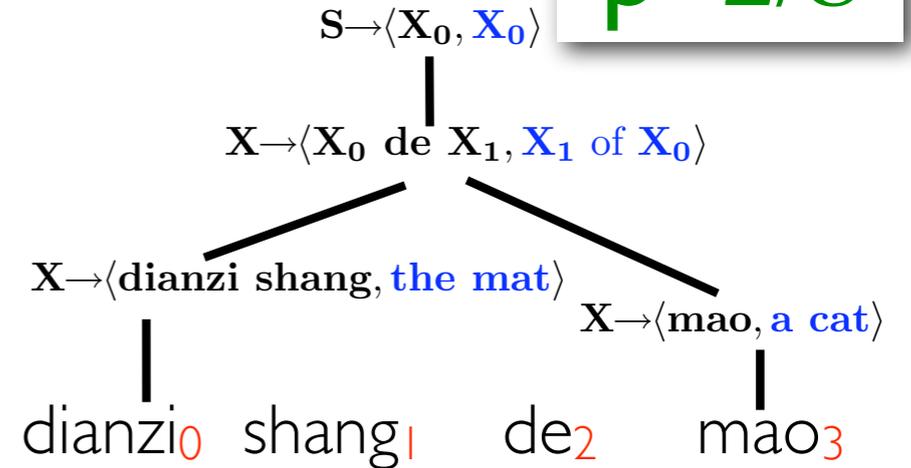
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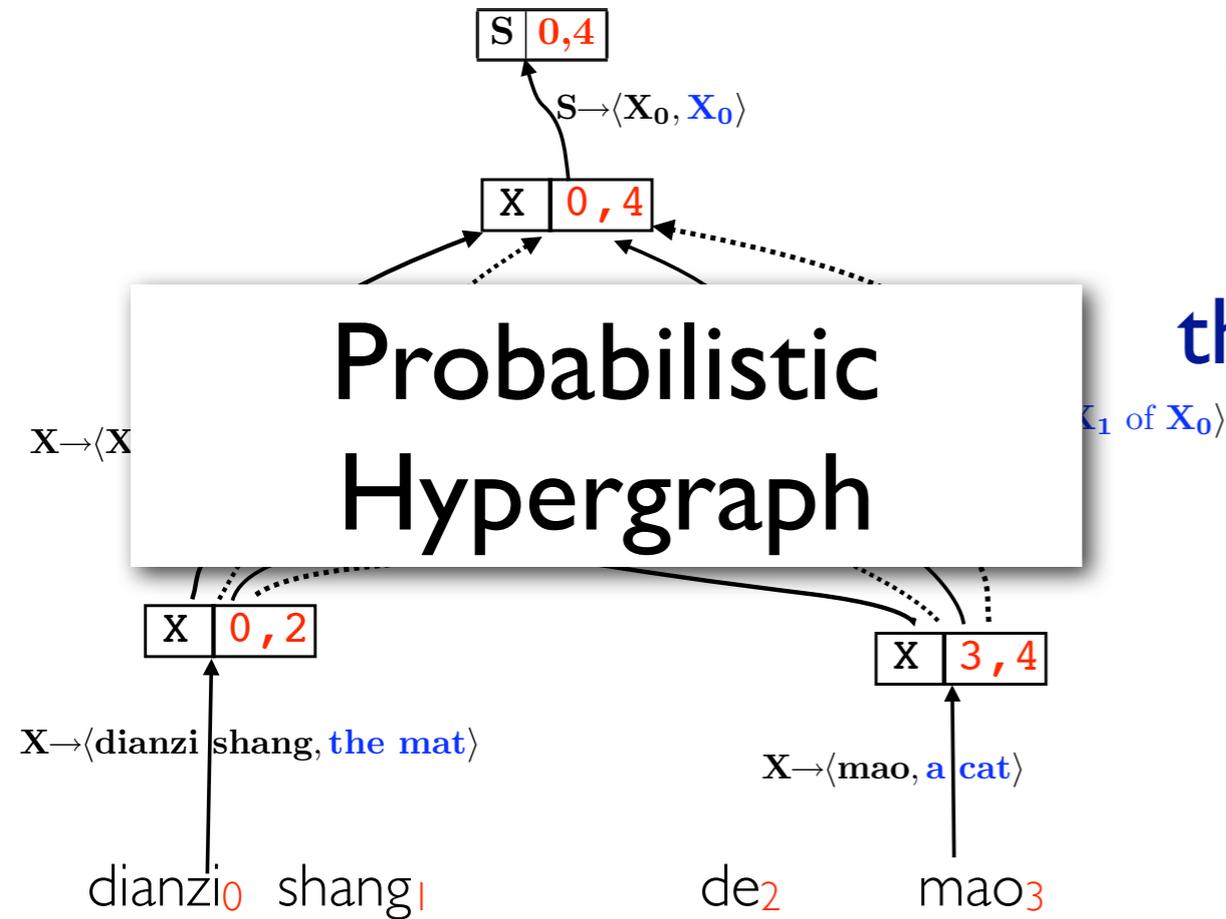
$p=2/8$



The hypergraph defines a probability distribution over **trees!**

the distribution is parameterized by  $\Theta$

# Probabilistic Hypergraph

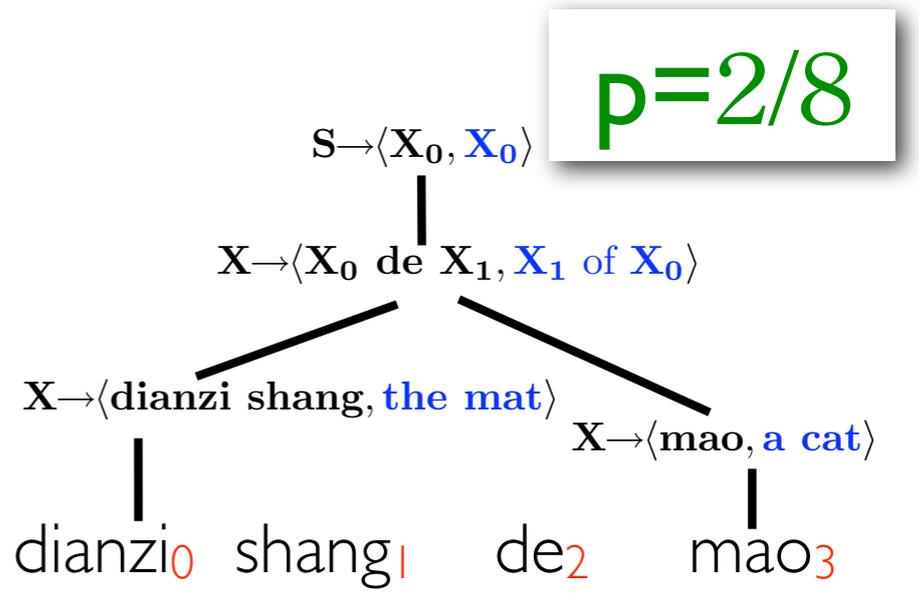
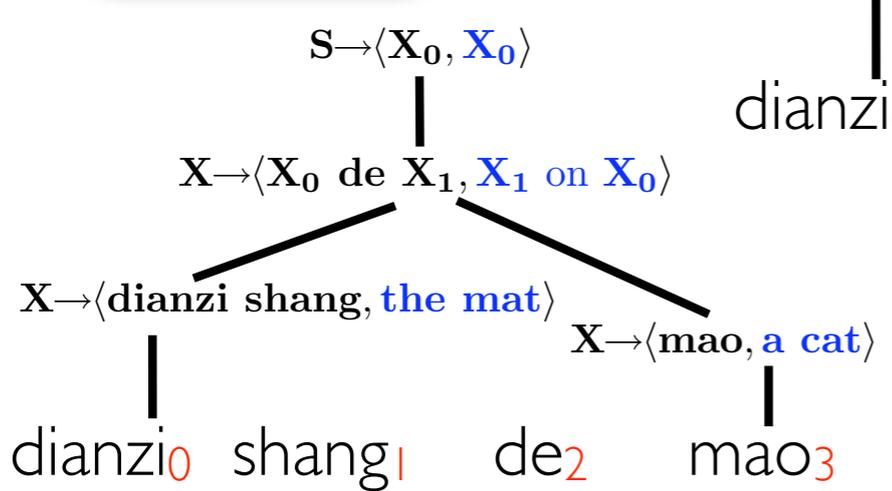
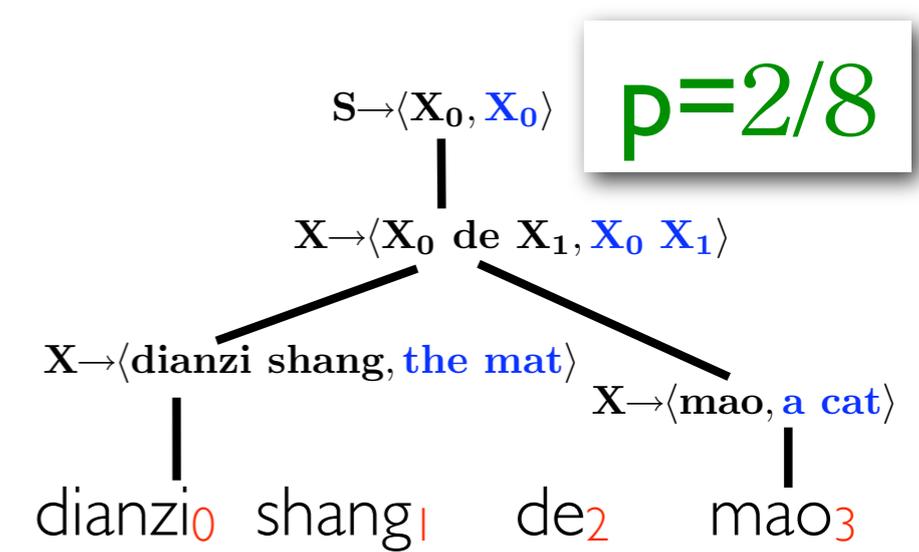
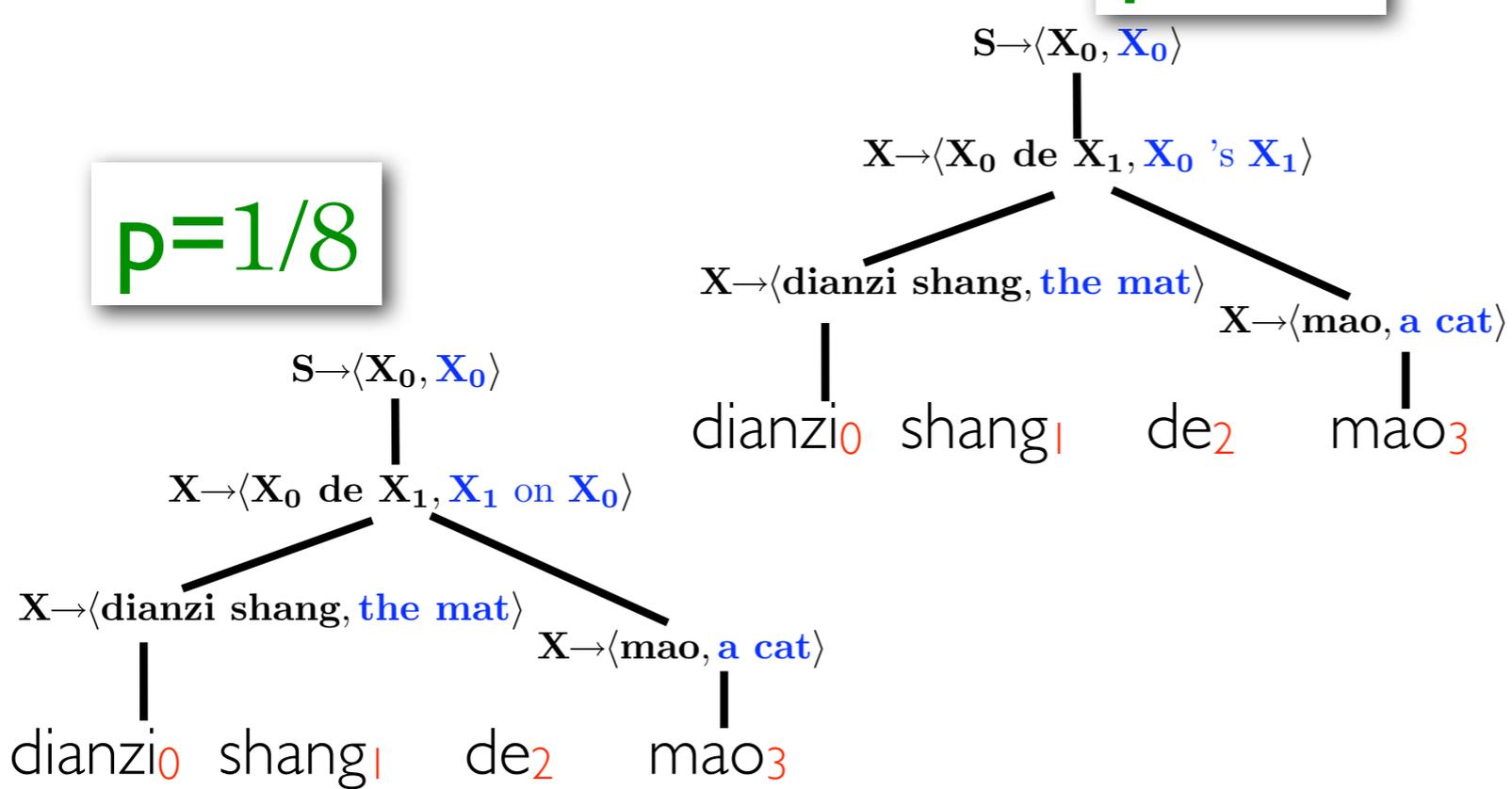


$p=3/8$

$p=2/8$

$p=1/8$

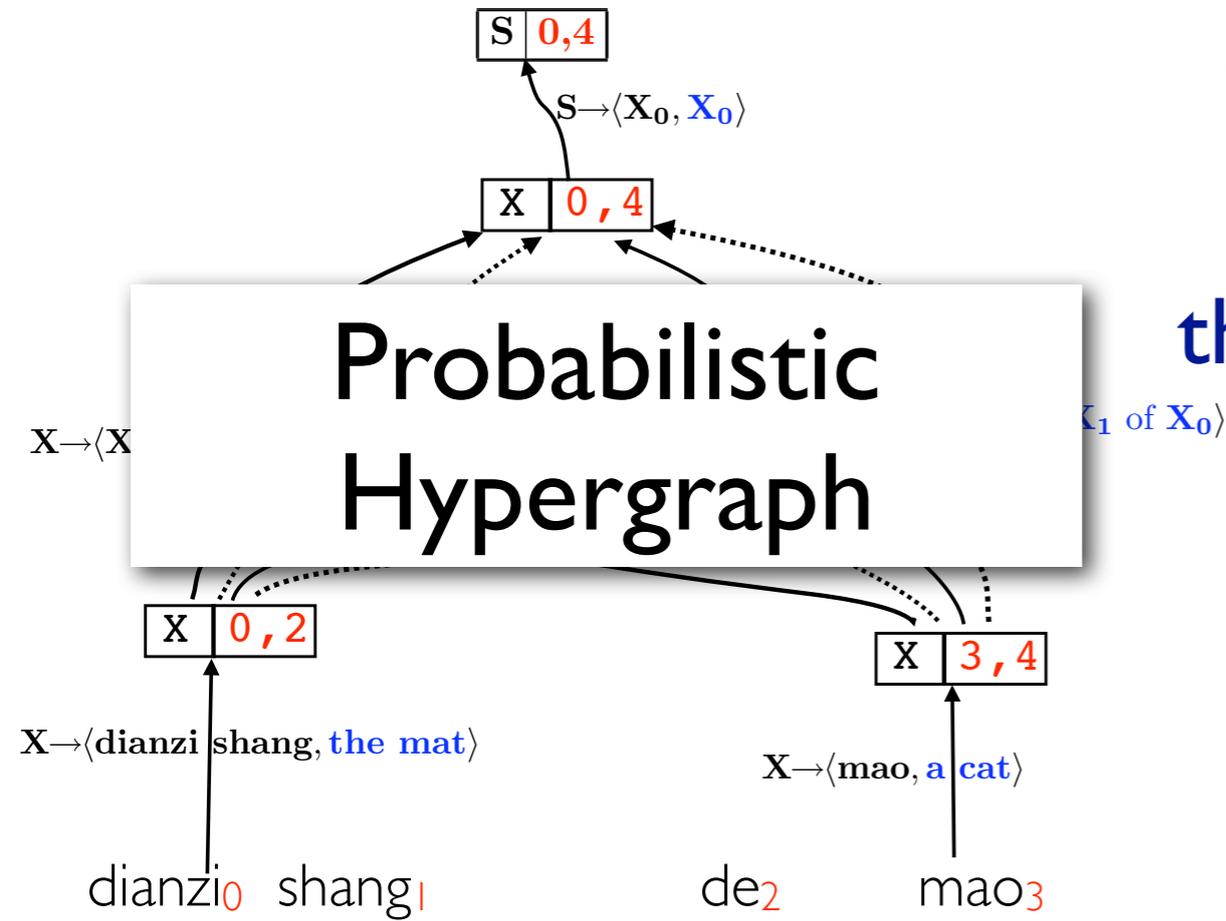
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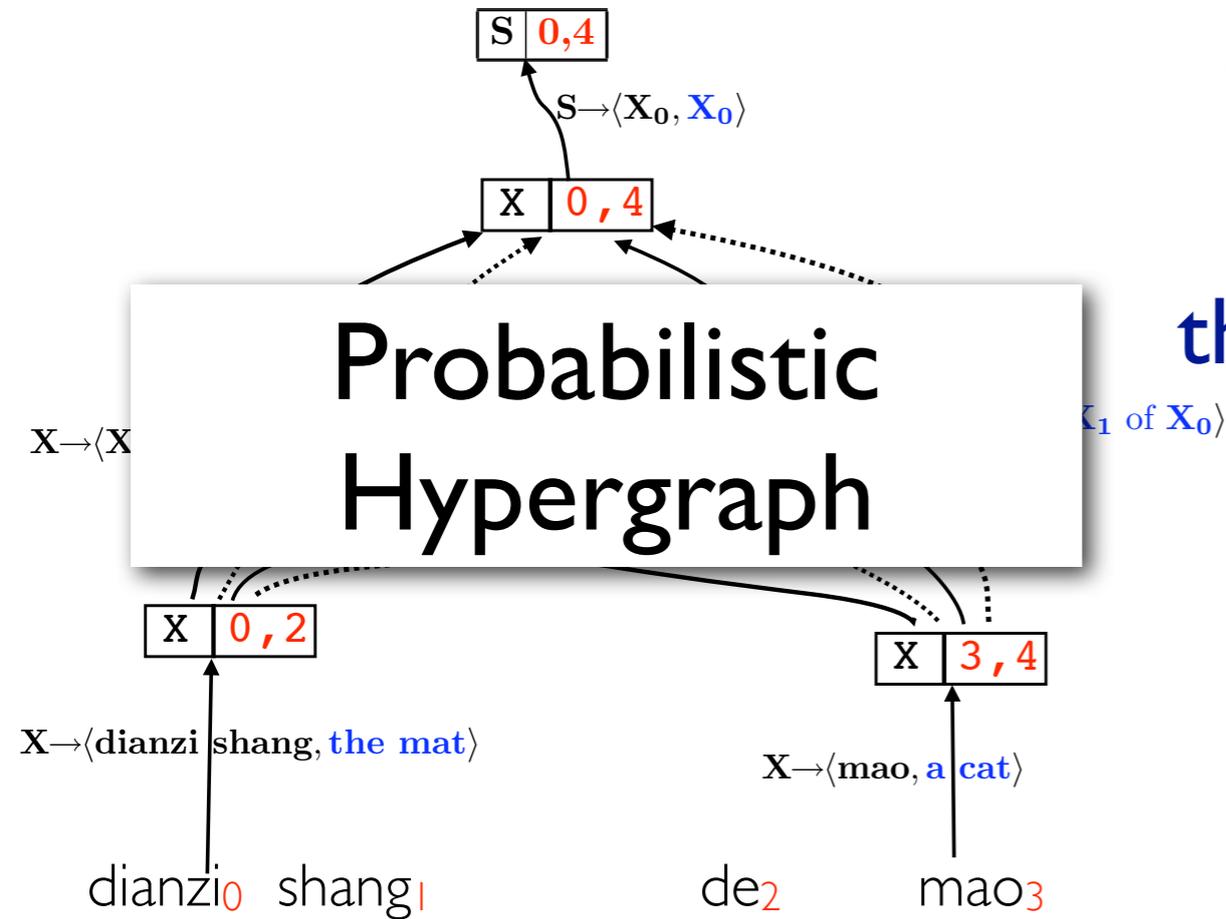
# Probabilistic Hypergraph



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# Probabilistic Hypergraph



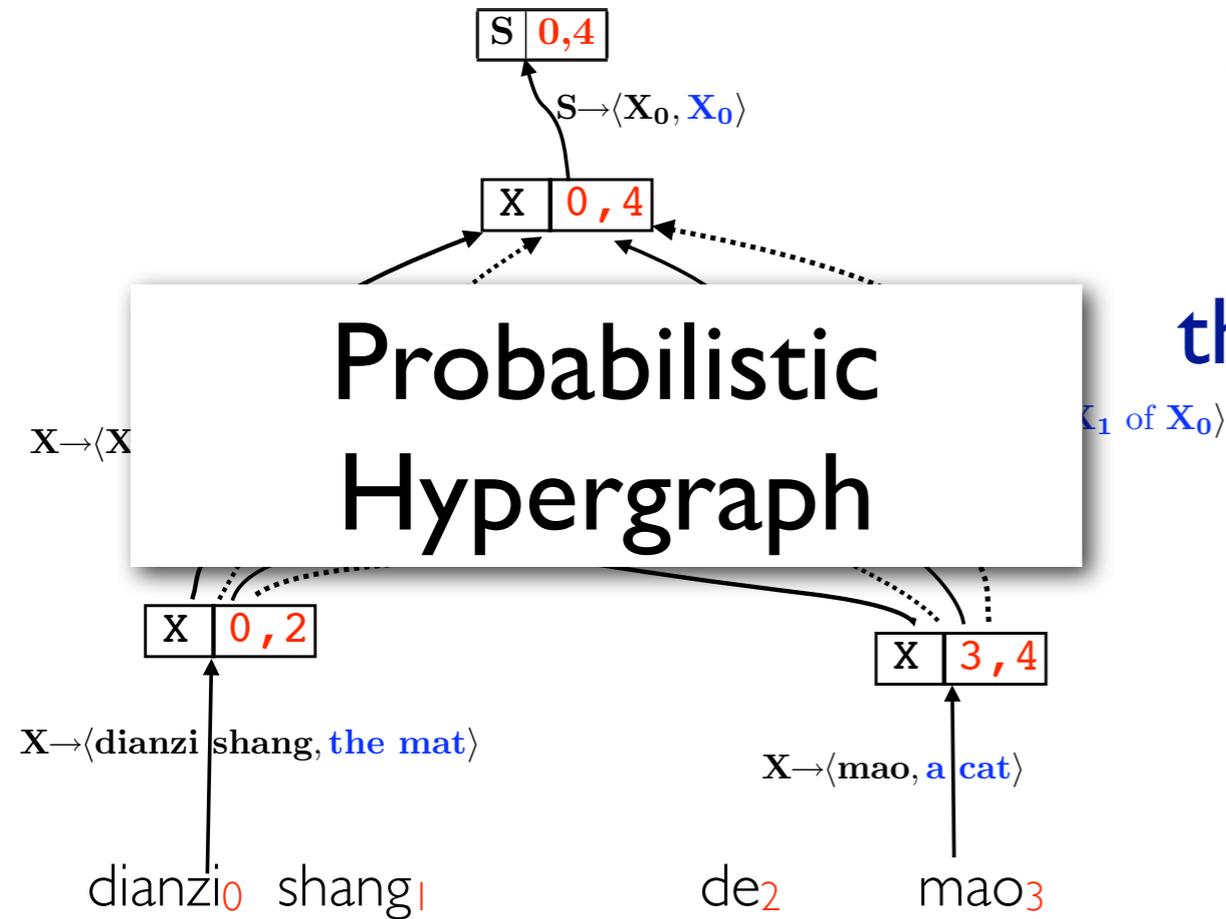
Which translation do we present to a user?

Decoding

The hypergraph defines a probability distribution over **trees!**

the distribution is parameterized by  $\Theta$

# Probabilistic Hypergraph



Which translation do we present to a user?

How do we set the parameters  $\Theta$ ?

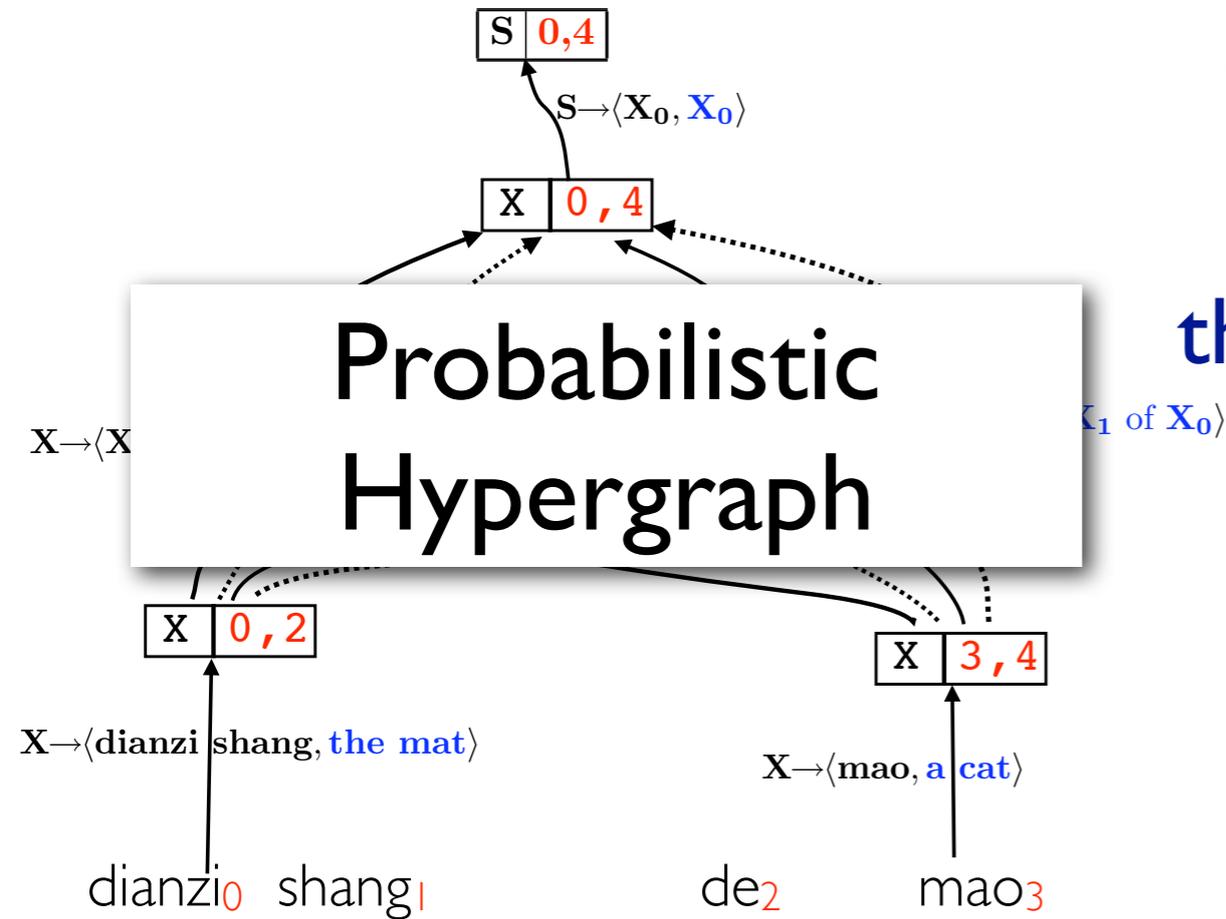
Decoding

Training

The hypergraph defines a probability distribution over **trees!**

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# Probabilistic Hypergraph



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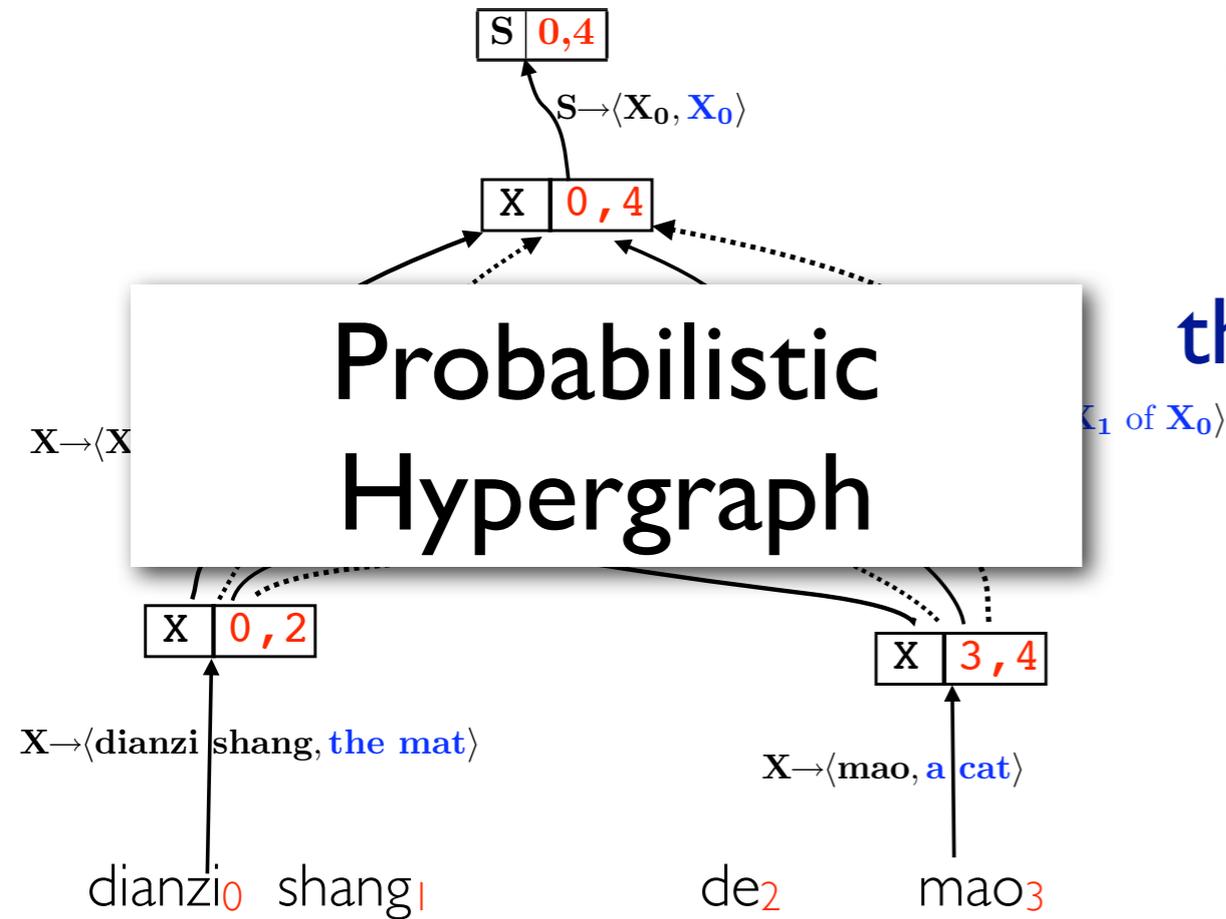
How do we set the parameters  $\Theta$ ?

Training

What atomic operations do we need to perform? Atomic Inference

The hypergraph defines a probability distribution over **trees!**

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<b>training</b> (e.g., mert)	<b>decoding</b> (e.g., mbr)
<b>atomic inference operations</b> (e.g., finding one-best, k-best or expectation, inference can be <i>exact</i> or <i>approximate</i> )	

Which translation do we present to a user?

Decoding

How do we set the parameters  $\Theta$ ?

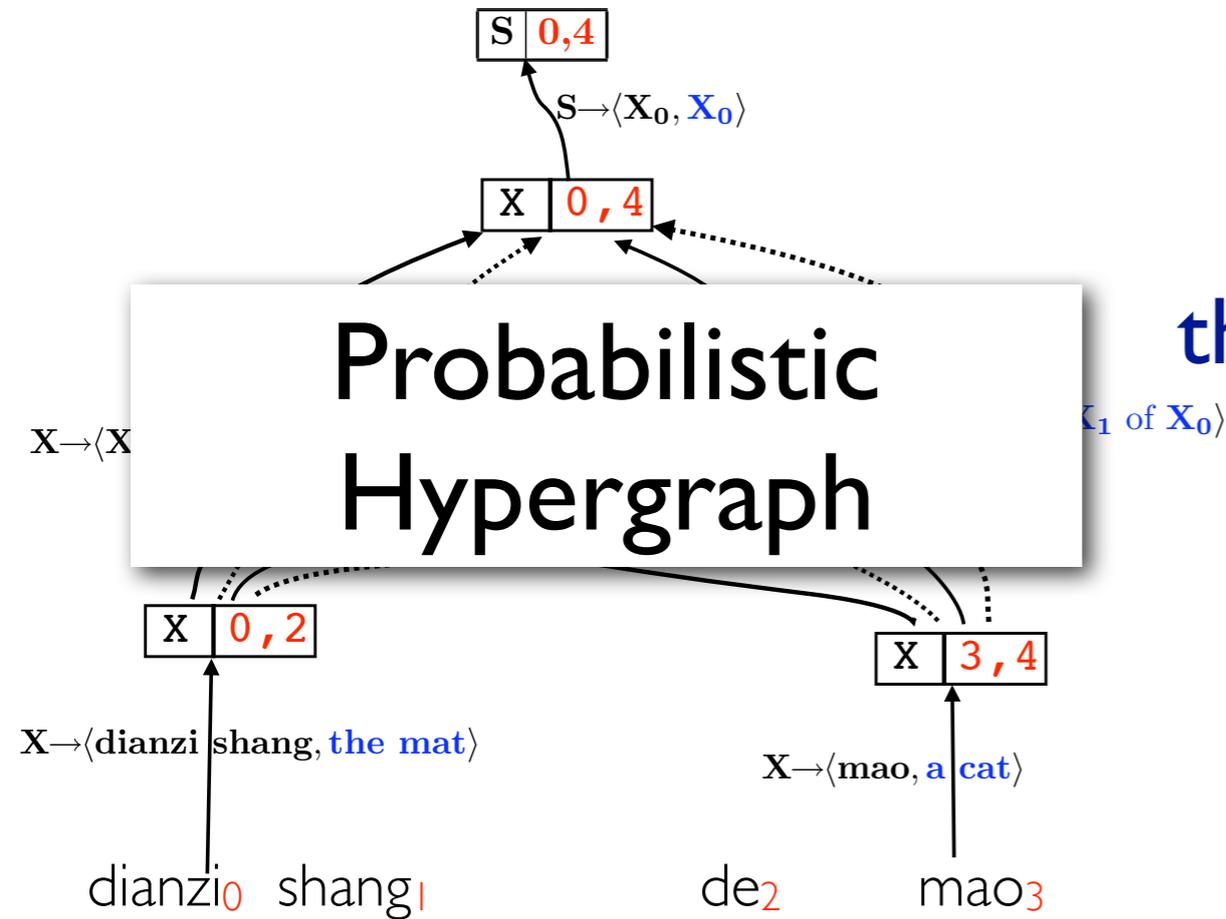
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# Probabilistic Hypergraph



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Decoding

How do we set the parameters  $\Theta$ ?

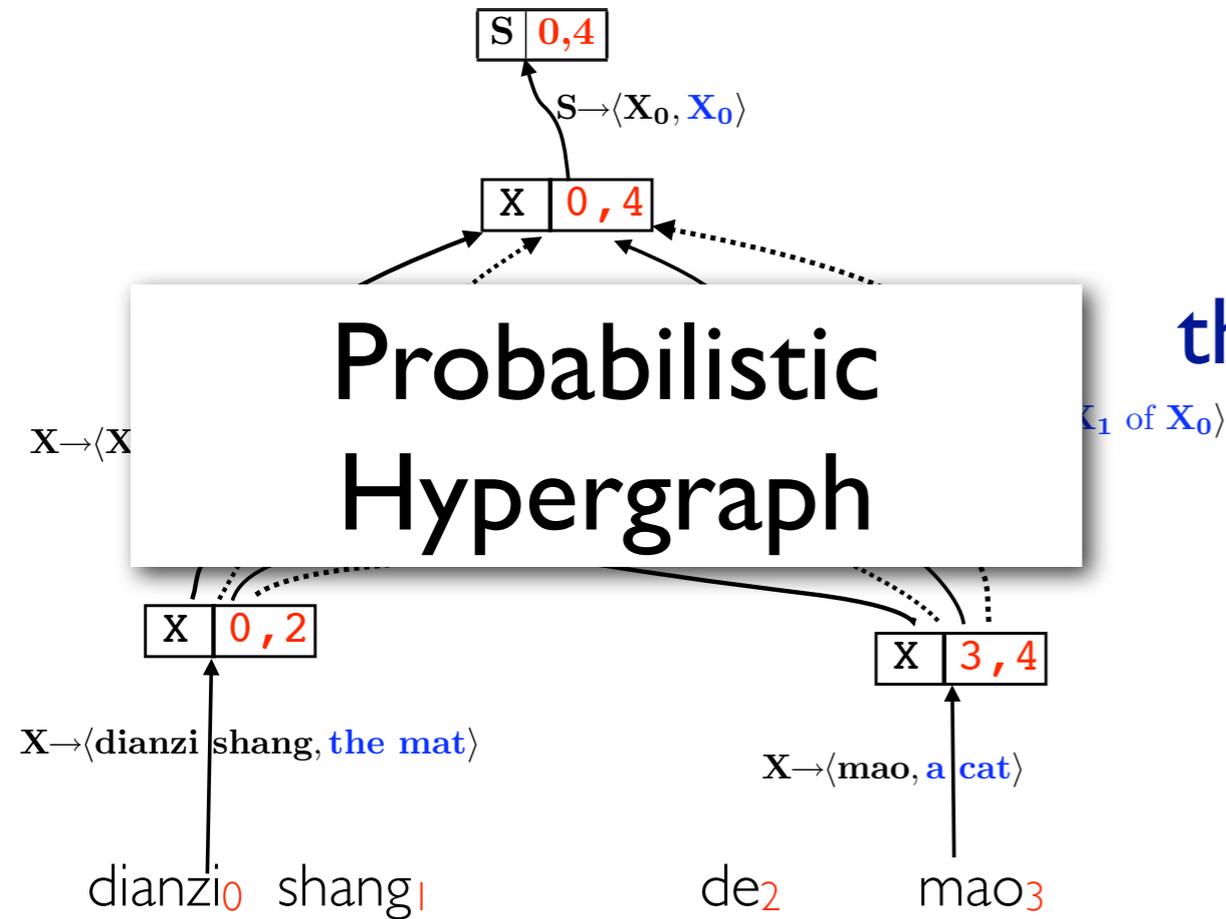
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What atomic operations do we need to perform? Atomic Inference

Why are the problems difficult?

The hypergraph defines a probability distribution over **trees!**

the distribution is parameterized by  $\Theta$



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Decoding

How do we set the parameters  $\Theta$ ?

Training

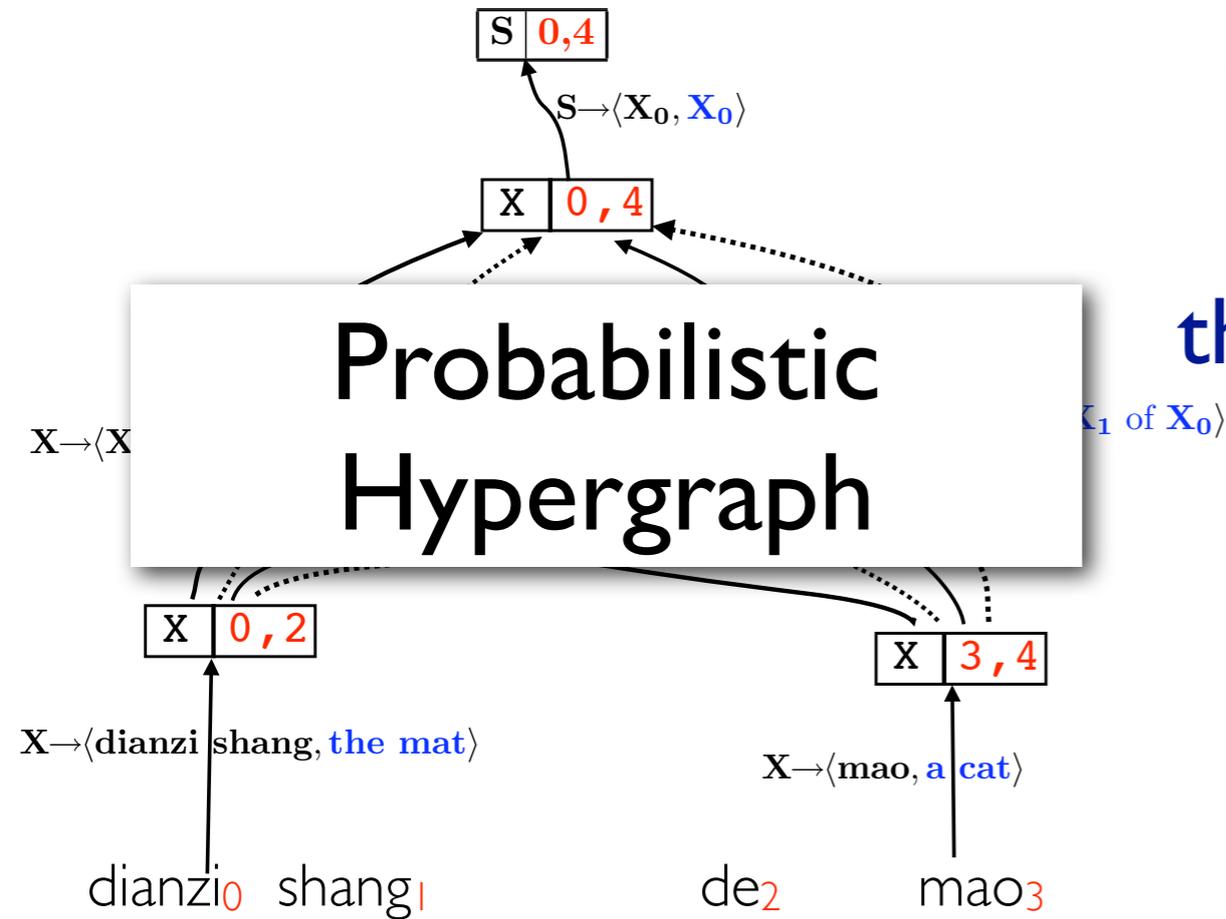
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Why are the problems difficult?

- brute-force will be too slow as there are exponentially many trees, so require sophisticated dynamic programs

The hypergraph defines a probability distribution over **trees!**

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<b>training</b> (e.g., mert)	<b>decoding</b> (e.g., mbr)
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Which translation do we present to a user?

Decoding

How do we set the parameters  $\Theta$ ?

Training

What atomic operations do we need to perform? Atomic Inference

Why are the problems difficult?

- brute-force will be too slow as there are exponentially many trees, so require sophisticated dynamic programs
- sometimes intractable, require approximations

# Inference, Training and Decoding on Hypergraphs

- Atomic Inference

- finding one-best derivations

Graph	Topological	Best-first		
		no heuristic	with heuristic	with hierarchy
FSA	Viterbi	Dijkstra	$A^*$	$HA^*$
Hypergraph	CYK	Knuth	Klein and Manning	Generalized $A^*$

- finding k-best derivations
- computing expectations (e.g., of features)

- Training

- Perceptron, conditional random field (CRF), minimum error rate training (MERT), minimum risk, and MIRA

- Decoding

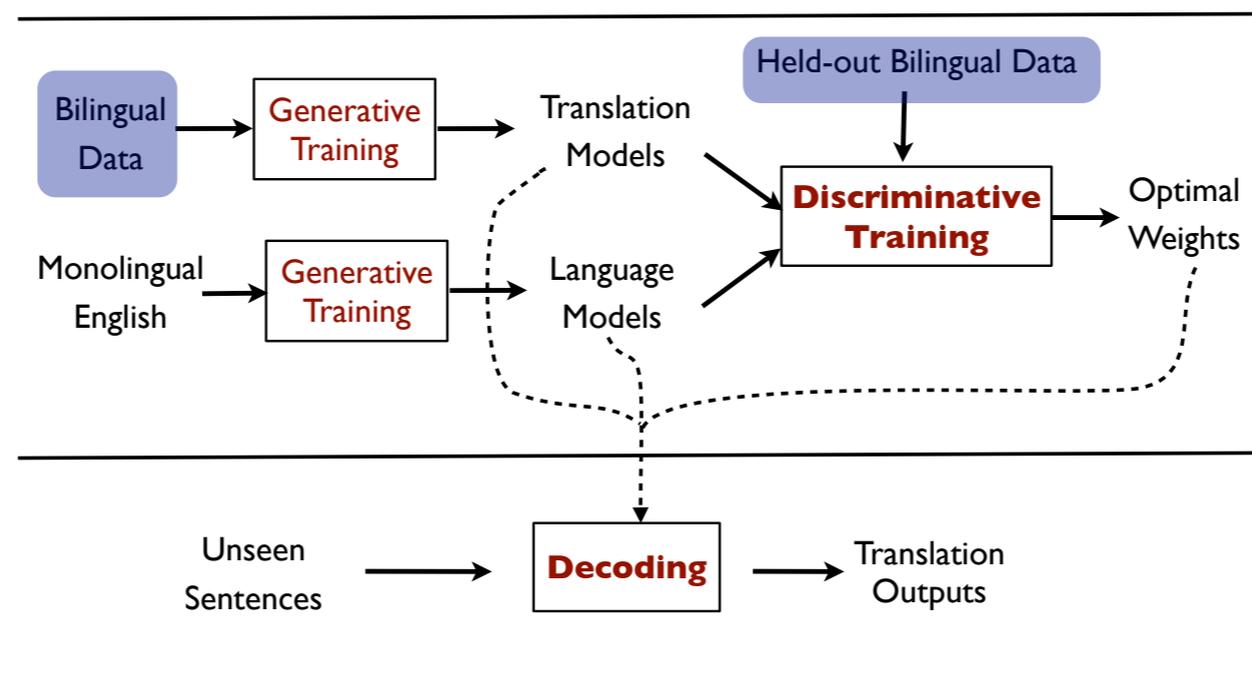
- Viterbi decoding, maximum a posterior (MAP) decoding, and minimum Bayes risk (MBR) decoding

# Outline

- Hypergraph as Hypothesis Space
- Unsupervised Discriminative Training
  - ▶ minimum imputed risk
  - ▶ contrastive language model estimation
- Variational Decoding
- First- and Second-order Expectation Semirings

# Outline

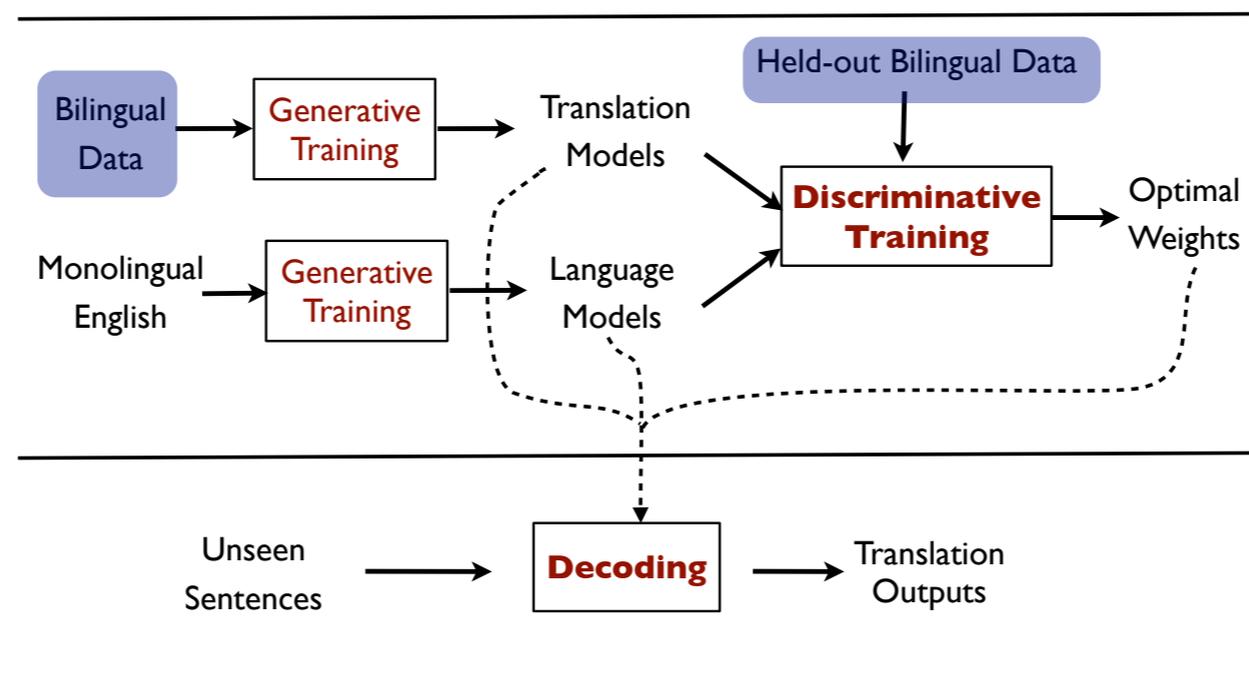
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# Outline

- Hypergraph as Hypothesis Space
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  - ▶ minimum imputed risk
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main  
focus

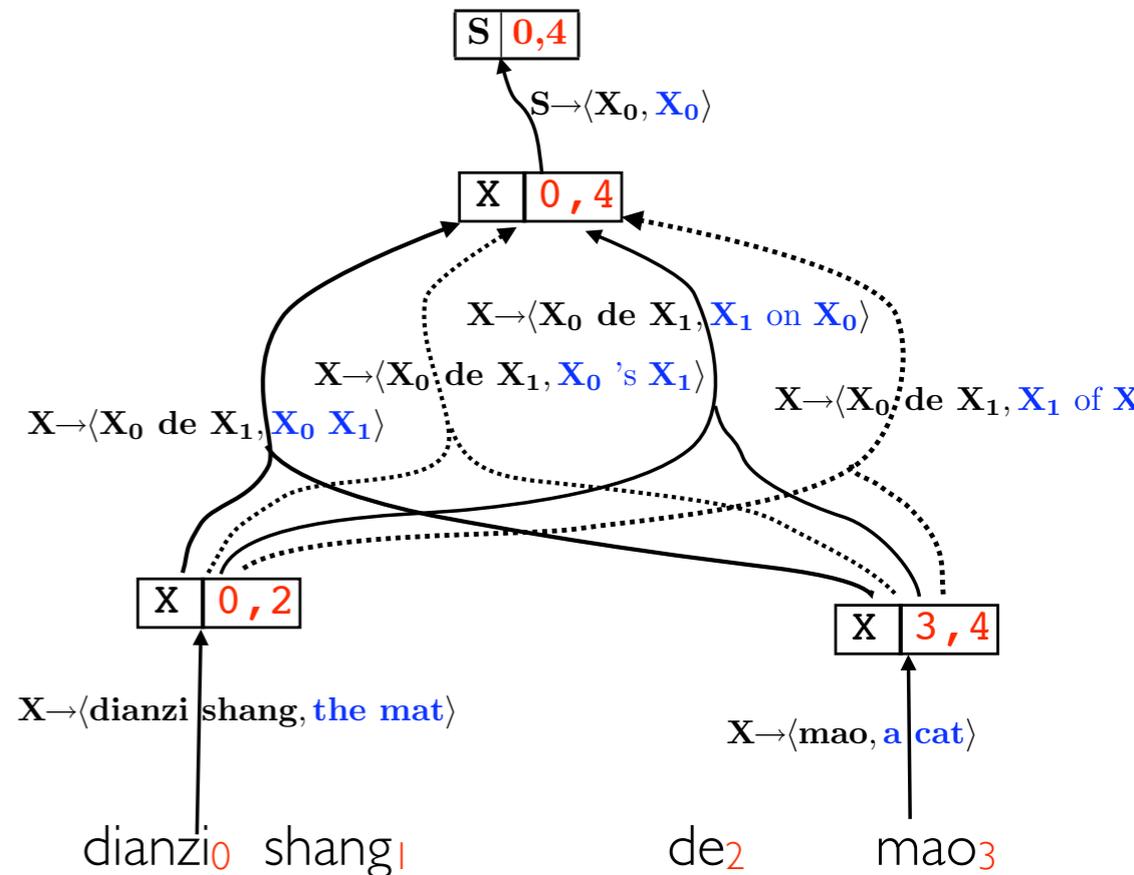


# Training Setup

- Each **training example** consists of
  - a foreign sentence (from which a **hypergraph** is generated to represent many possible translations)
  - a reference translation

**x:** dianzi shang de mao

**y:** a cat on the mat



## • Training

- adjust the parameters  $\Theta$  so that the reference translation is preferred by the model

# Supervised: Minimum Empirical Risk

# Supervised: Minimum Empirical Risk

- Minimum Empirical Risk Training

$$\theta^* = \arg \min_{\theta} \sum_{x,y} \tilde{p}(x,y) L(\delta_{\theta}(x), y)$$

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empirical  
distribution



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empirical  
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x

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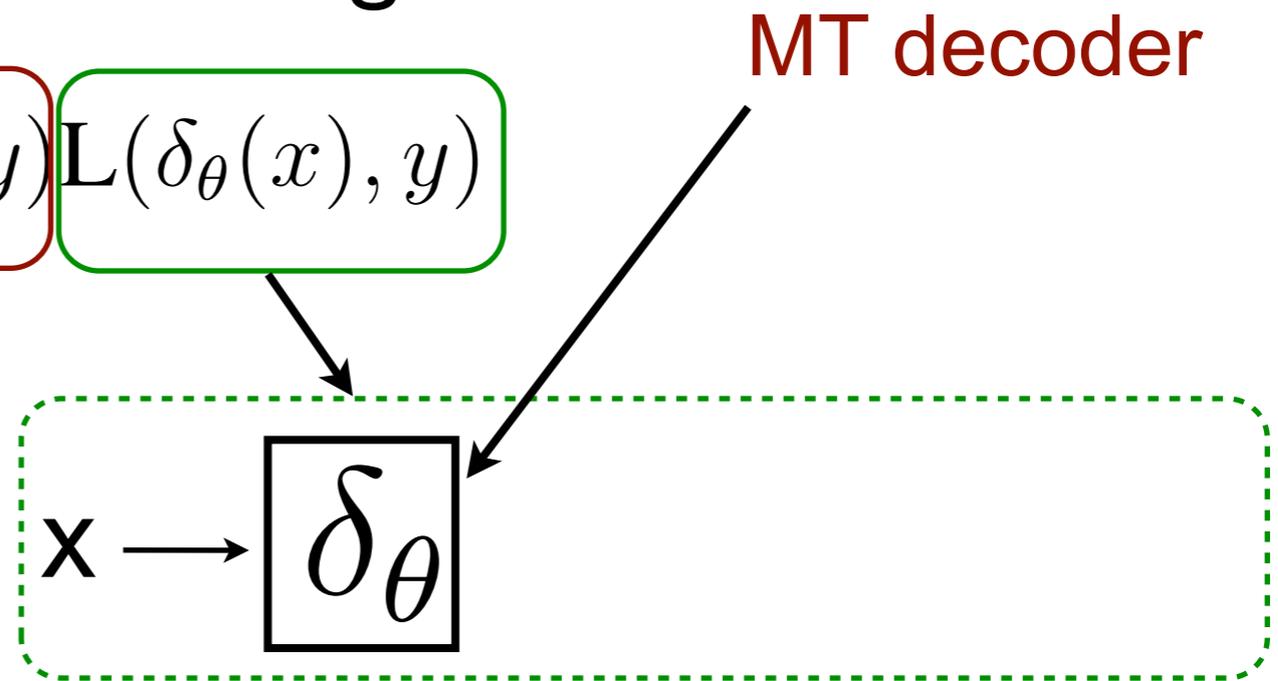


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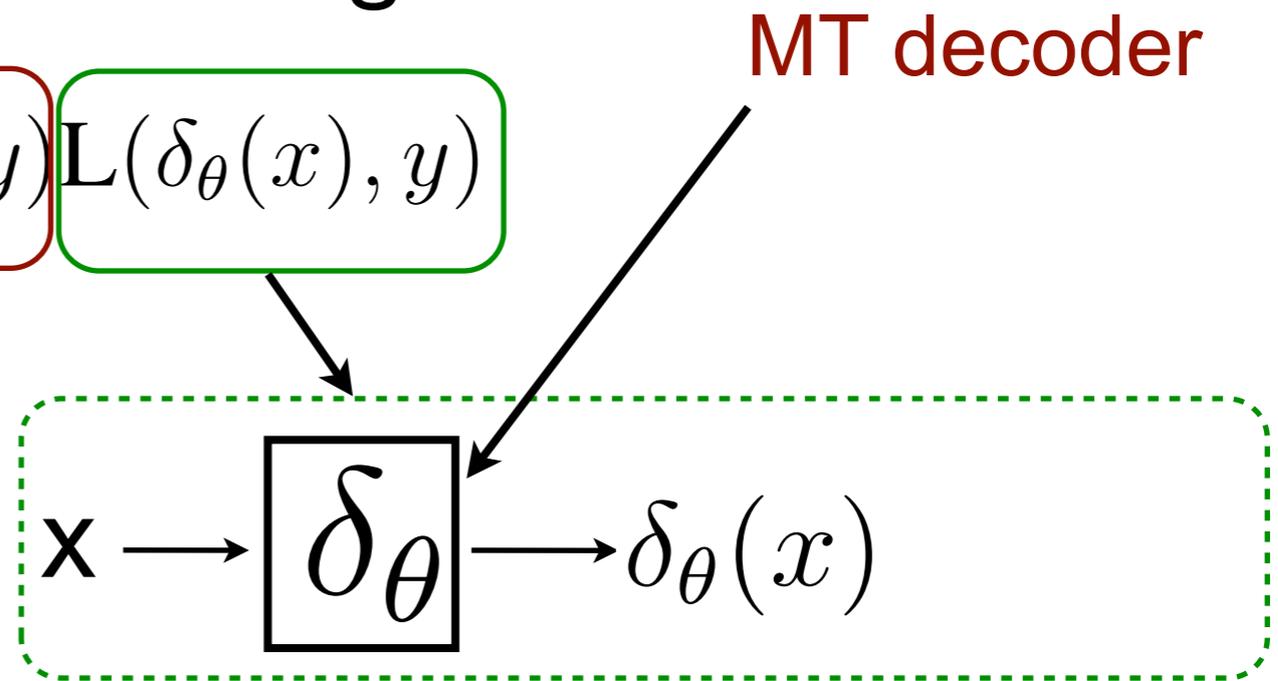


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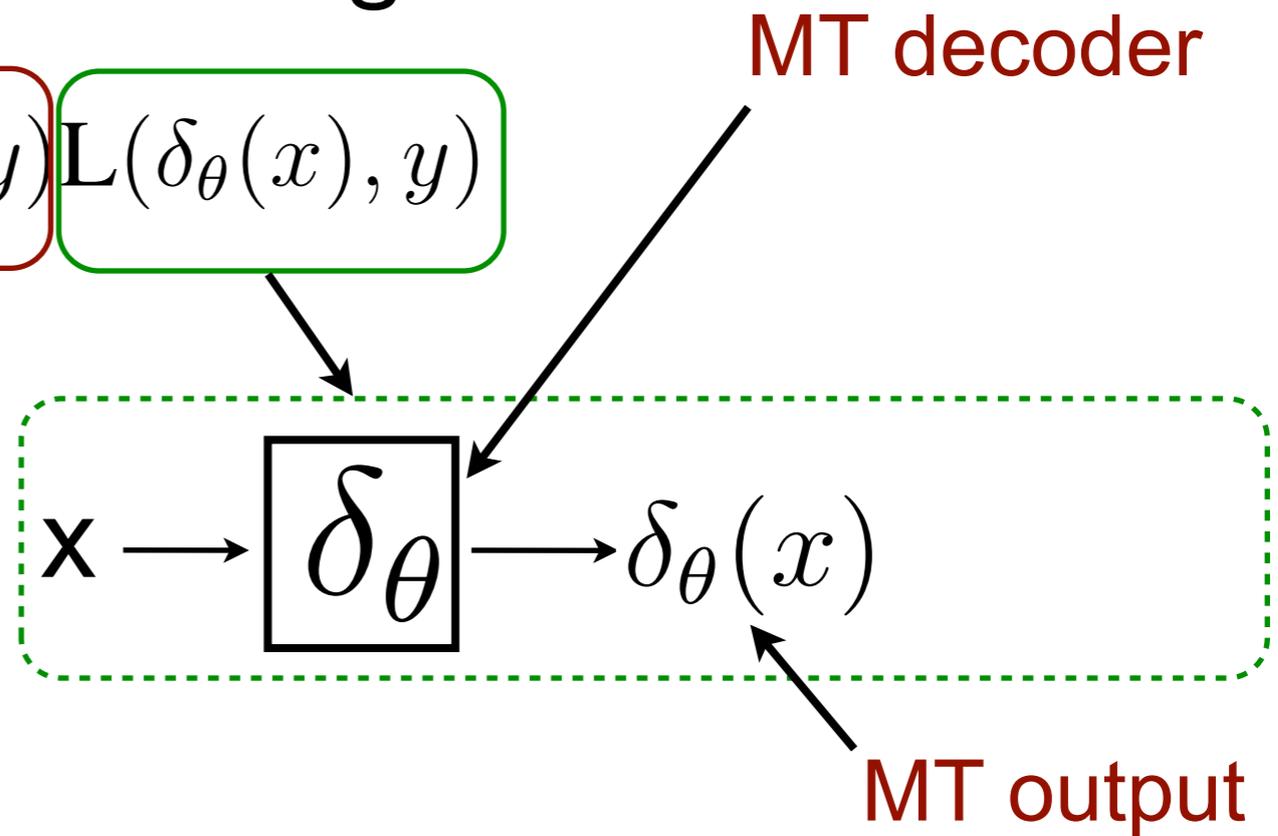


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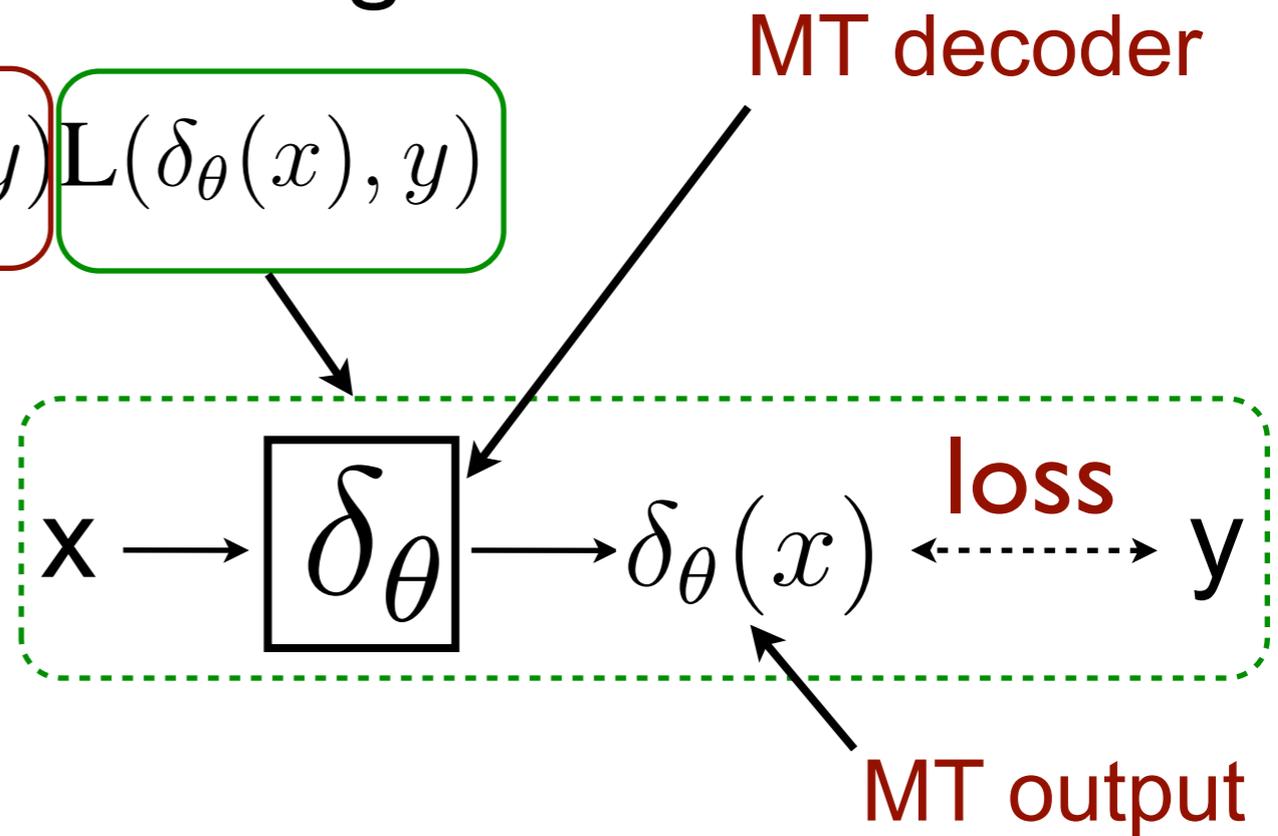


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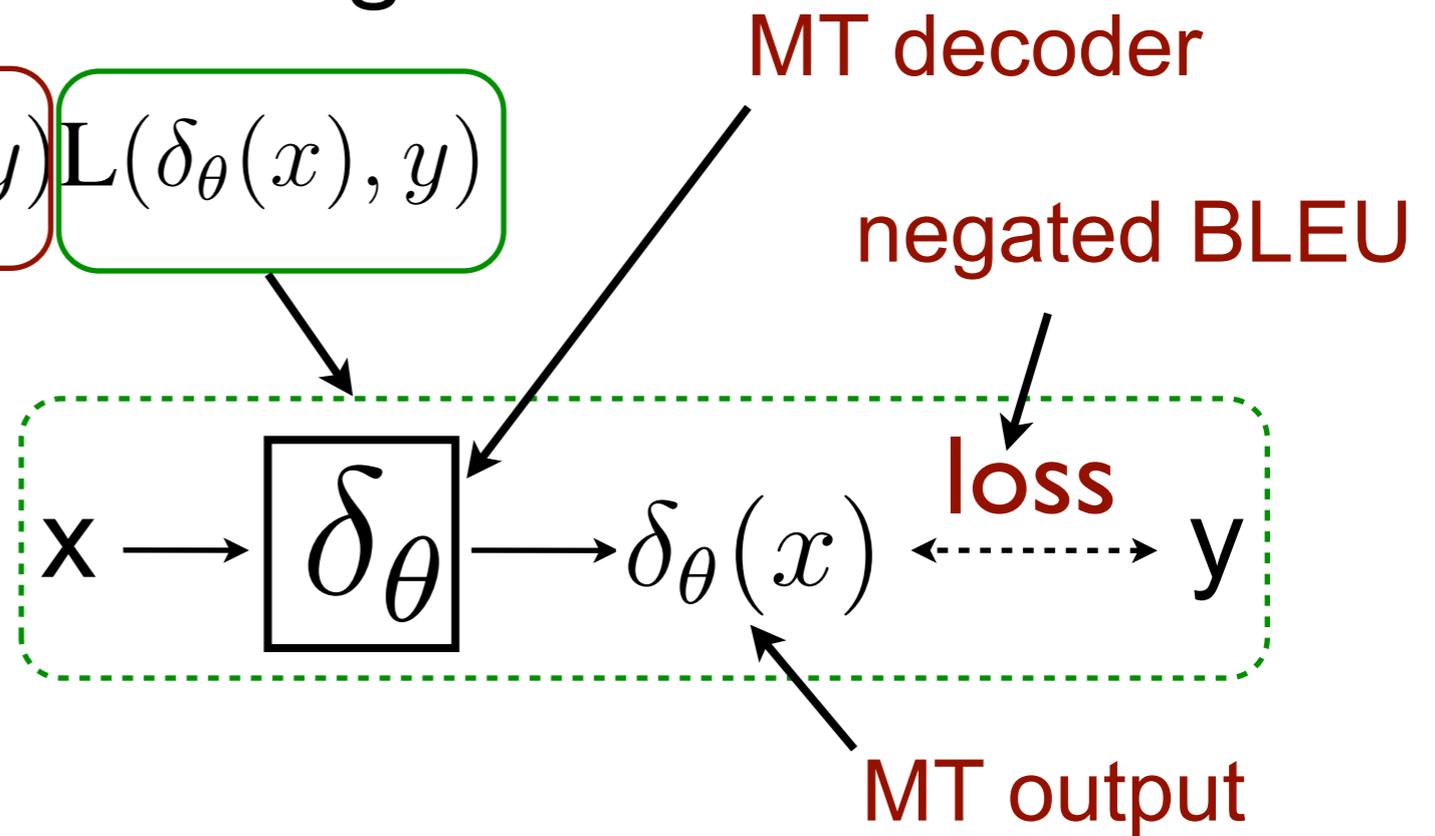


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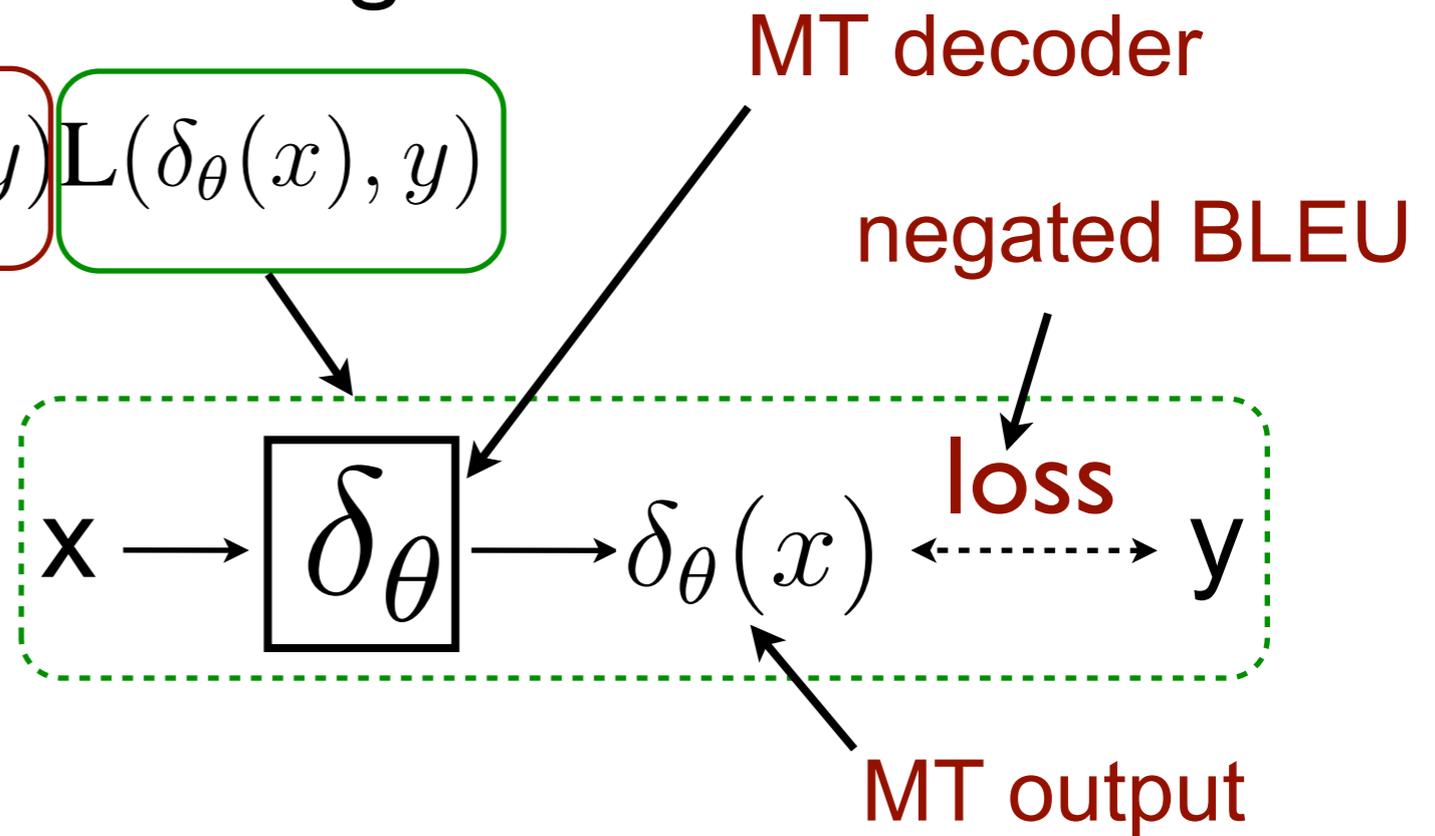


# Supervised: Minimum Empirical Risk

- Minimum Empirical Risk Training

$$\theta^* = \arg \min_{\theta} \sum_{x,y} \tilde{p}(x,y) \mathbf{L}(\delta_{\theta}(x), y)$$

empirical distribution



- Uniform Empirical Distribution

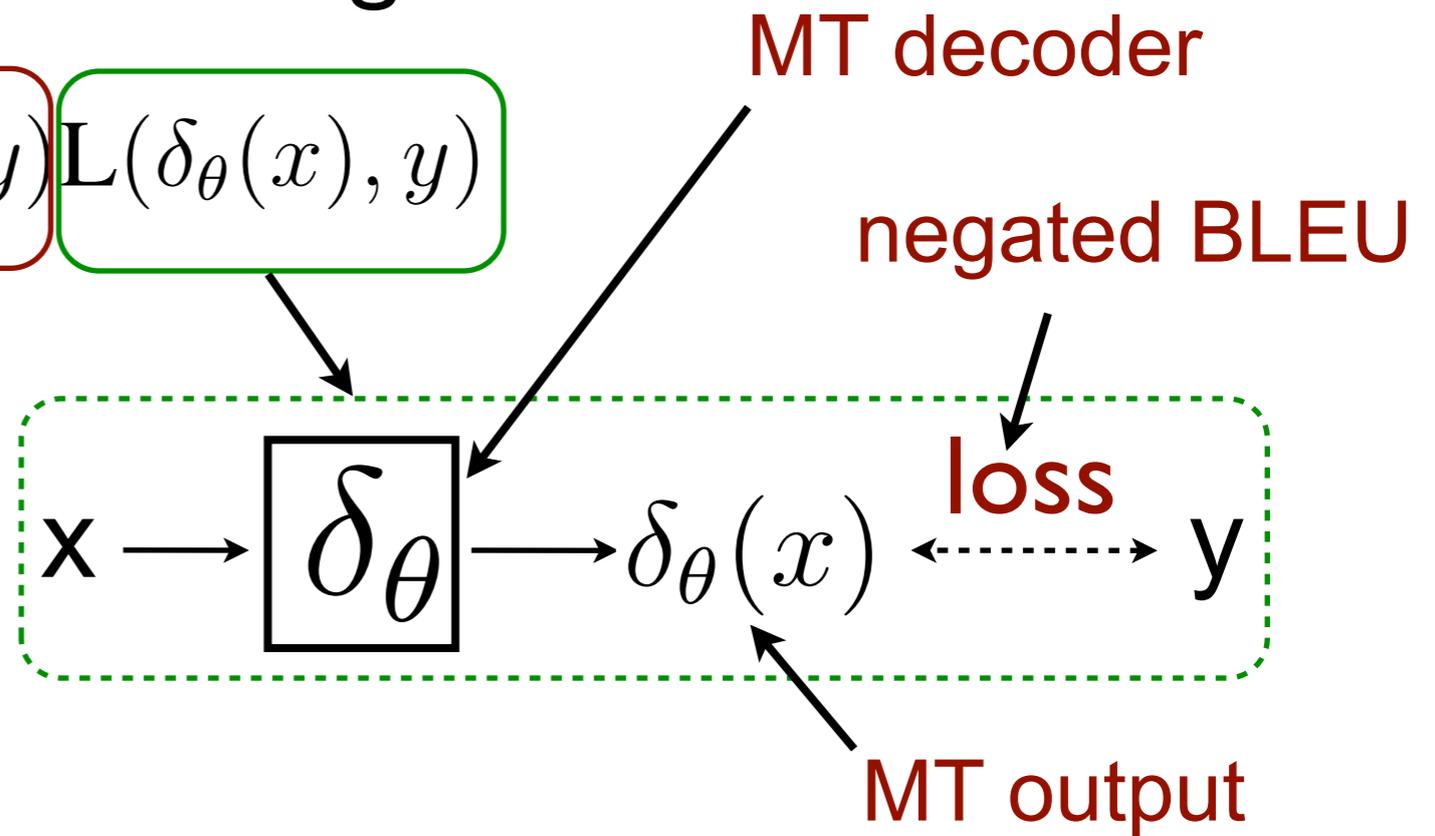
$$\theta^* = \arg \min_{\theta} \frac{1}{N} \sum_{i=1}^N \mathbf{L}(\delta_{\theta}(x_i), \tilde{y}_i)$$

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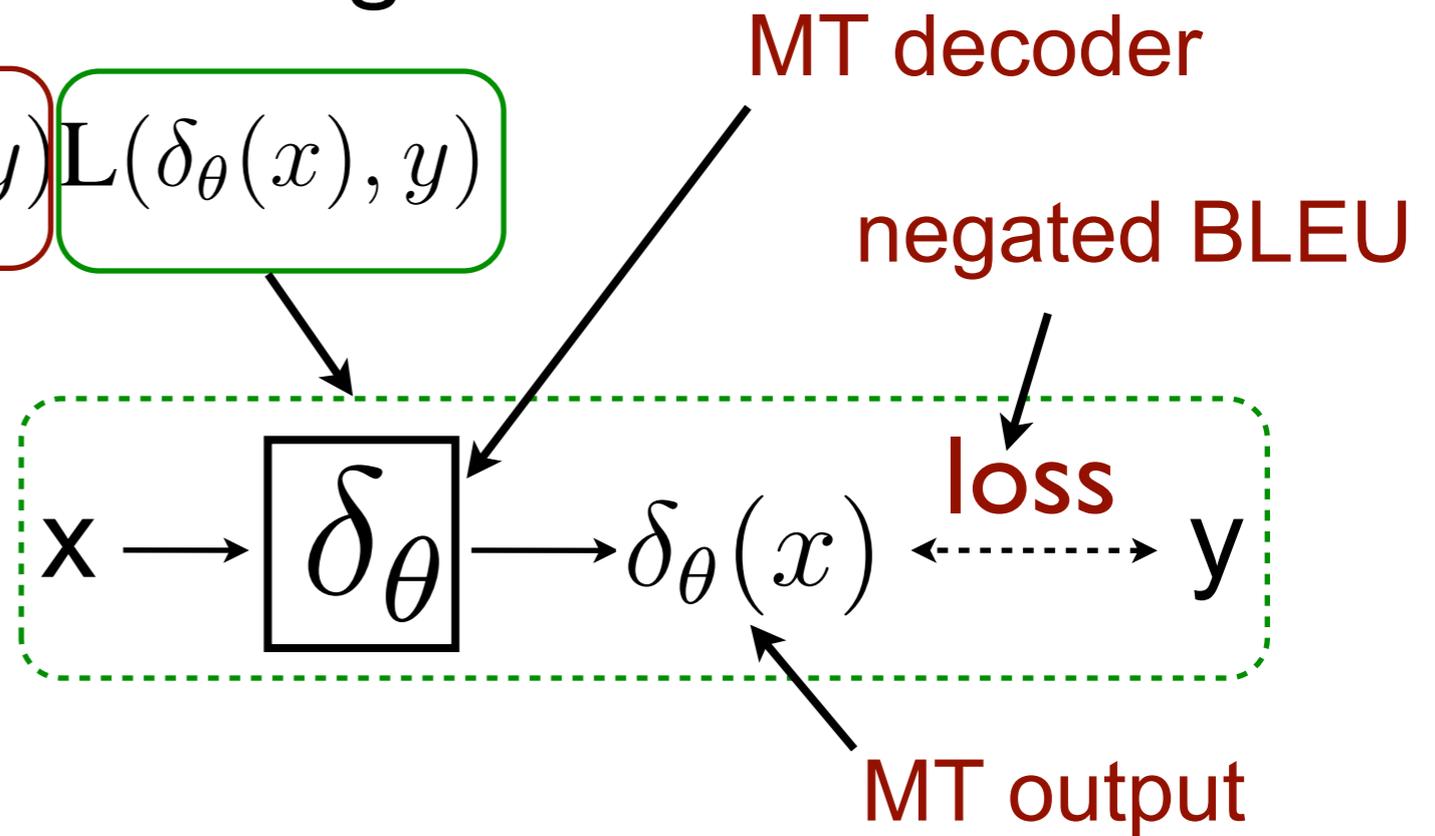
- MERT  
- CRF  
- Peceptron

# Supervised: Minimum Empirical Risk

- Minimum Empirical Risk Training

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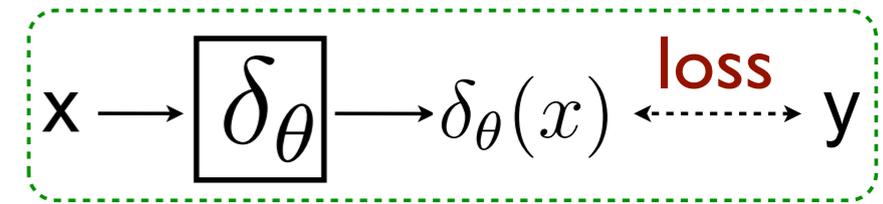
- MERT  
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**What if the input  $x$  is missing?**

# Unsupervised: Minimum Imputed Risk

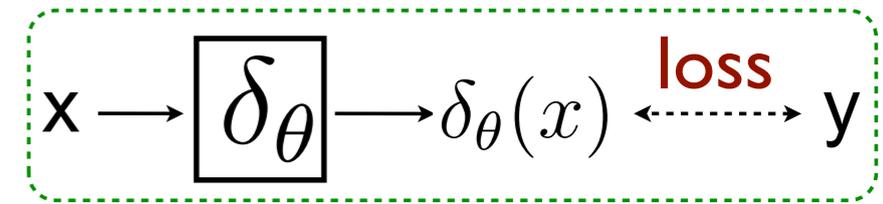
- Minimum Empirical Risk Training

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# Unsupervised: Minimum **Imputed** Risk

- Minimum **Empirical** Risk Training



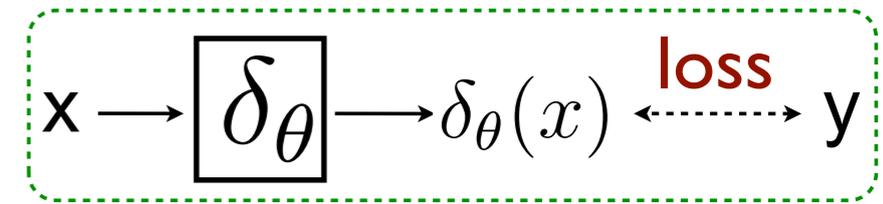
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- Minimum **Imputed** Risk Training

$$\theta^* = \arg \min_{\theta} \frac{1}{N} \sum_{i=1}^N \sum_x p_\phi(x | \tilde{y}_i) \mathbf{L}(\delta_\theta(x), \tilde{y}_i)$$

# Unsupervised: Minimum **Imputed Risk**

- Minimum **Empirical Risk Training**



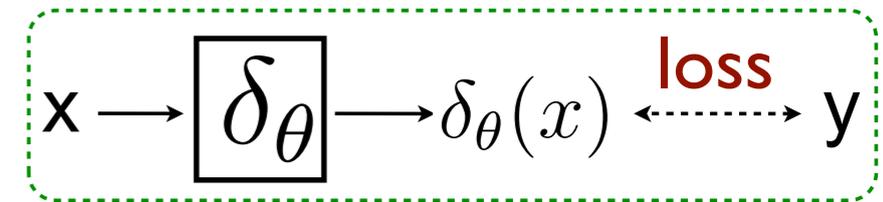
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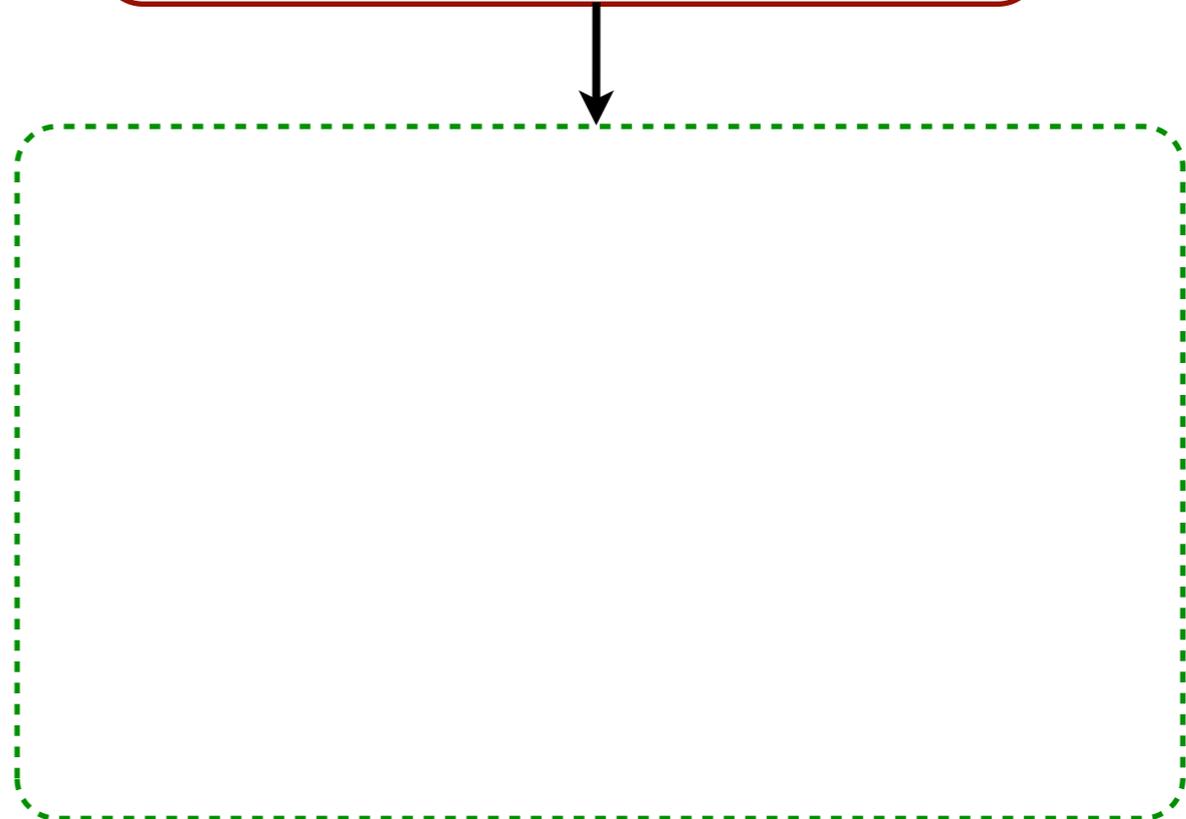
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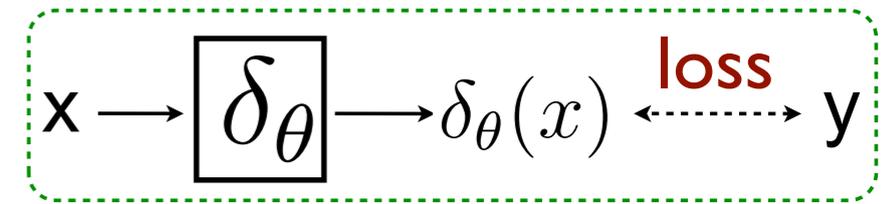
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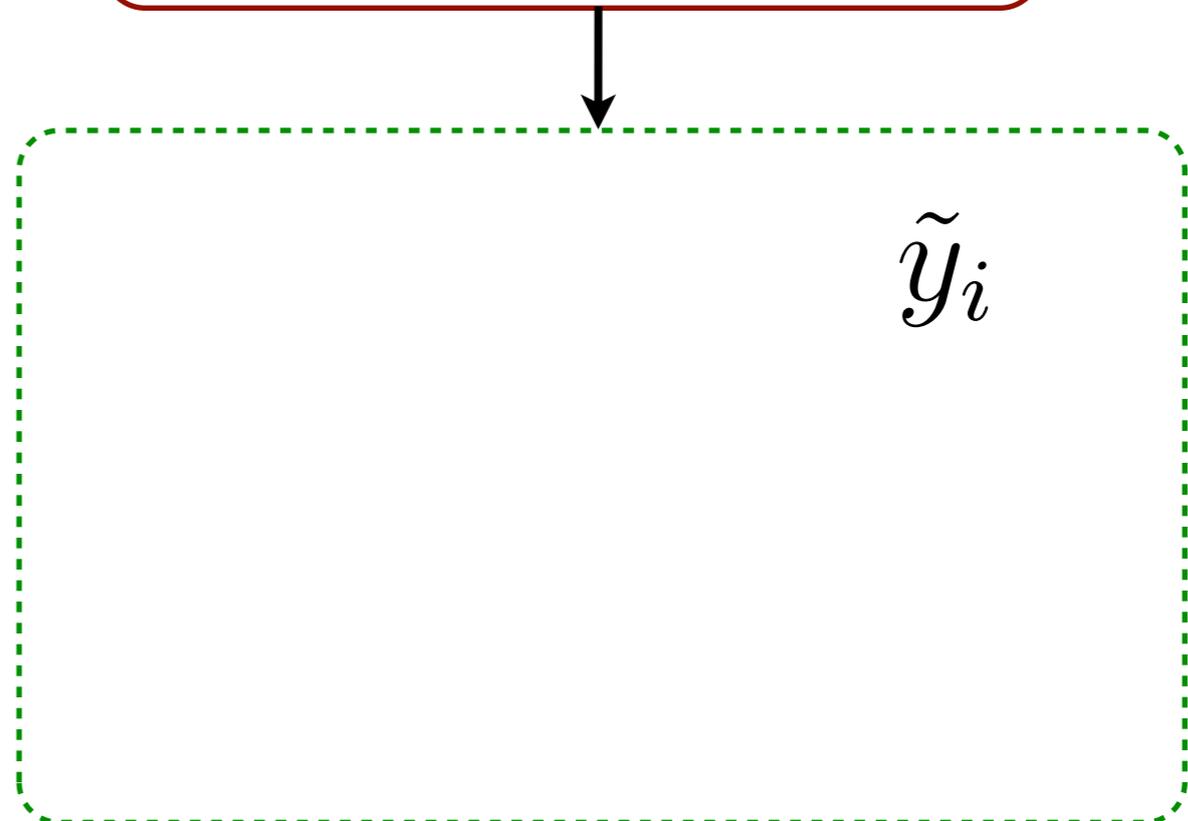
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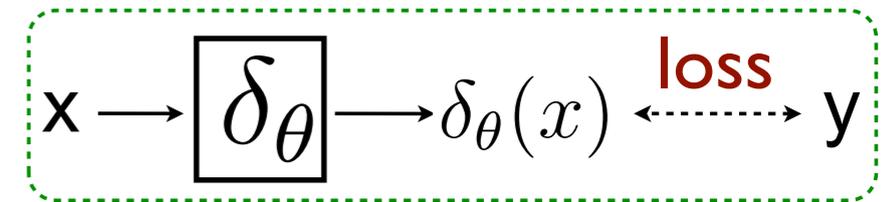
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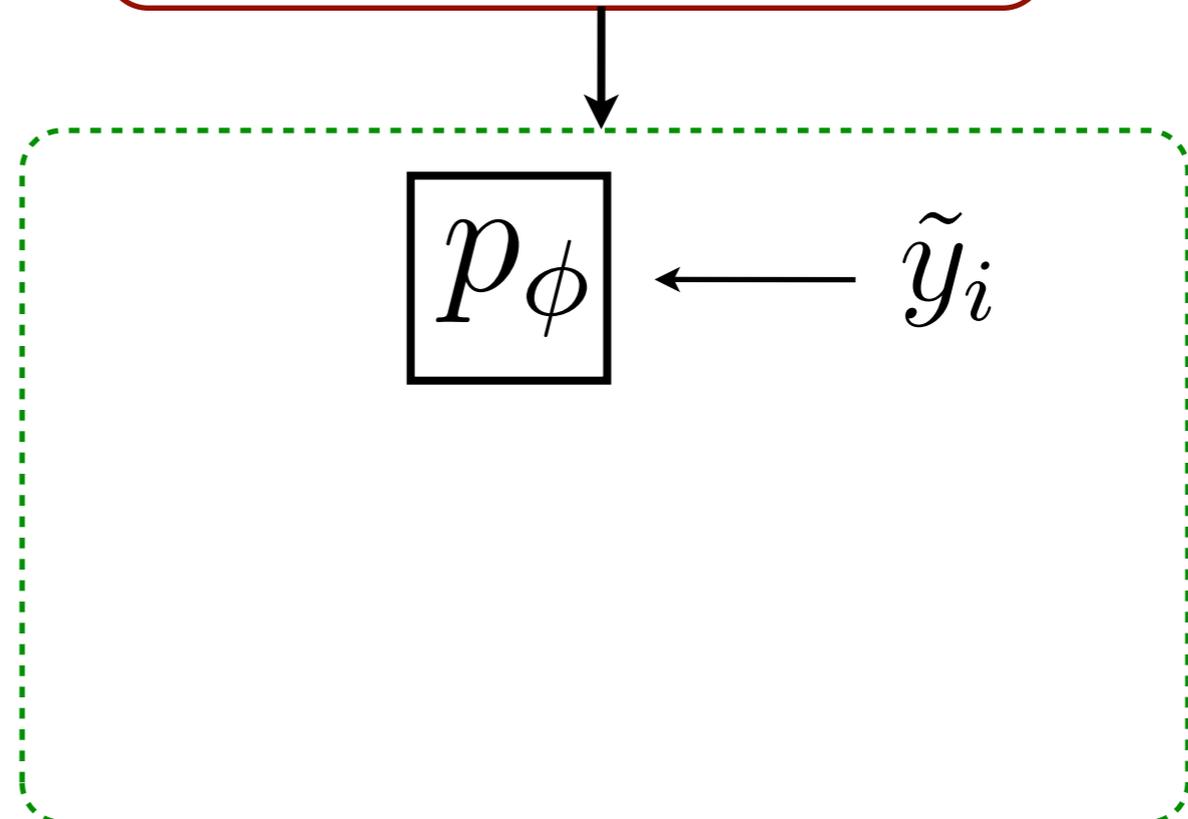


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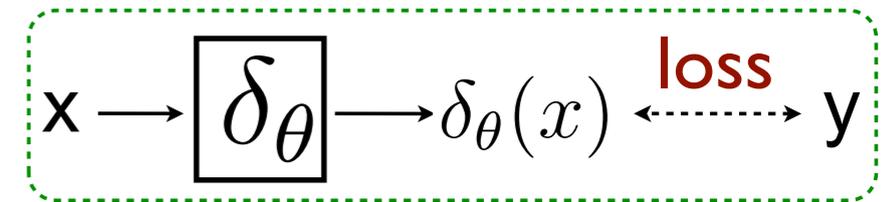
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$p_\phi$ : reverse model



# Unsupervised: Minimum Imputed Risk

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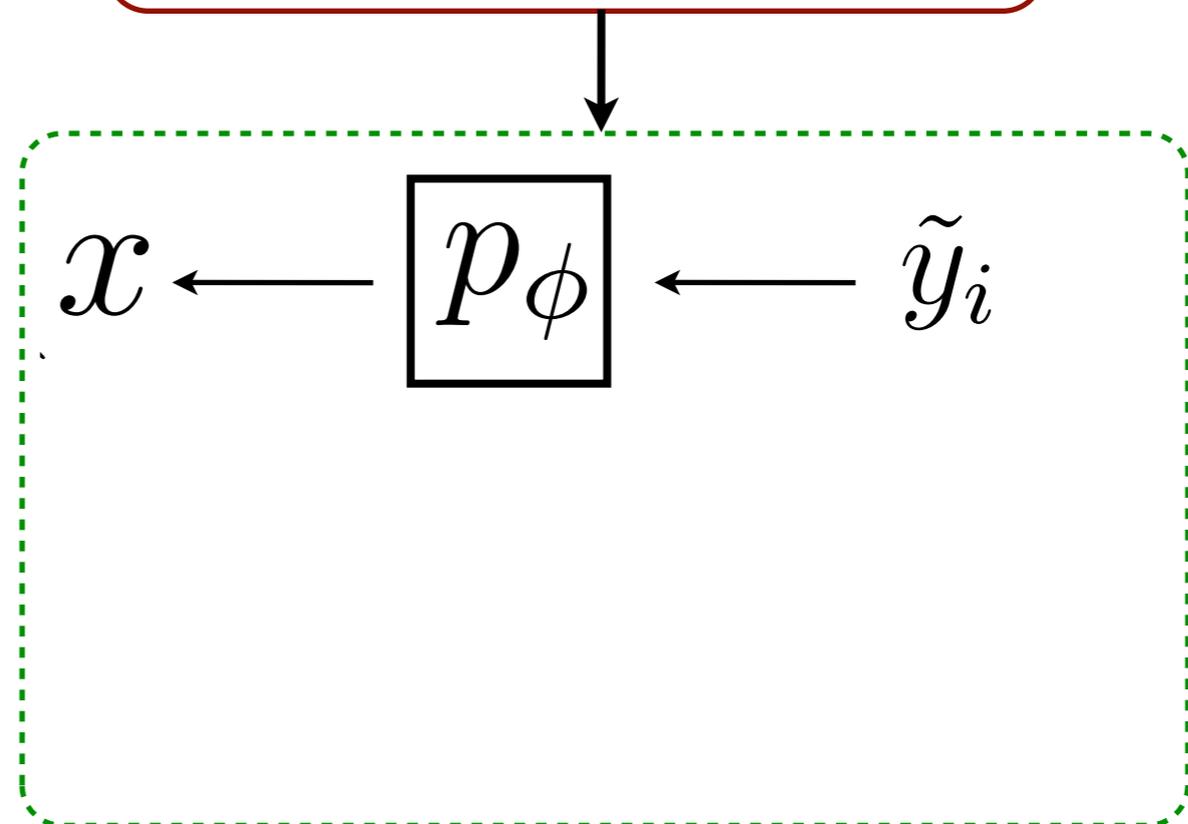
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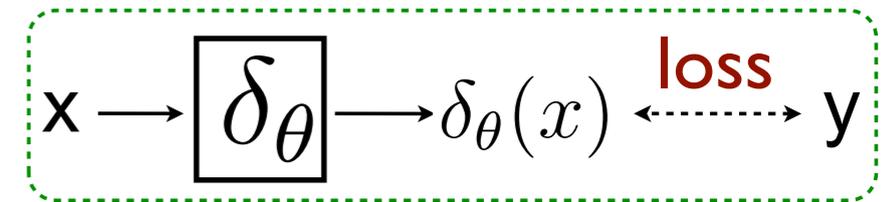
$p_\phi$ : reverse model

$x$ : imputed input



# Unsupervised: Minimum Imputed Risk

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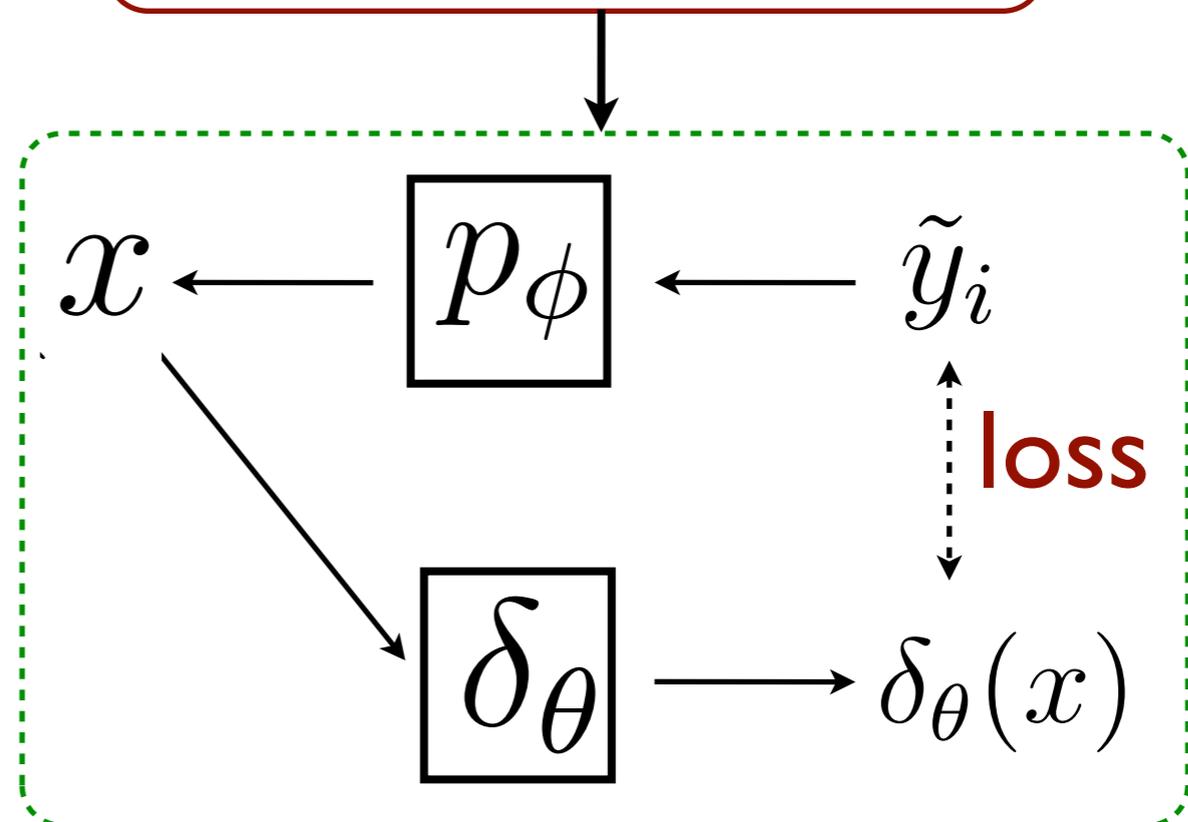
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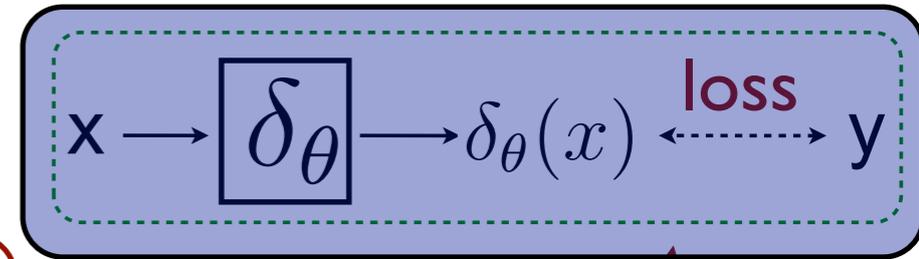
$\delta_\theta$ : forward system



# Unsupervised: Minimum Imputed Risk

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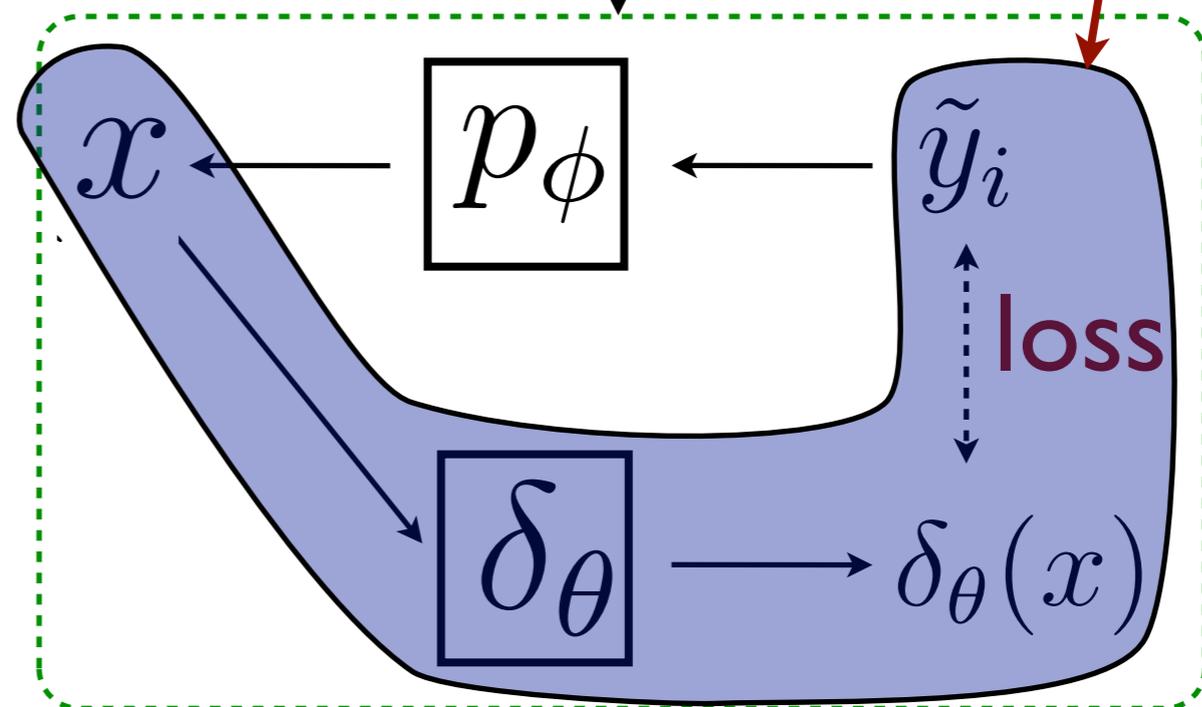
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$p_{\phi}$ : reverse model

$x$ : imputed input

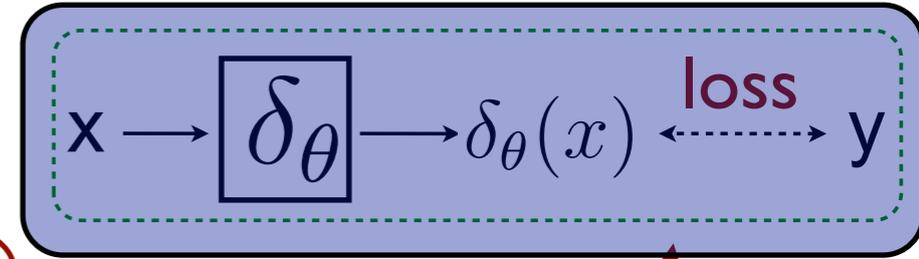
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# Unsupervised: Minimum Imputed Risk

- Minimum **Empirical** Risk Training

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- Minimum **Imputed** Risk Training

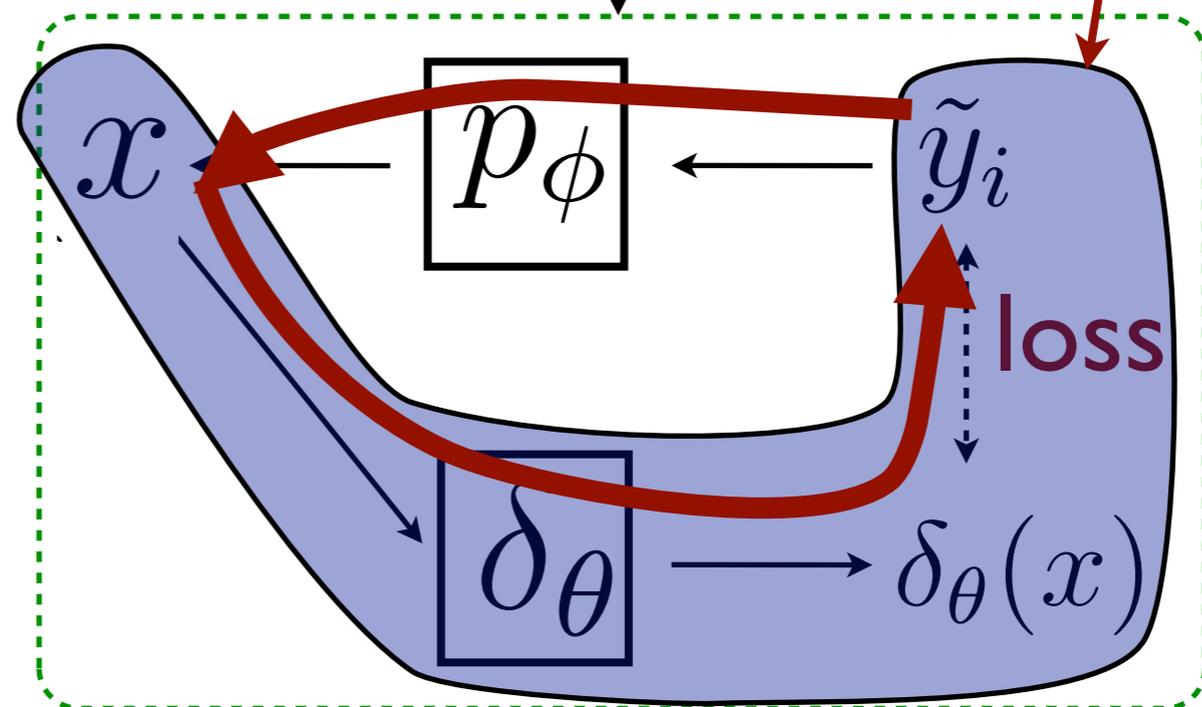
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$p_{\phi}$ : reverse model

$x$ : imputed input

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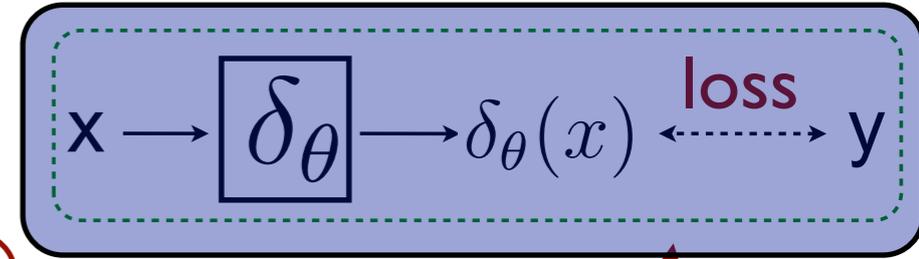
Round trip translation



# Unsupervised: Minimum Imputed Risk

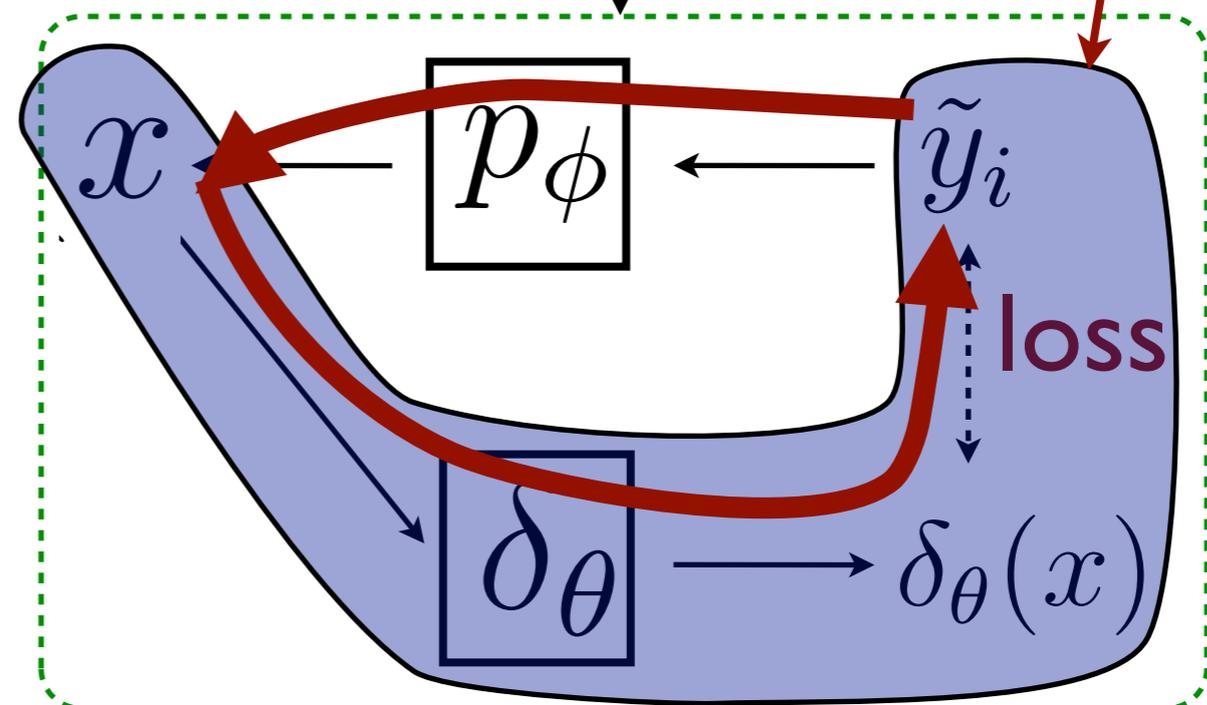
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- Minimum **Imputed** Risk Training

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$p_{\phi}$ : reverse model

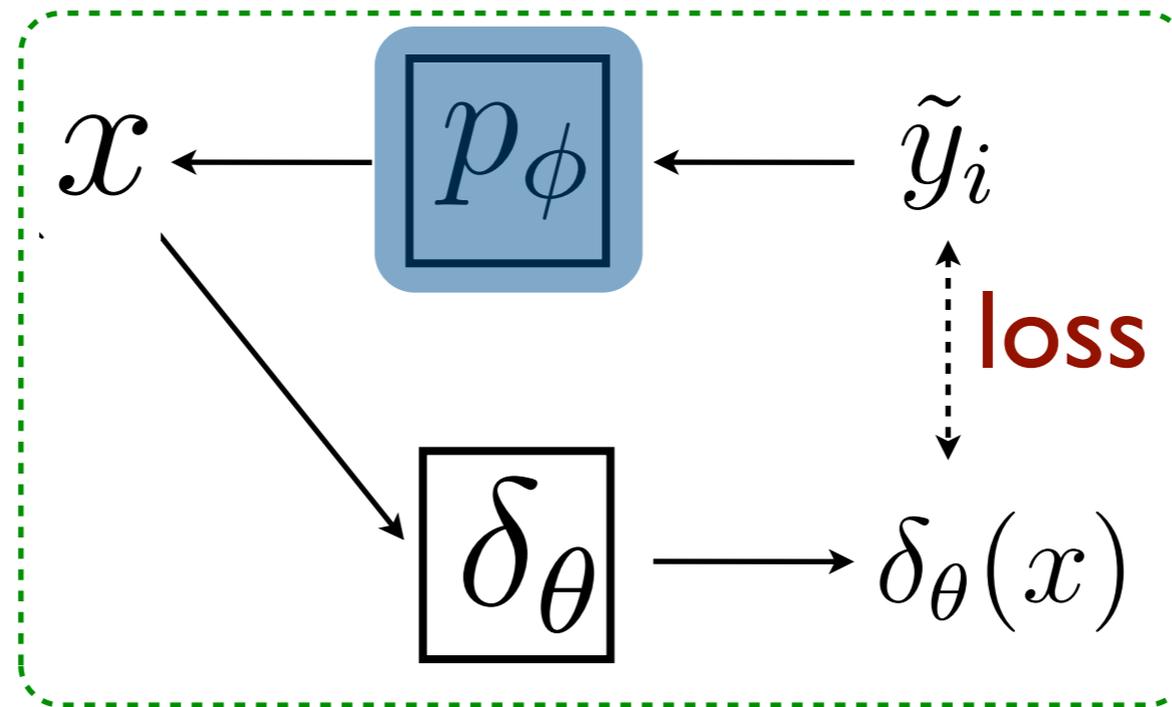
$x$ : imputed input

$\delta_{\theta}$ : forward system

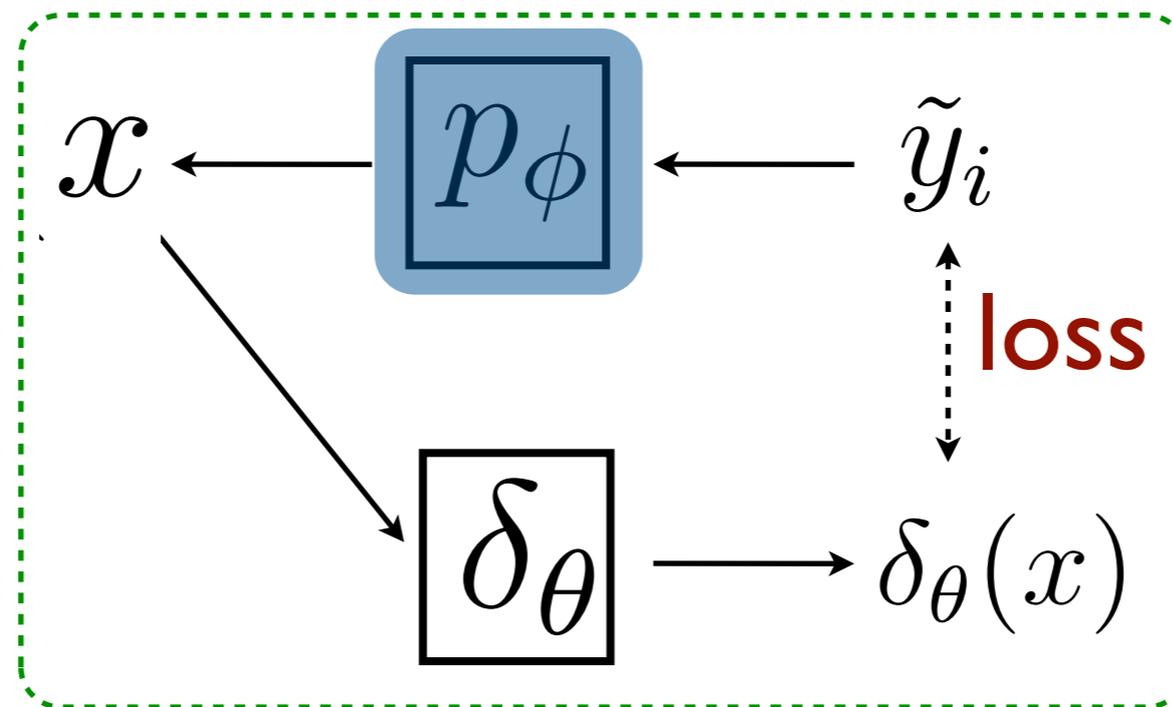
Round trip translation

Speech recognition?

# Training Reverse Model $p_\phi$

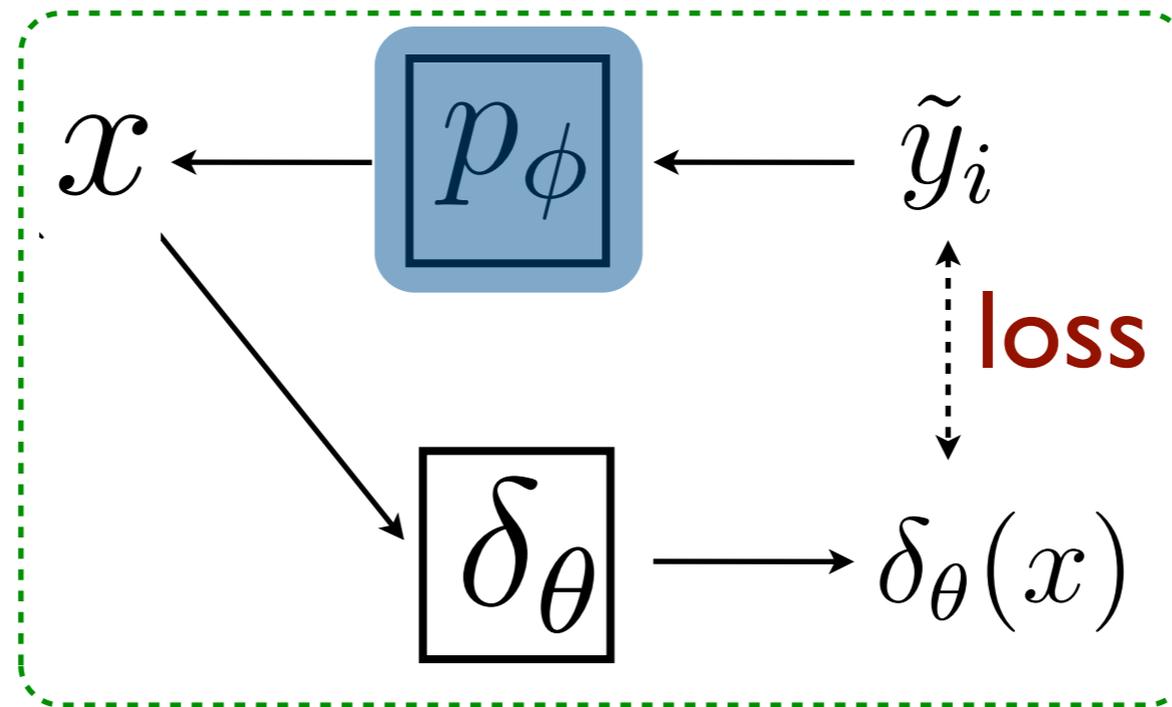


# Training Reverse Model $\mathcal{P}_\phi$



Our goal is to train a good forward system  $\delta_\theta$

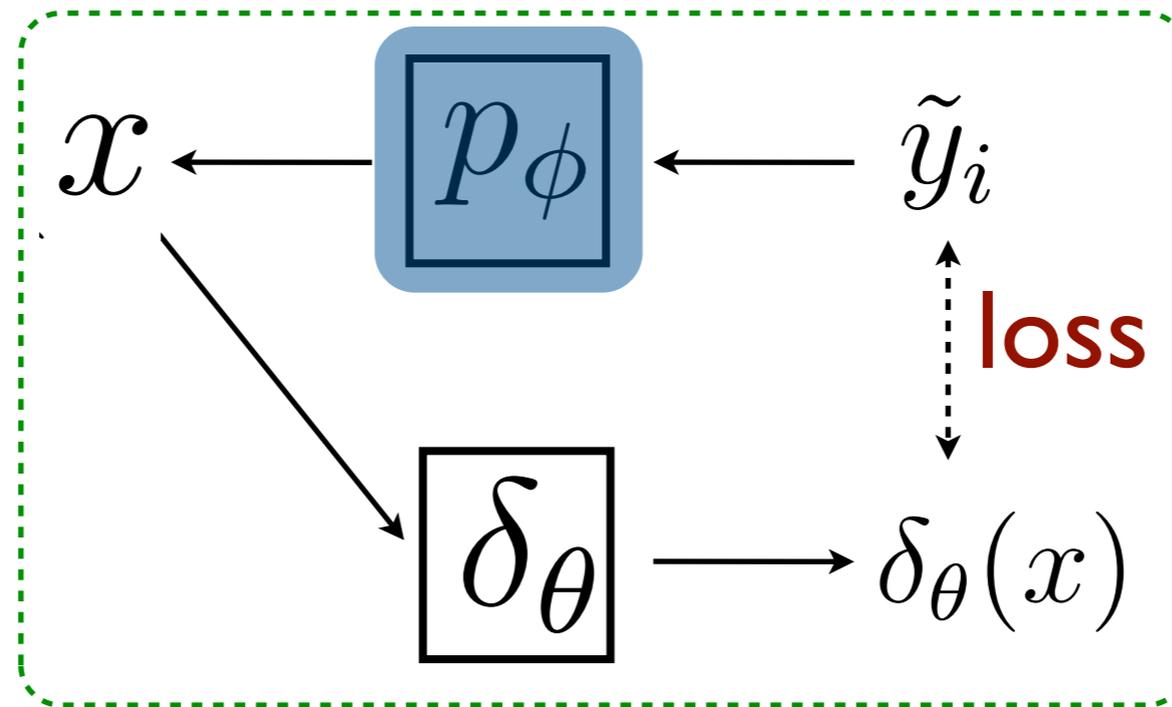
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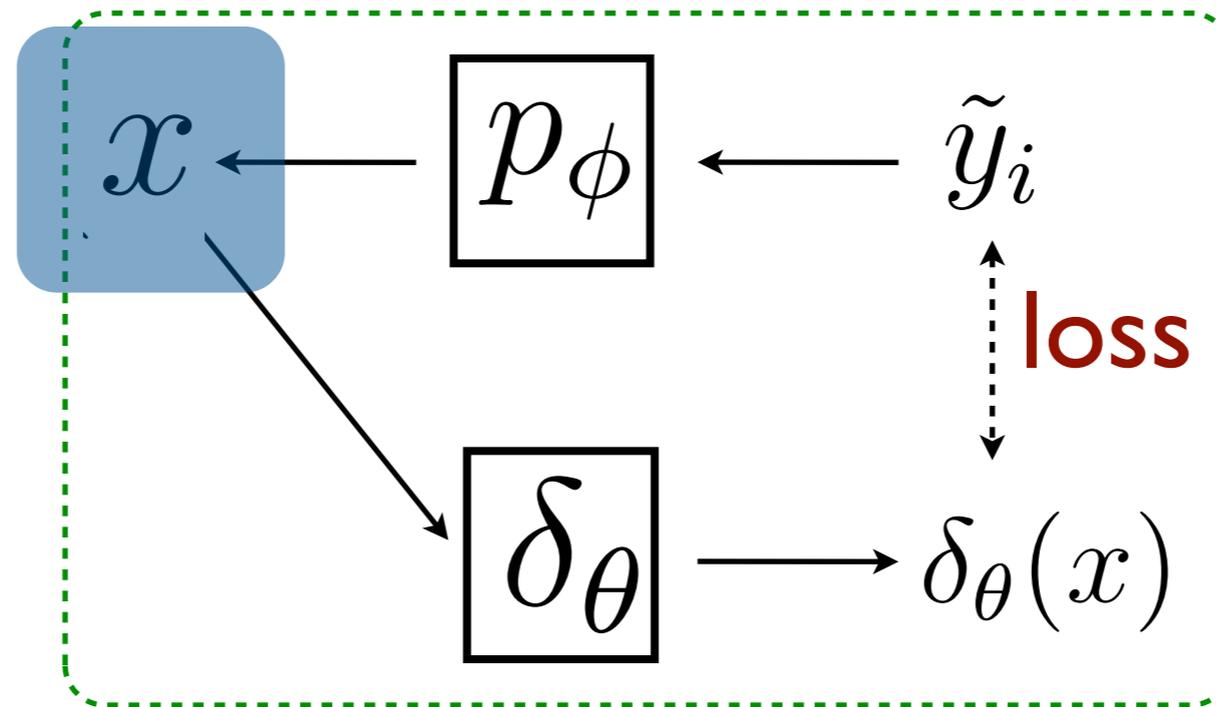


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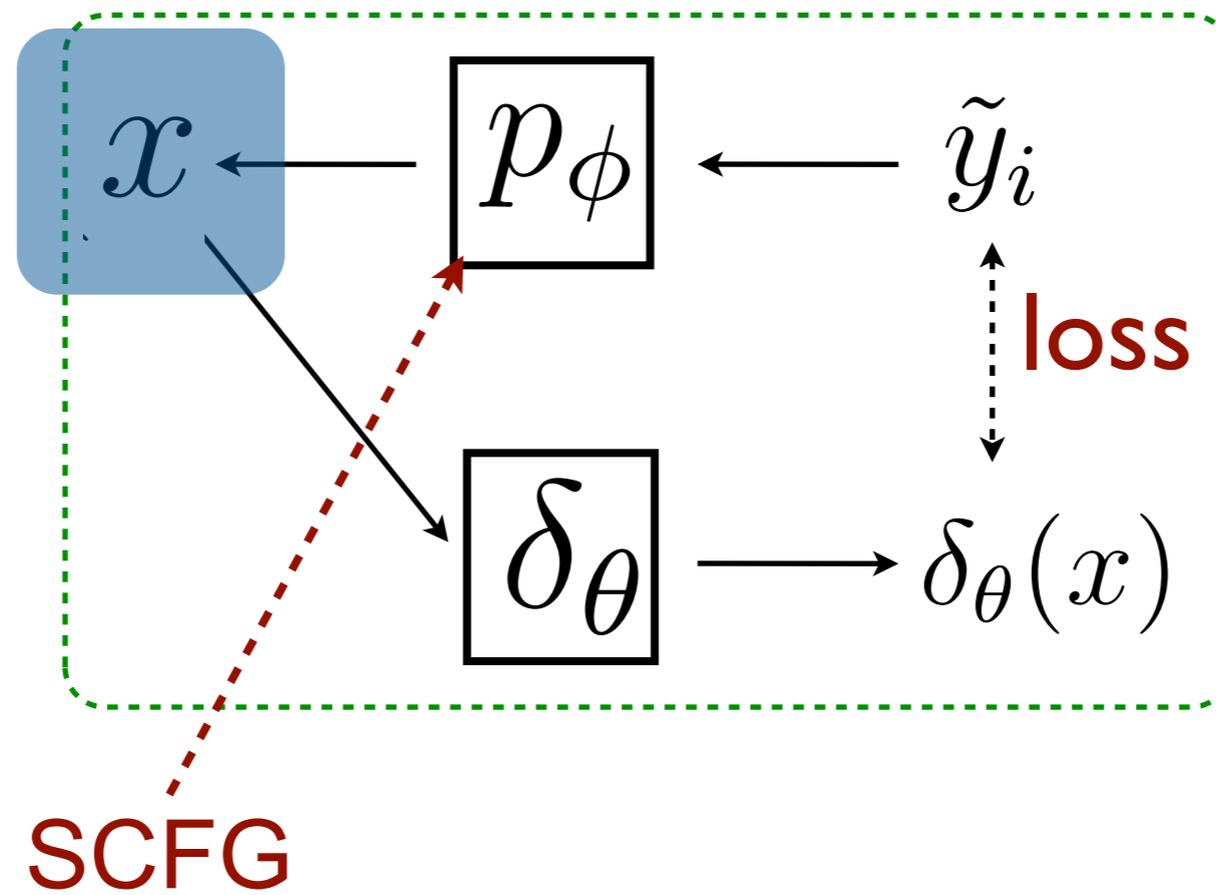
$\mathcal{P}_\phi$  and  $\delta_\theta$  are parameterized and trained separately

$\mathcal{P}_\phi$  is fixed when training  $\delta_\theta$

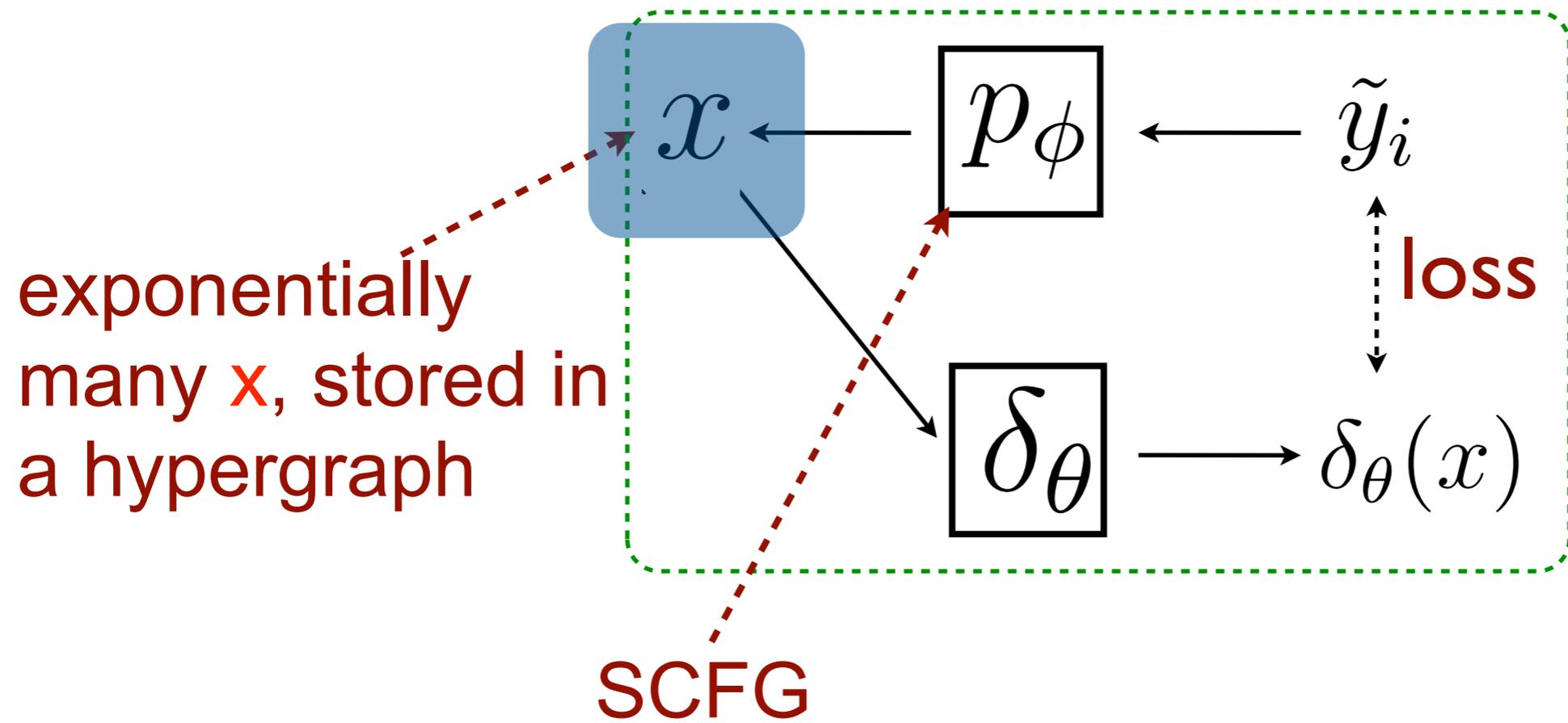
# Approximating $p_\phi(x | \tilde{y}_i)$



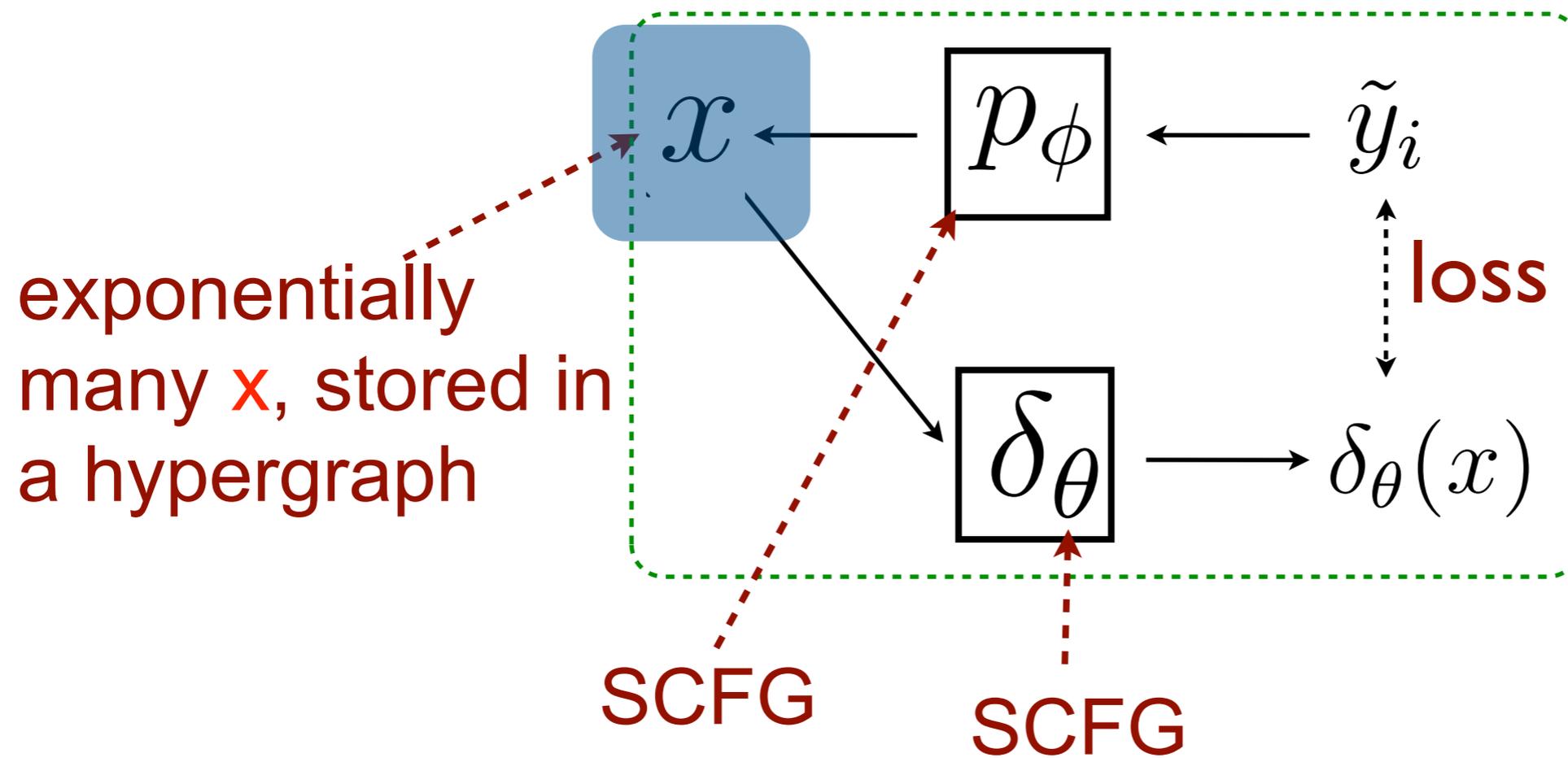
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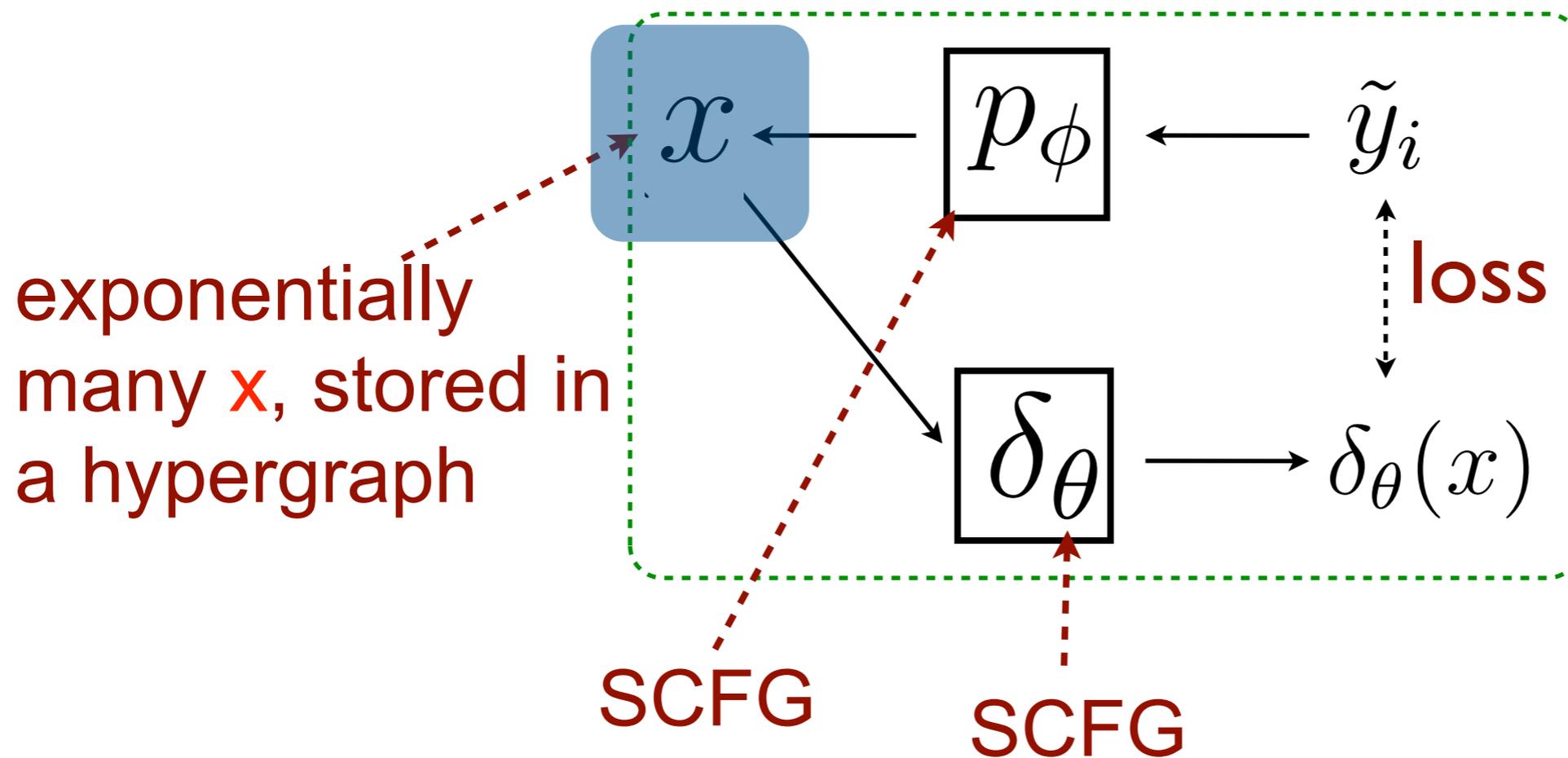
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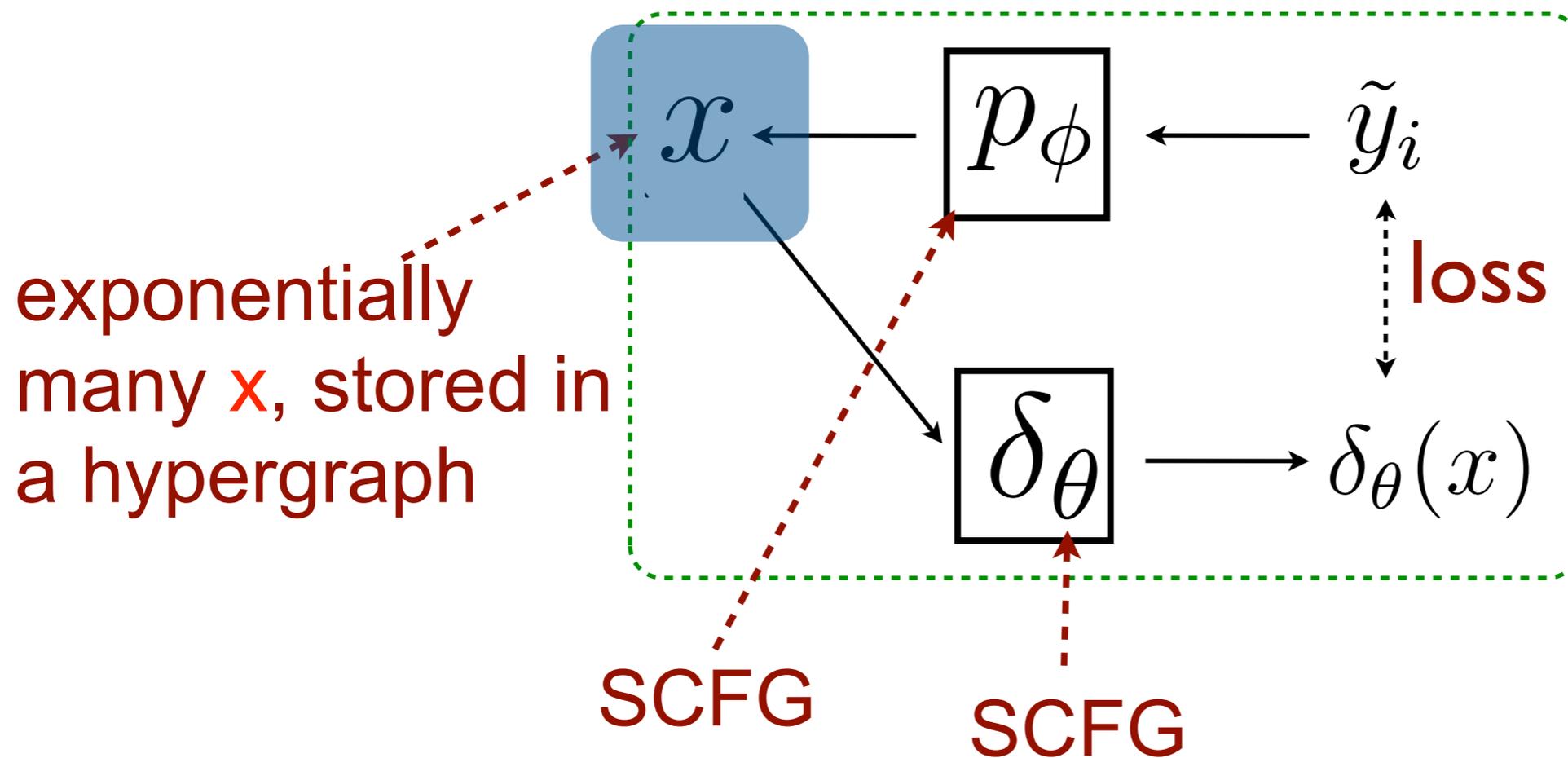


# Approximating $p_\phi(x | \tilde{y}_i)$



CFG is not closed under composition!

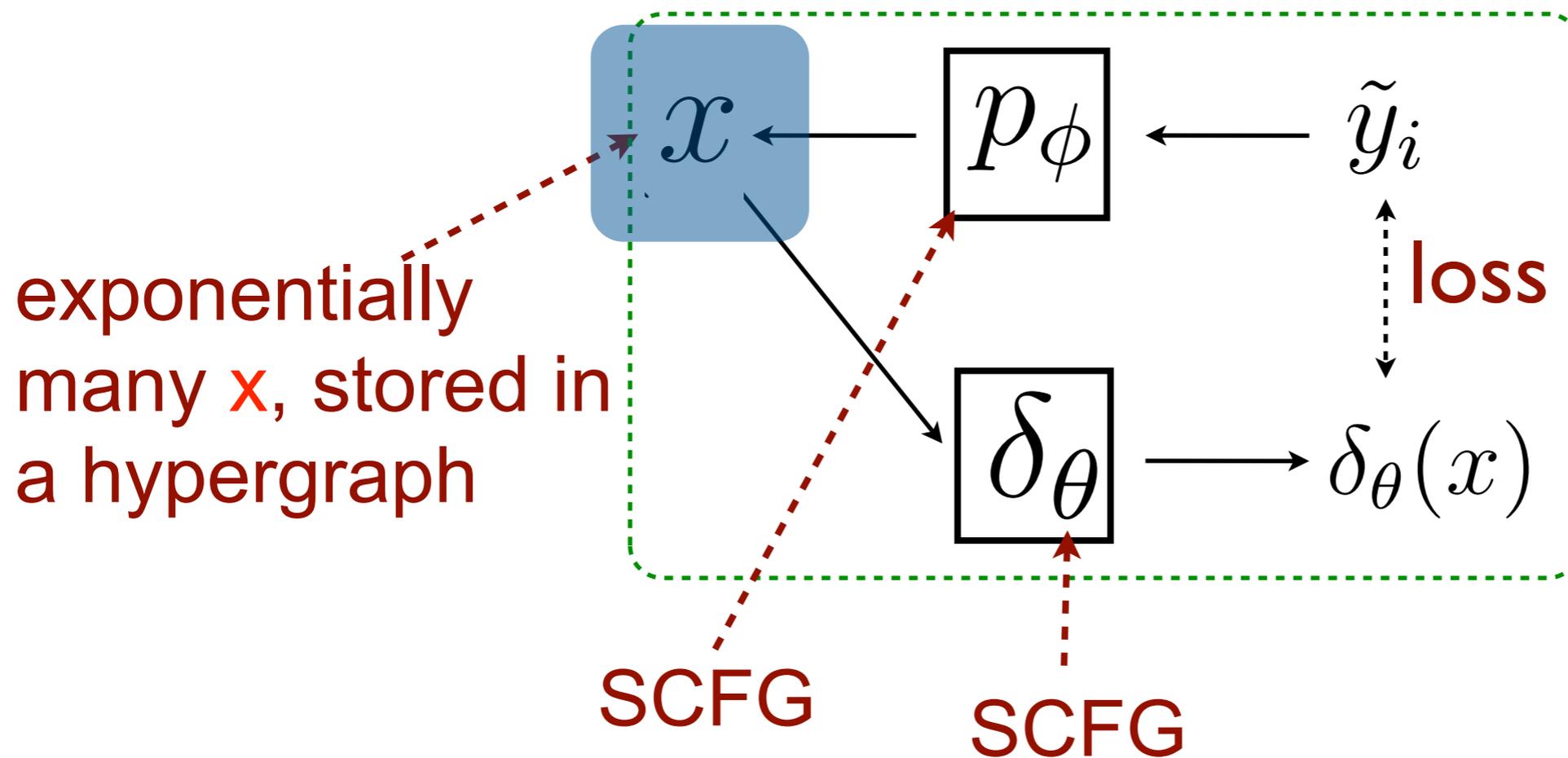
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CFG is not closed under composition!

- Approximations
  - k-best
  - sampling
  - lattice

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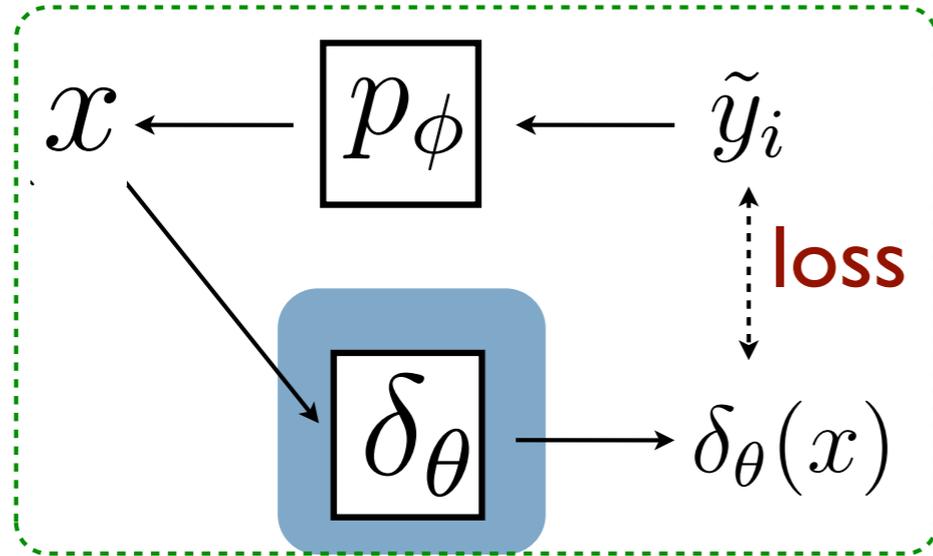
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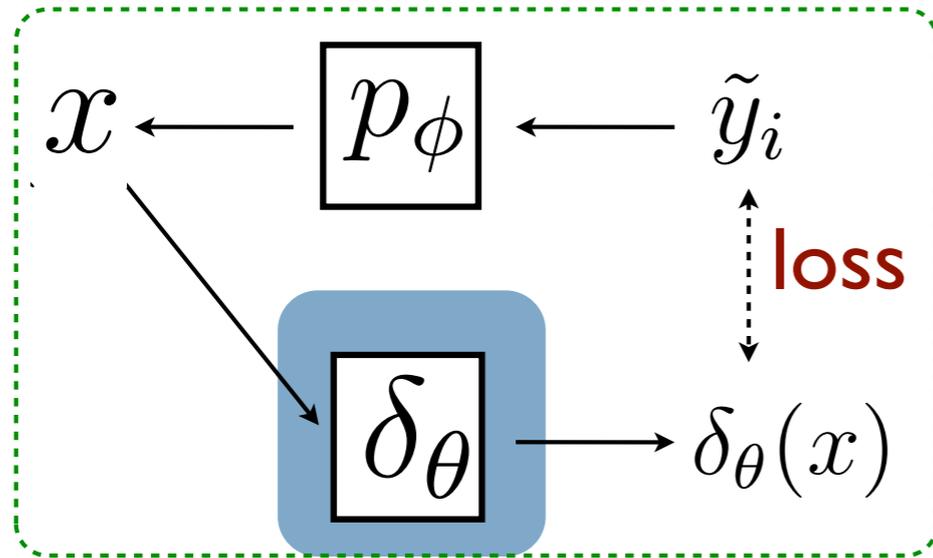
- k-best
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- lattice

variational approximation  
+  
lattice decoding (Dyer et al., 2008)

# The Forward System $\delta_\theta(x)$



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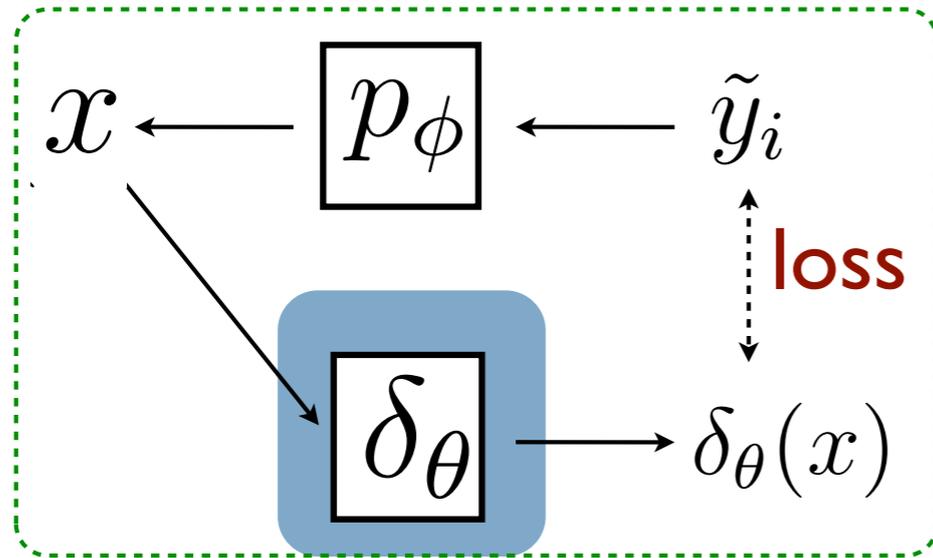


$$\delta_\theta(x) = \operatorname{argmax}_y p_\theta(y | x)$$

- **Deterministic Decoding**
- use **one-best** translation

$$\theta^* = \operatorname{arg min}_\theta \frac{1}{N} \sum_{i=1}^N \sum_x p_\phi(x | \tilde{y}_i) \mathbf{L}(\delta_\theta(x), \tilde{y}_i)$$

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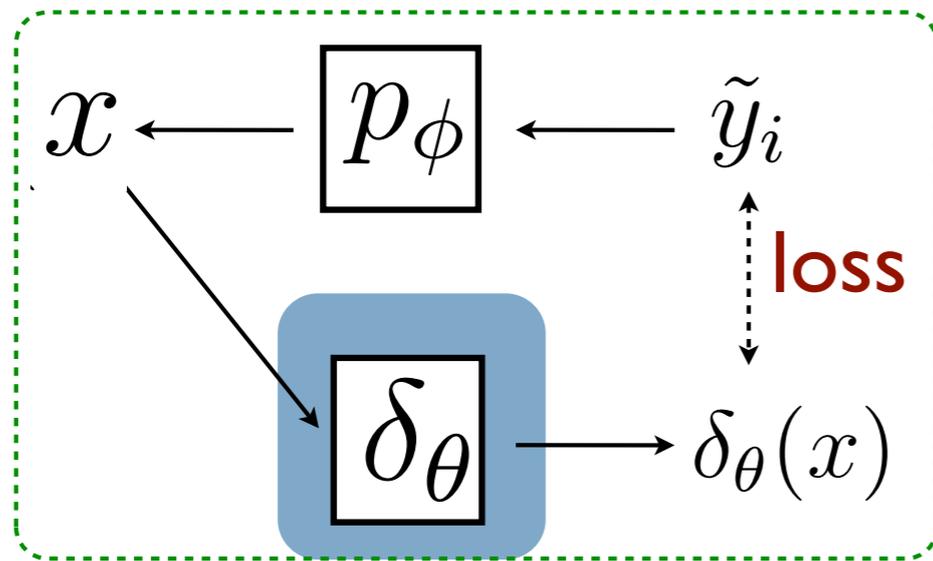
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the objective is not differentiable



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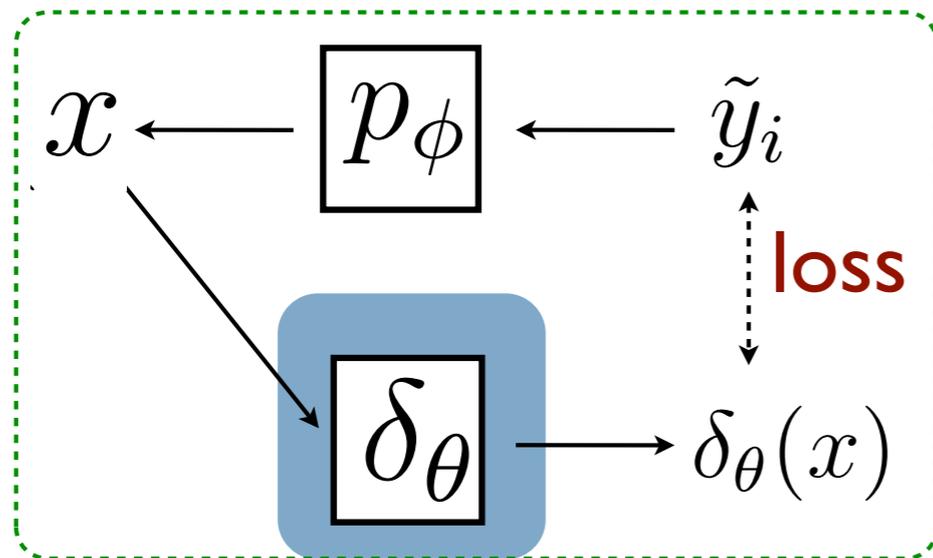


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- **Randomized Decoding**
- use a **distribution** of translations

$$\theta^* = \operatorname{argmin}_\theta \frac{1}{N} \sum_{i=1}^N \sum_x p_\phi(x | \tilde{y}_i) \sum_y p_\theta(y | x) \mathbf{L}(y, \tilde{y}_i)$$

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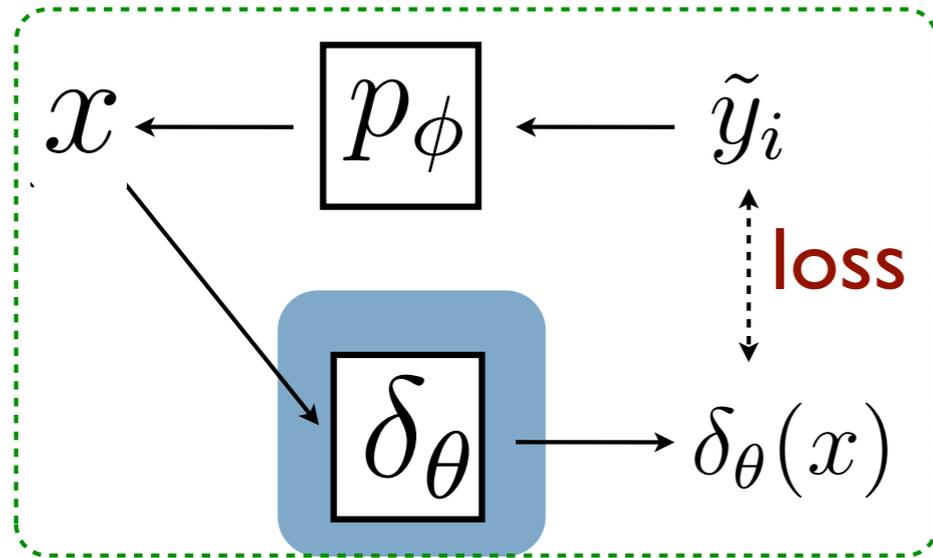


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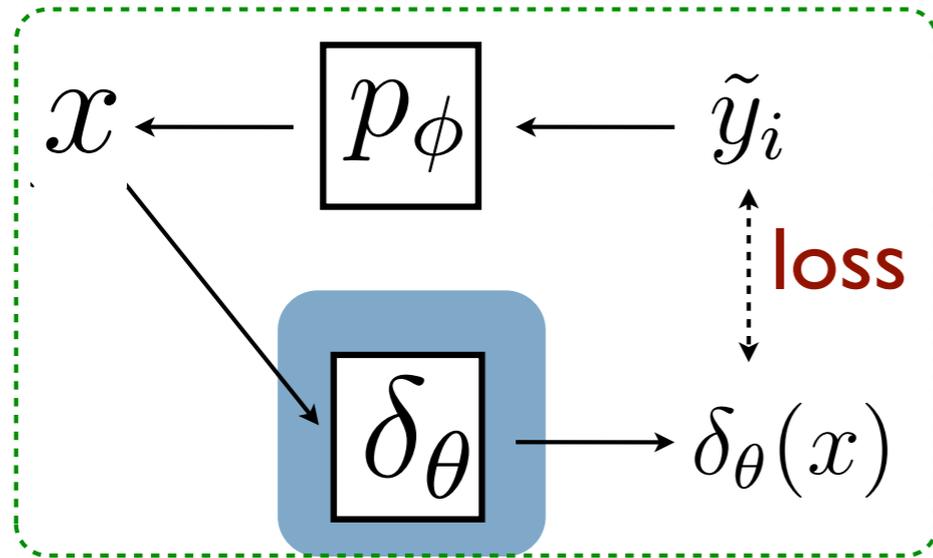
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expected loss

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- **Randomized Decoding**
- use a **distribution** of translations

differentiable



expected loss

$$\theta^* = \operatorname{argmin}_\theta \frac{1}{N} \sum_{i=1}^N \sum_x p_\phi(x | \tilde{y}_i) \sum_y p_\theta(y | x) \mathbf{L}(y, \tilde{y}_i)$$

# Experiments

- Supervised Training
  - require bitext
- Unsupervised Training
  - require monolingual English
- Semi-supervised Training
  - interpolation of supervised and unsupervised

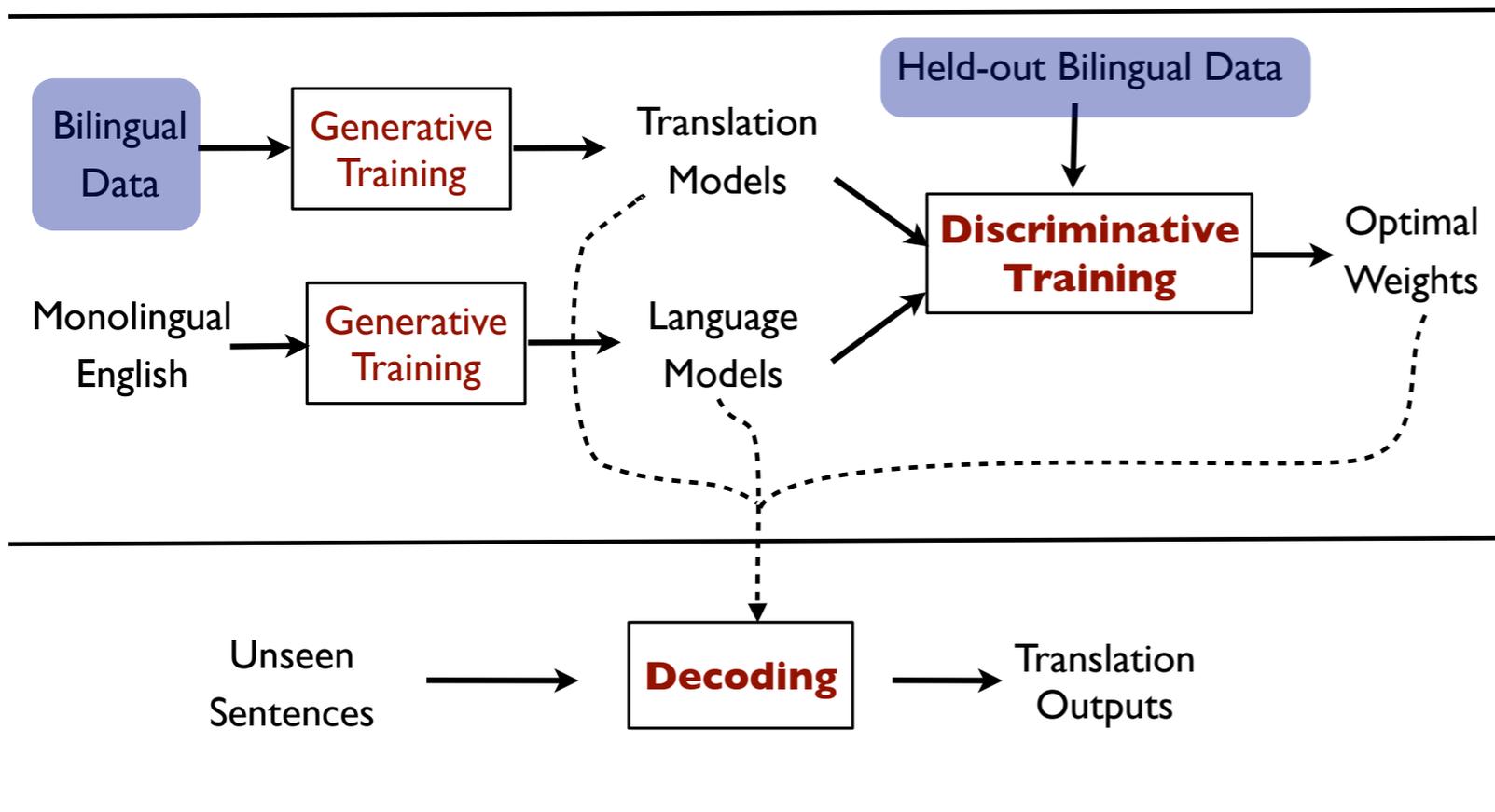
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Training scenario	Test BLEU
Sup, (200, 200*16)	47.6

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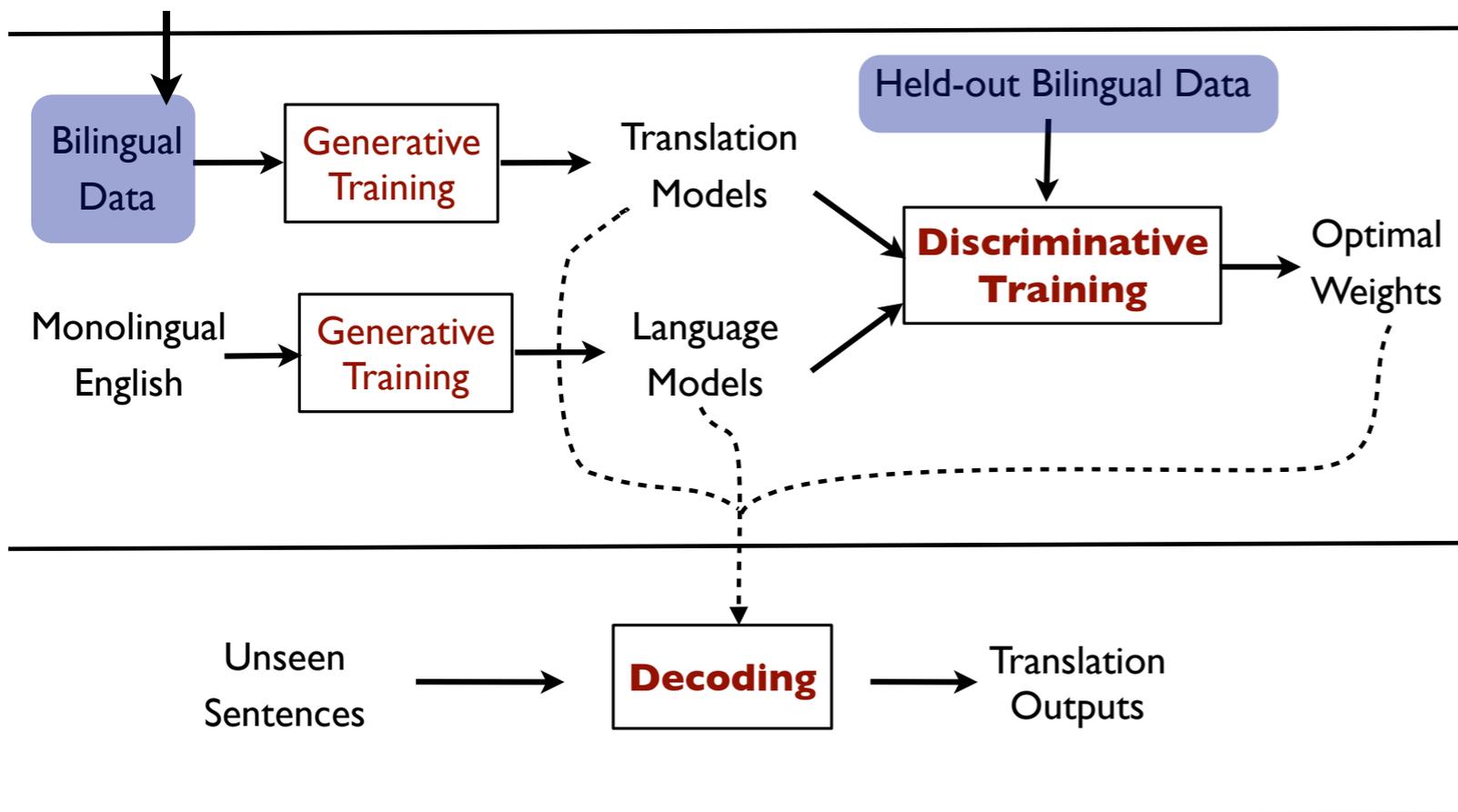
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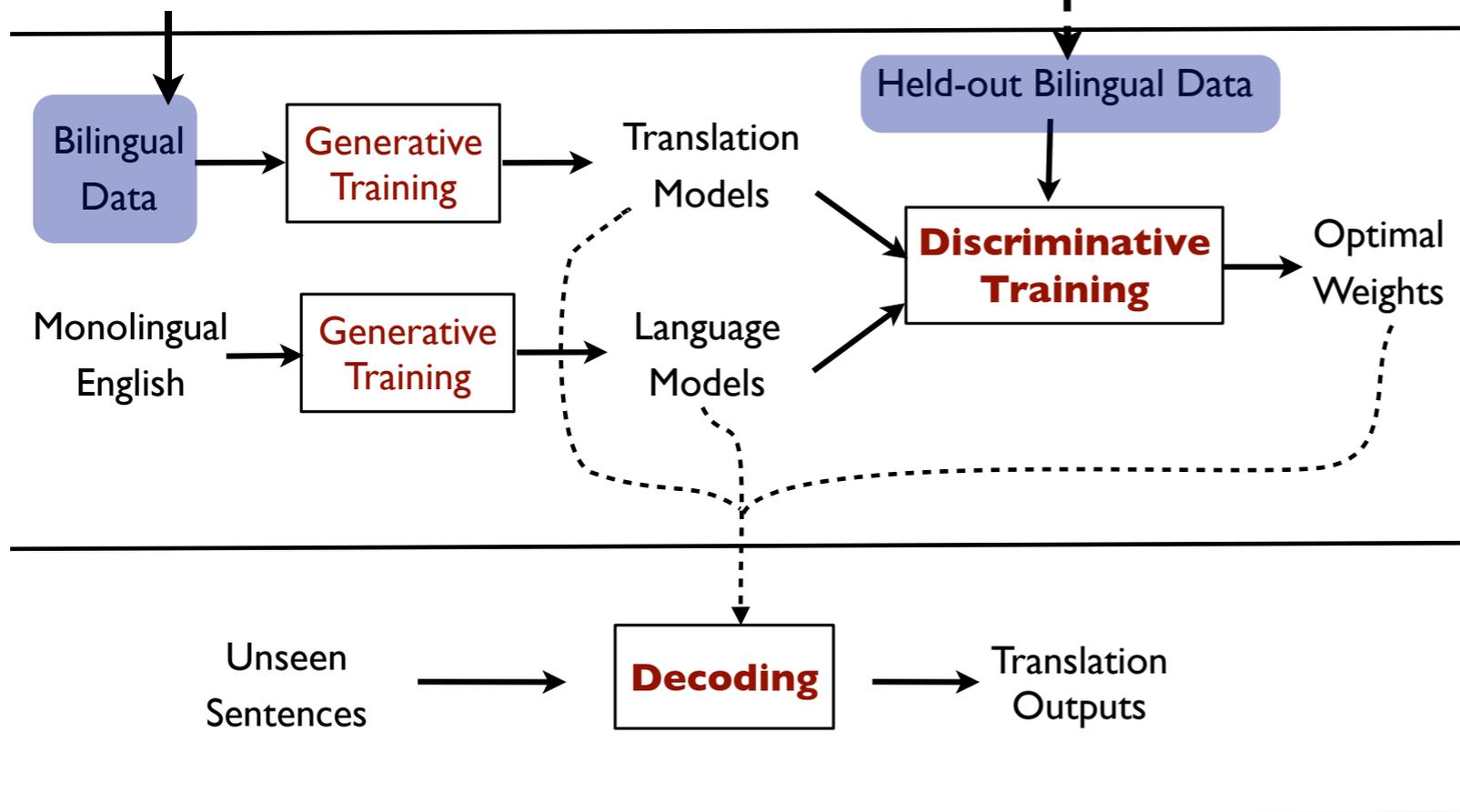
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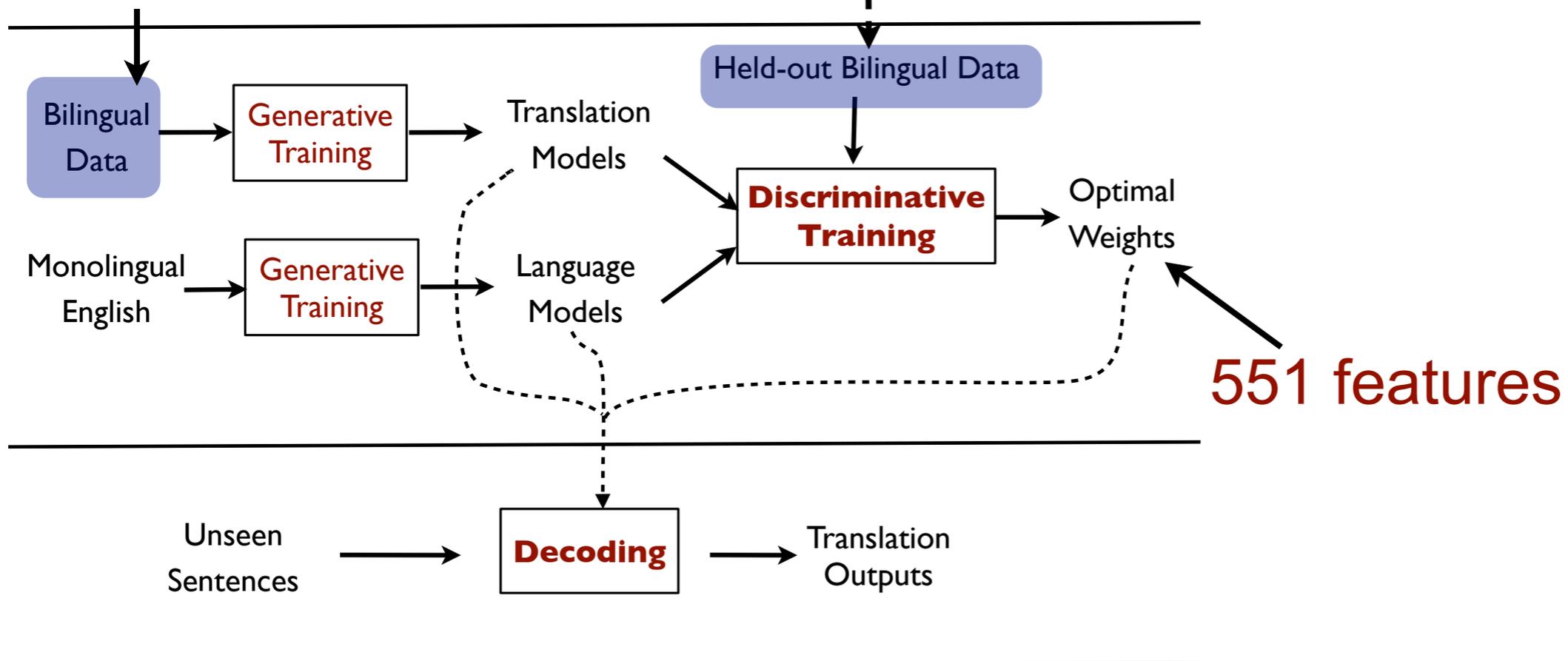
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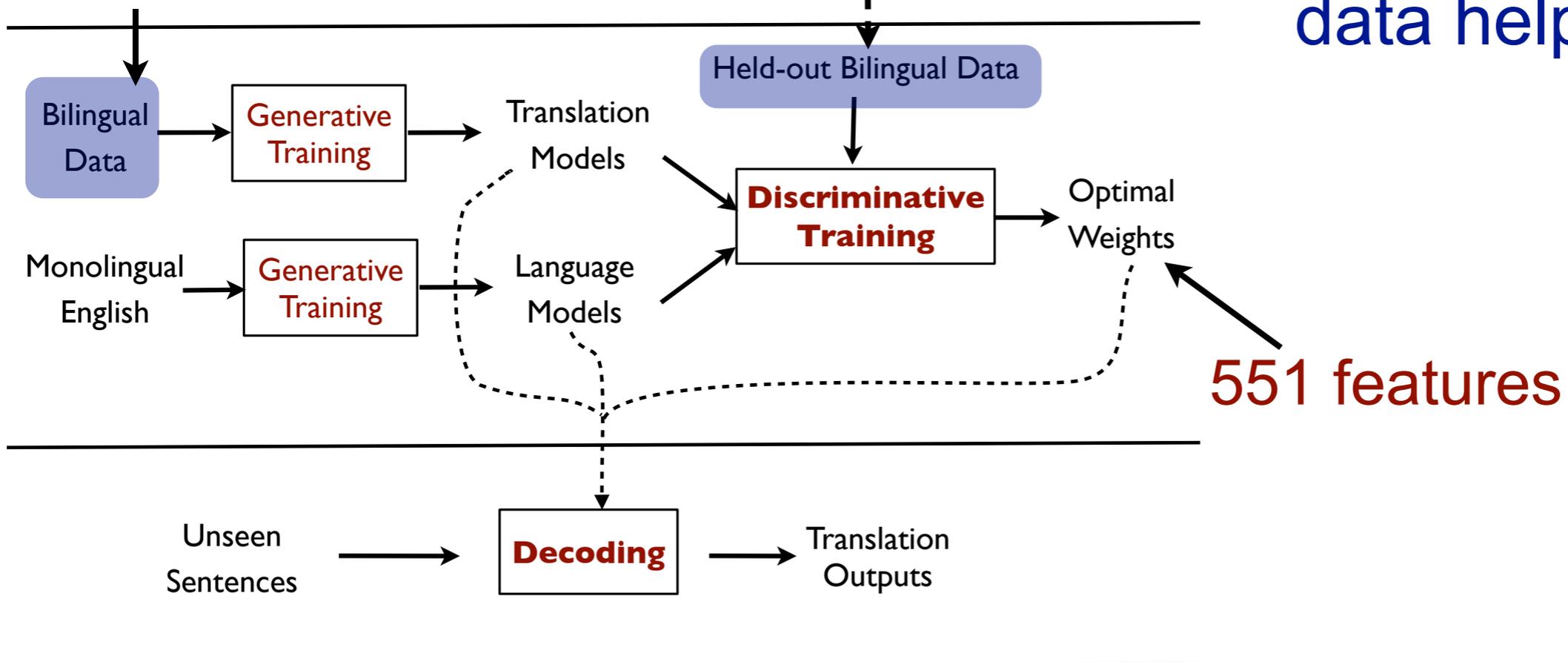


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Training scenario	Test BLEU
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+Unsup, 100*16 Eng sentences	49.0
+Unsup, 200*16 Eng sentences	48.9

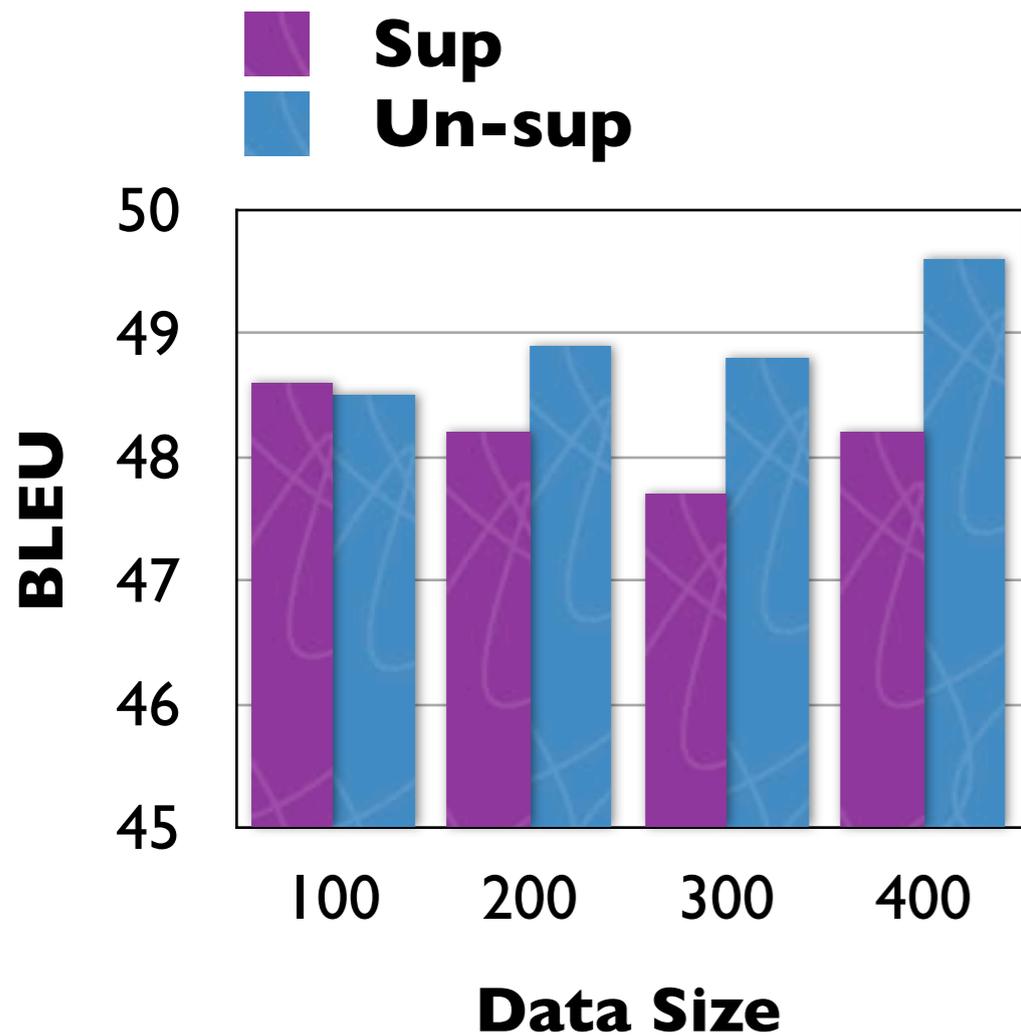
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Adding unsupervised data helps!



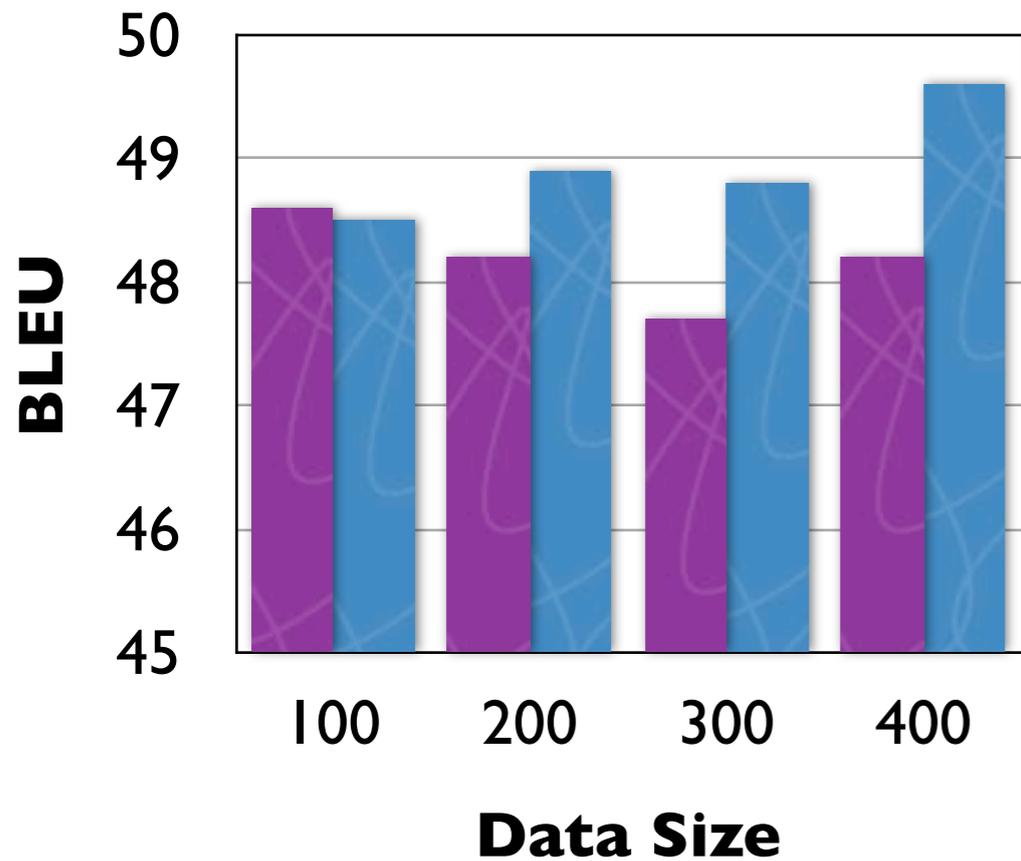
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Unsupervised training performs as well as (and often better than) the supervised one!



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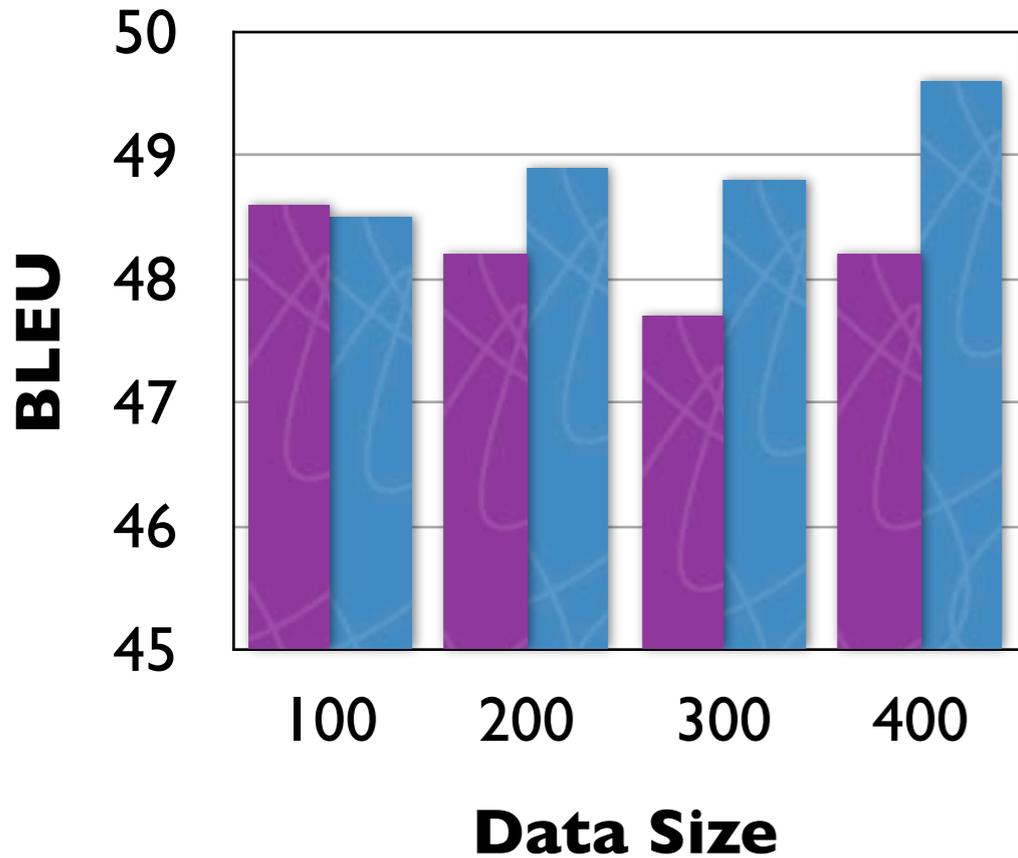


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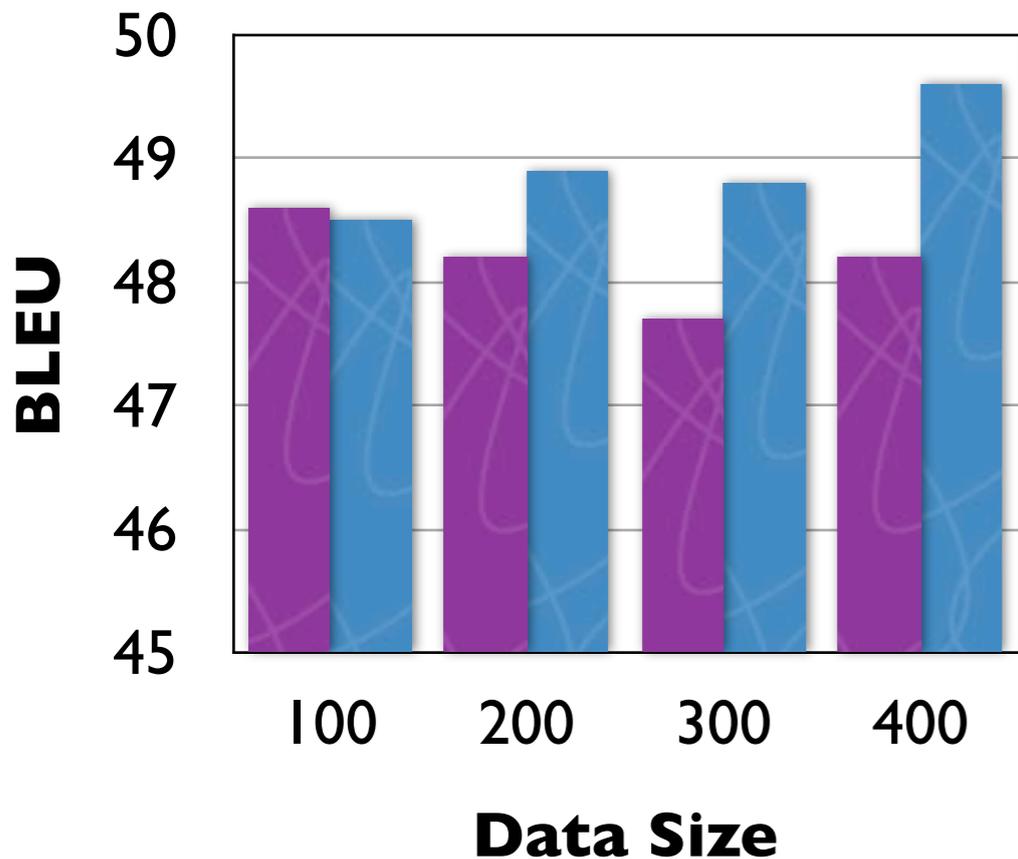
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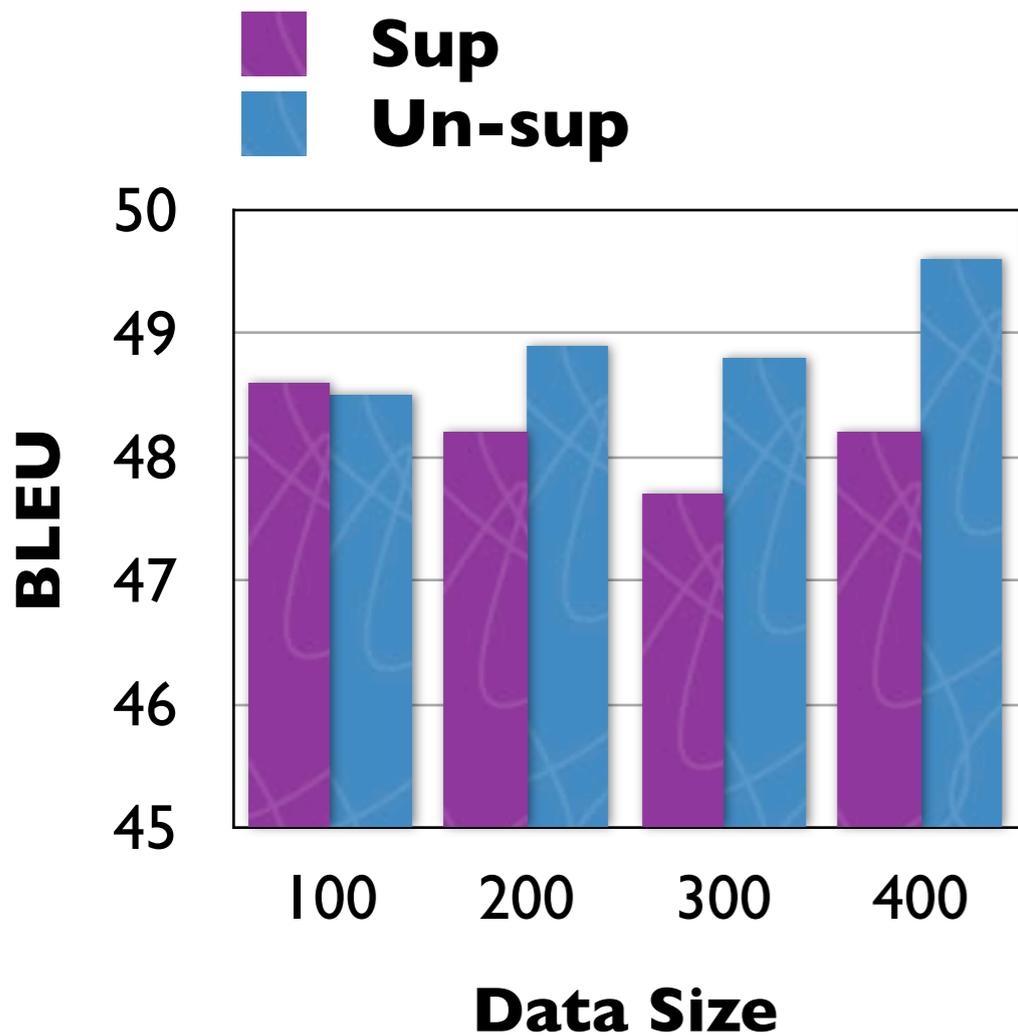


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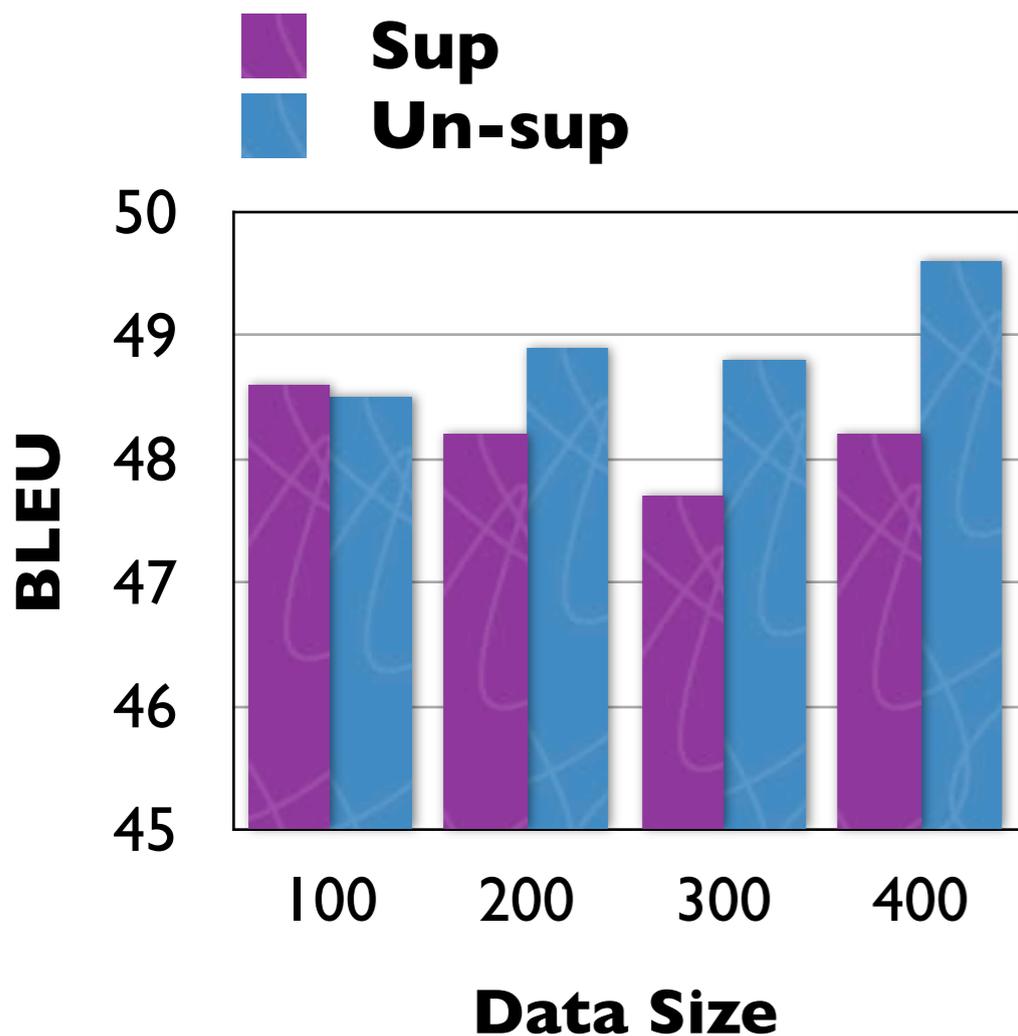
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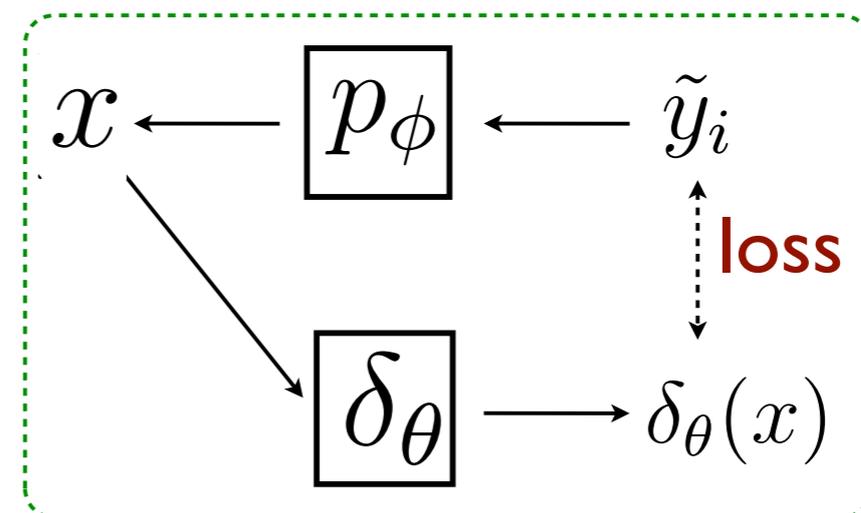
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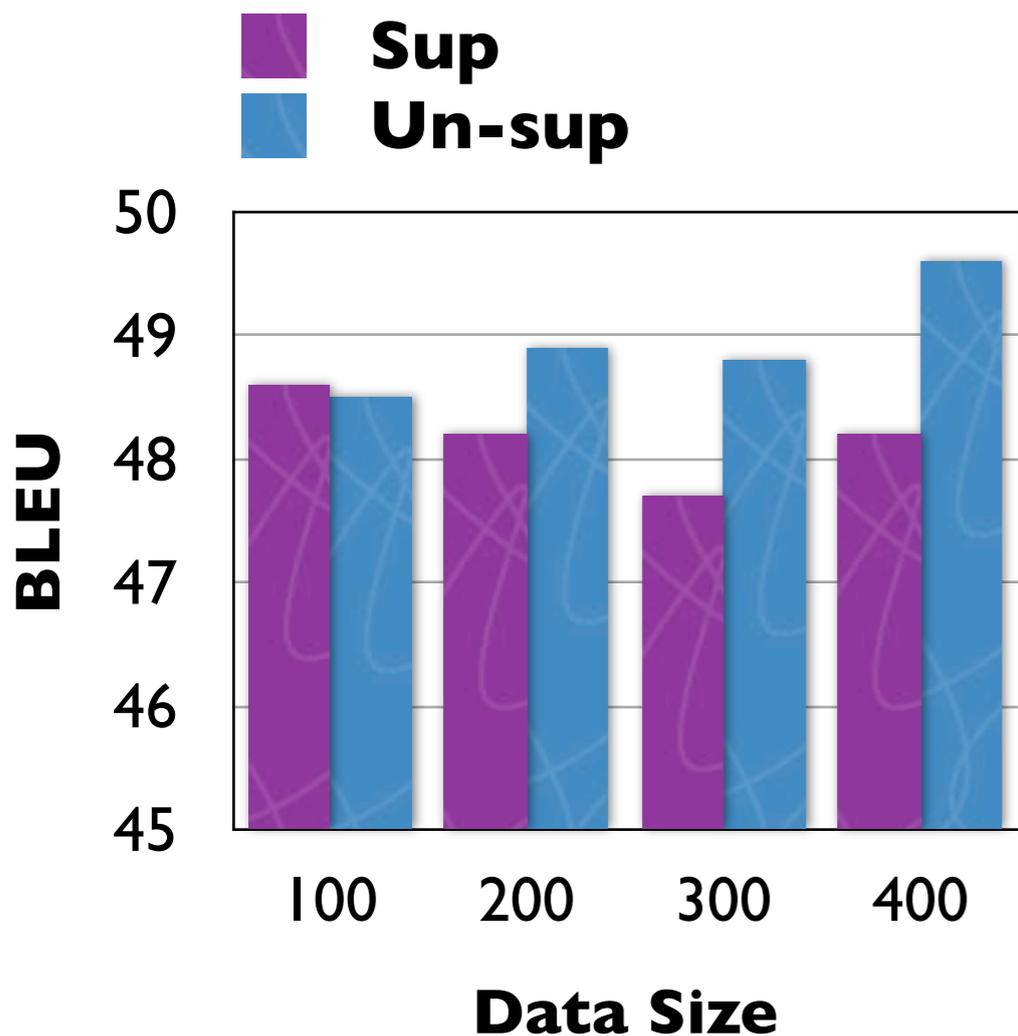
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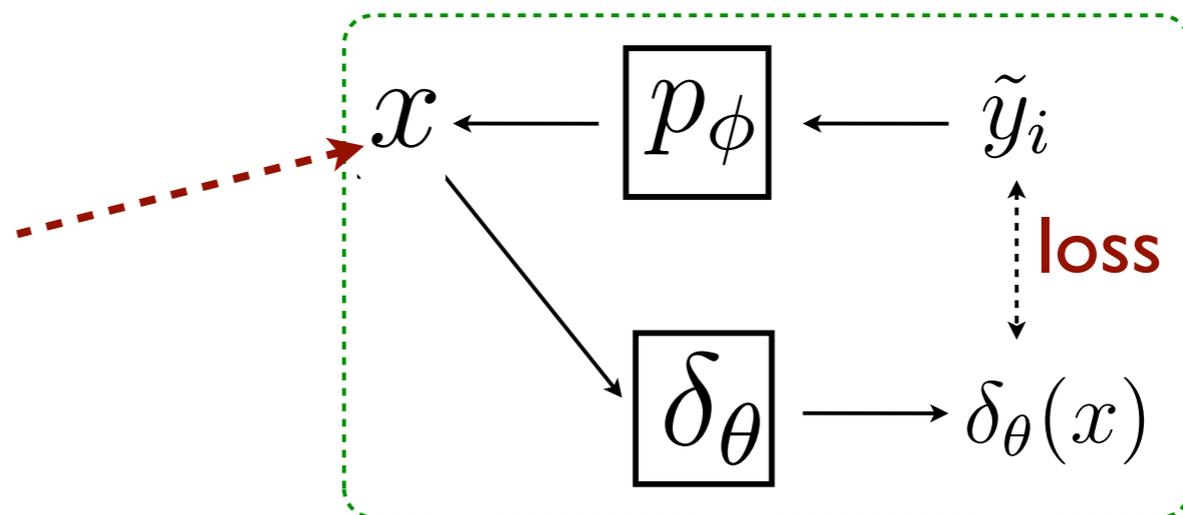
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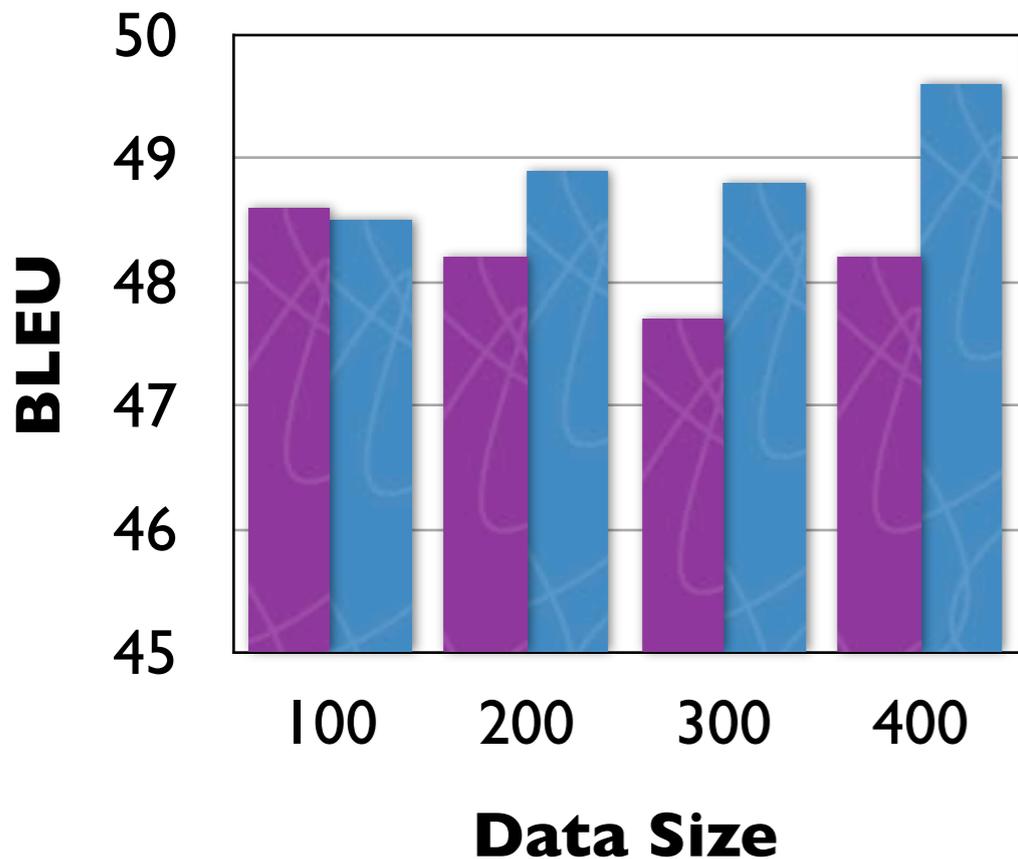
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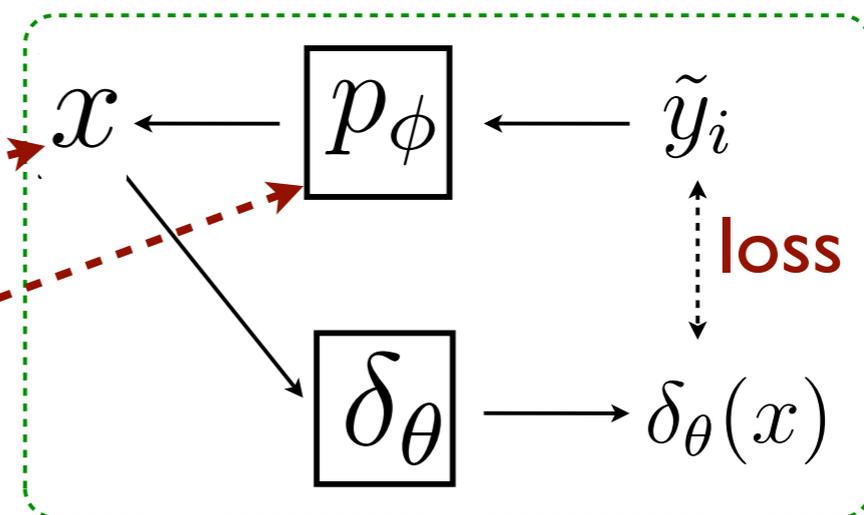
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  - different reverse model



# Outline

- Hypergraph as Hypothesis Space
- Unsupervised Discriminative Training
  - ▶ minimum imputed risk
  - ▶ **contrastive language model estimation**
- Variational Decoding
- First- and Second-order Expectation Semirings

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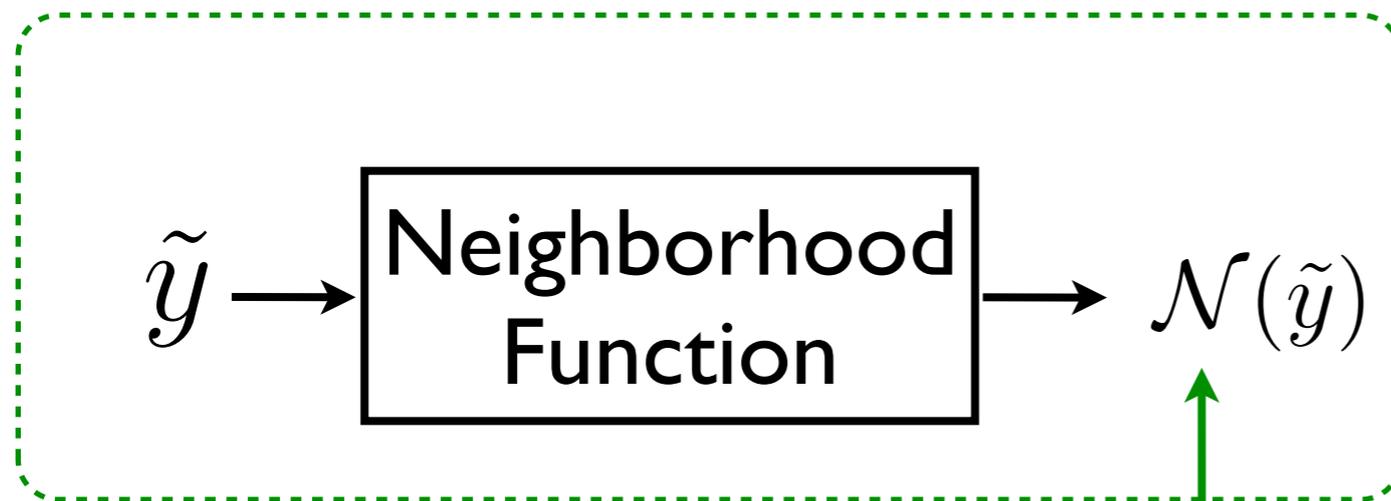
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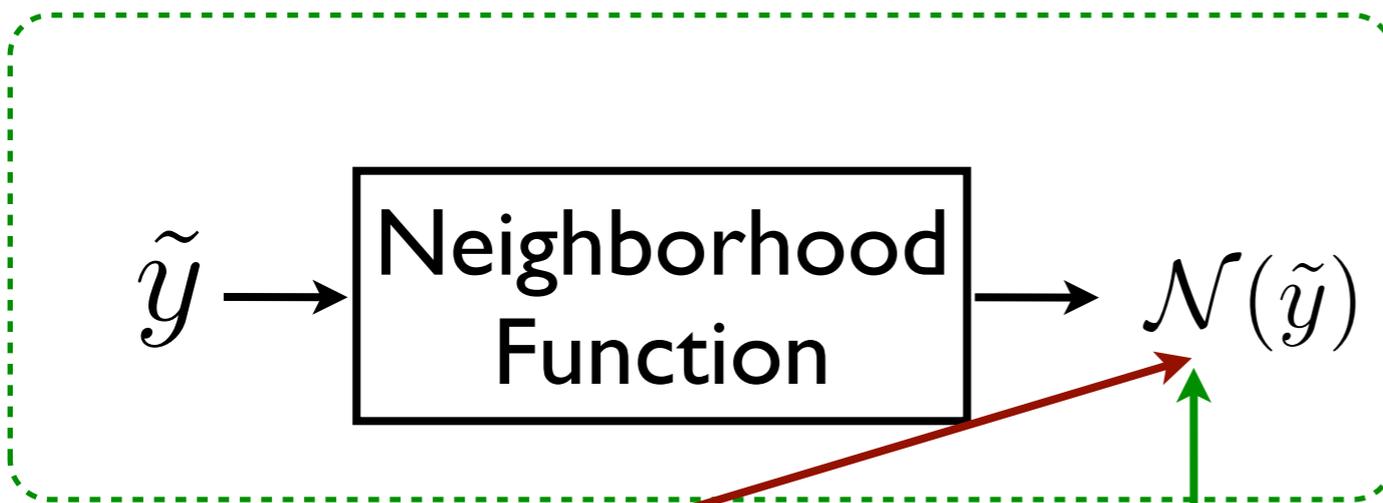
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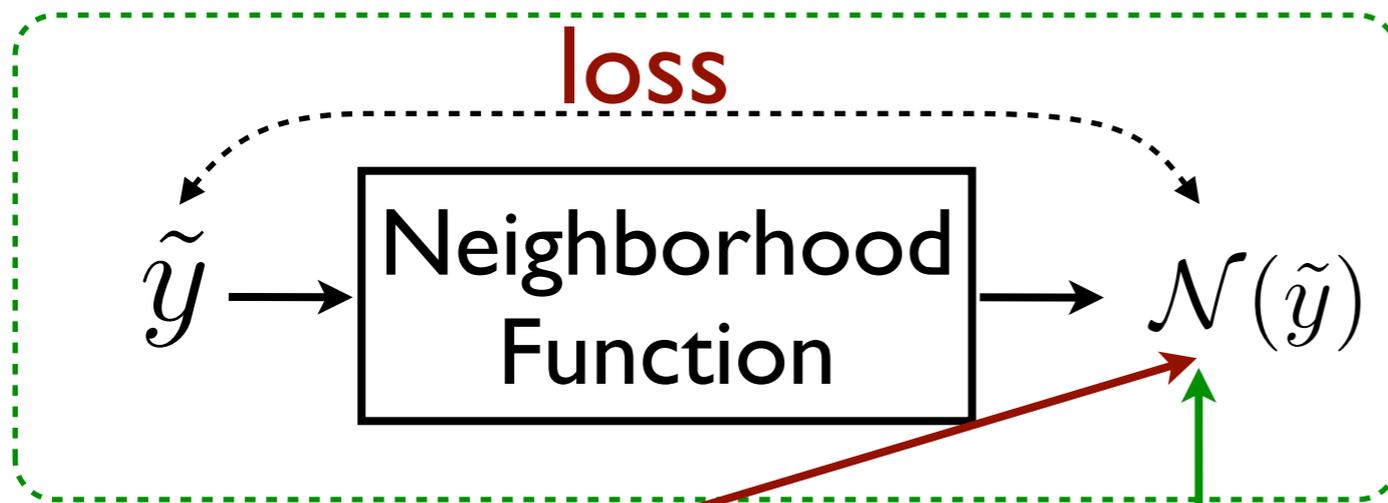
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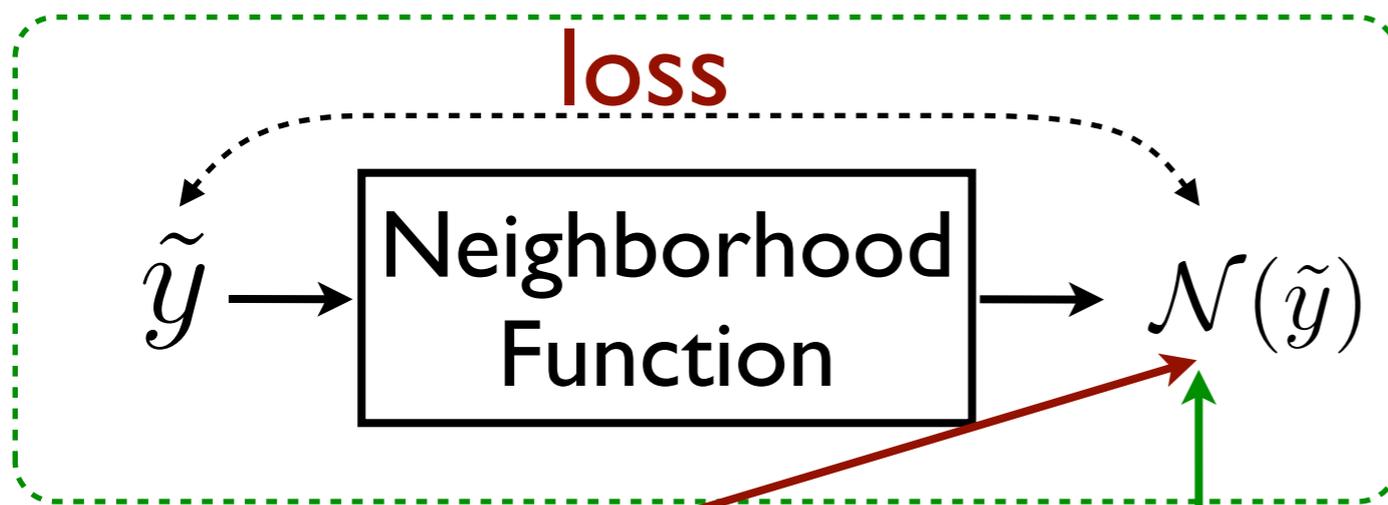
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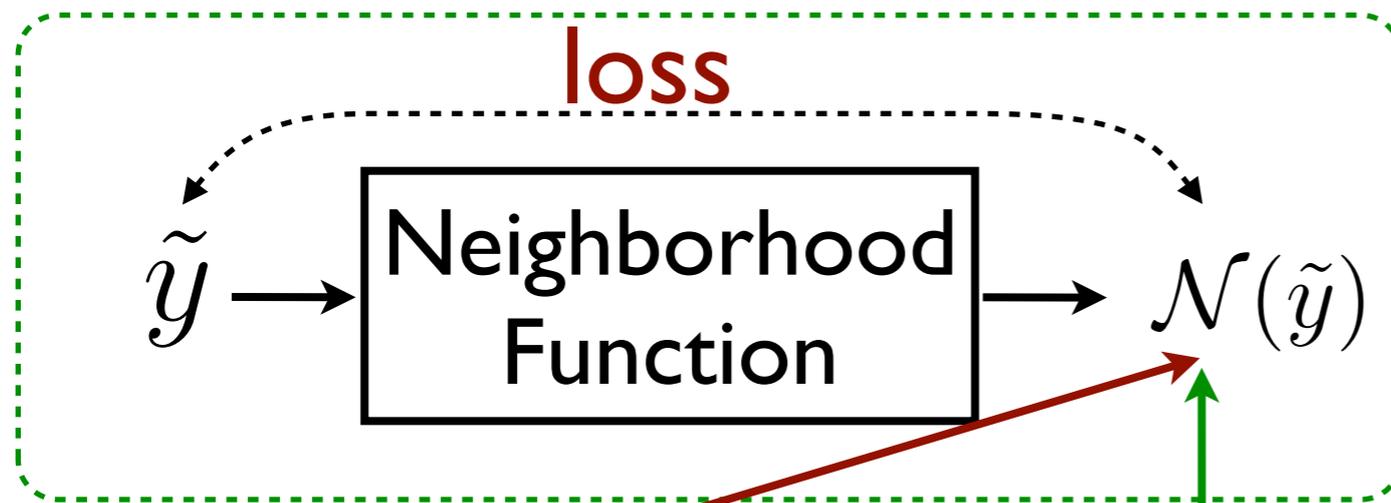
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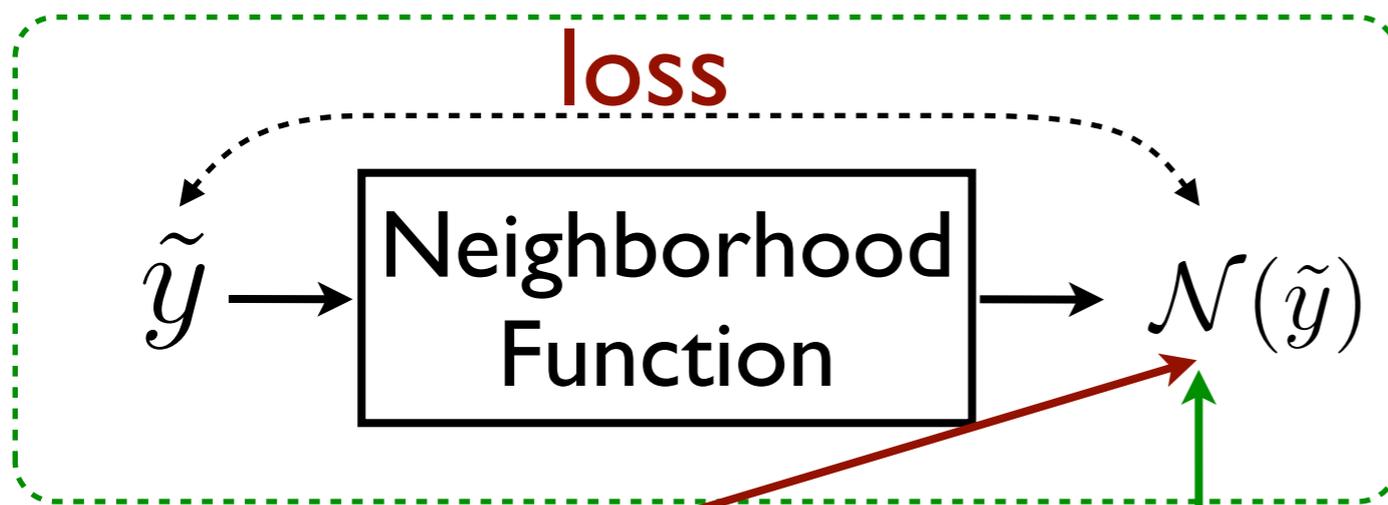
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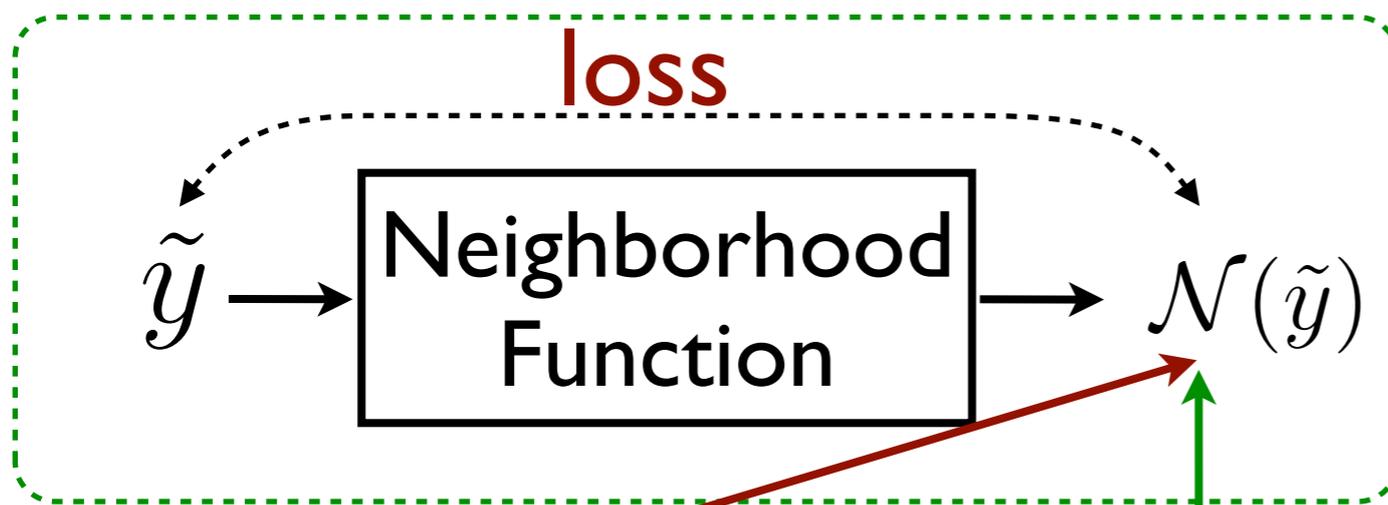
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train to recover the original English as much as possible



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- **Step-1**: extract a **confusion grammar (CG)**
  - an English-to-English SCFG

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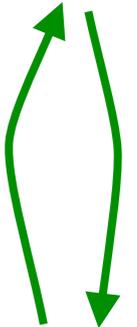
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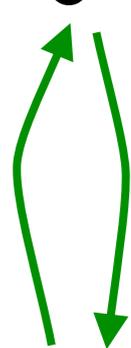
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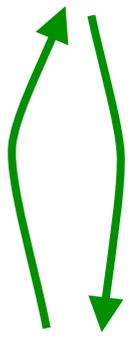
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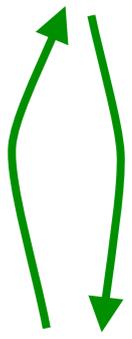
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# Contrastive Language Model Estimation

- **Step-1**: extract a **confusion grammar** (CG)
  - an English-to-English SCFG

neighborhood function

$X \rightarrow \langle \text{lead to}, \text{result in} \rangle$

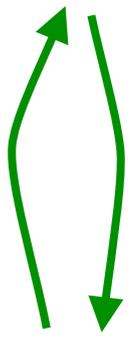
paraphrase

$X \rightarrow \langle X_0 \text{ at beijing}, \text{beijing 's } X_0 \rangle$

$X \rightarrow \langle X_0 \text{ of } X_1, X_0 \text{ of the } X_1 \rangle$

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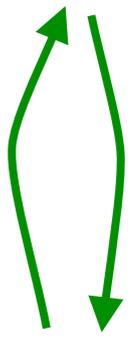
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insertion

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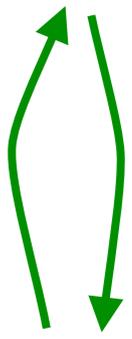
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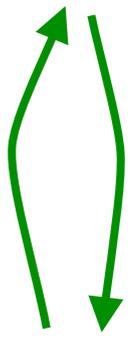
insertion

$X \rightarrow \langle X_0 \text{ 's } X_1, X_1 \text{ of } X_0 \rangle$

re-ordering

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# Step-1: Extracting a Confusion Grammar (CG)

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- Deriving a CG from a bilingual grammar
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Bilingual Rule

Confusion Rule

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**Bilingual Rule**

$X \rightarrow \langle \text{mao}, \text{a cat} \rangle$

$X \rightarrow \langle \text{mao}, \text{the cat} \rangle$

**Confusion Rule**

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- Deriving a CG from a bilingual grammar
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## Confusion Rule

$$X \rightarrow \langle \text{a cat}, \text{the cat} \rangle$$
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$X \rightarrow \langle \text{a cat, the cat} \rangle$

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**CG captures the confusion an MT system will have when translating an input.**

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CG captures the confusion an MT system will have when translating an input.

Our neighborhood function is **learned** and **MT-specific**.

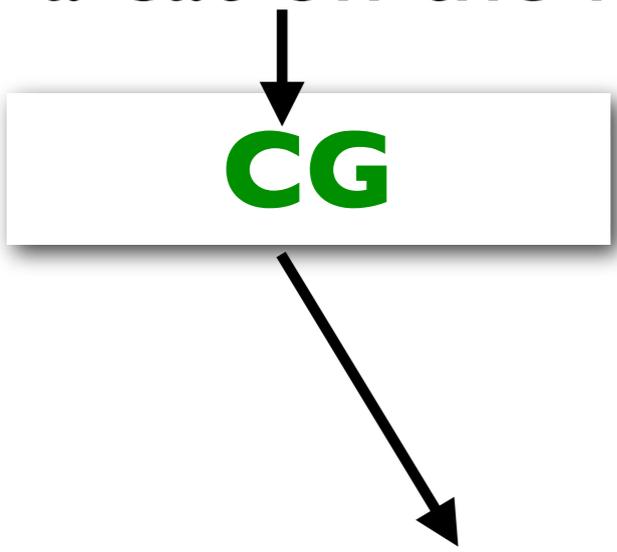
# Step-2: Generating Contrastive Sets

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a cat on the mat

# Step-2: Generating Contrastive Sets

a cat on the mat



**CG**

# Step-2: Generating Contrastive Sets

a cat on the mat

**CG**

S | 0, 5

$S \rightarrow \langle X_0, X_0 \rangle$

X | 0, 5

$X \rightarrow \langle X_0 \text{ on } X_1, X_1 \text{ on } X_0 \rangle$

$X \rightarrow \langle X_0 \text{ on } X_1, X_0 \text{'s } X_1 \rangle$

$X \rightarrow \langle X_0 \text{ on } X_1, X_1 \text{ of } X_0 \rangle$

$X \rightarrow \langle X_0 \text{ on } X_1, X_0 X_1 \rangle$

X | 0, 2

$X \rightarrow \langle \text{a cat}, \text{the cat} \rangle$

a<sub>0</sub> cat<sub>1</sub>

X | 3, 5

$X \rightarrow \langle \text{the mat}, \text{the mat} \rangle$

the<sub>3</sub> mat<sub>4</sub>

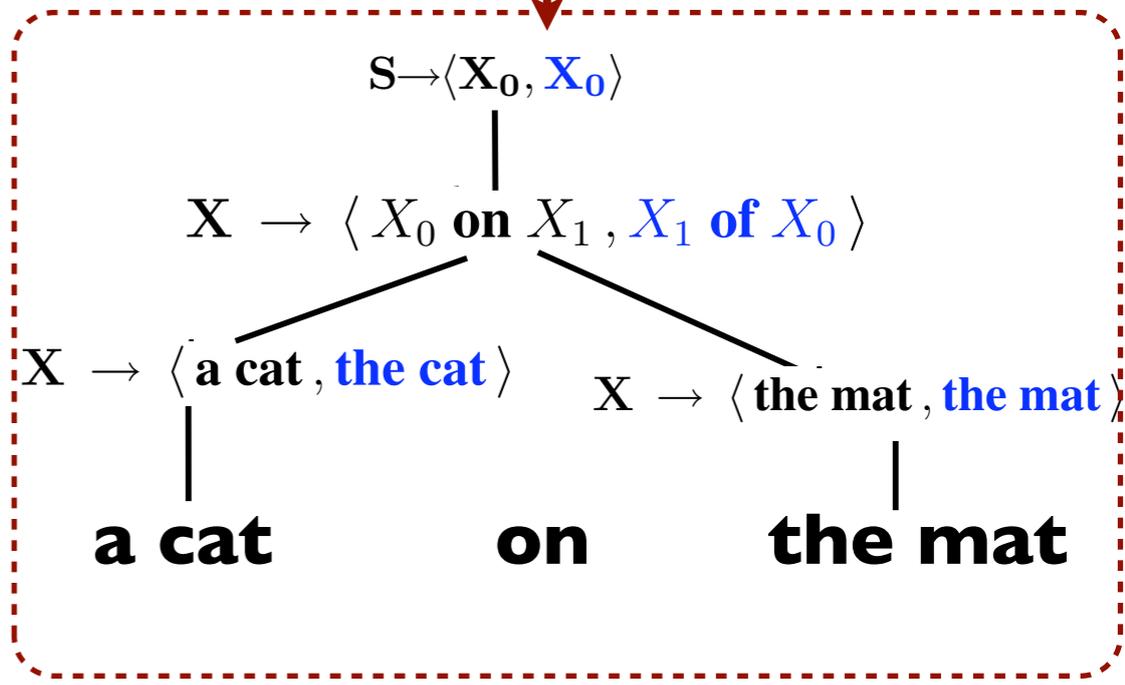
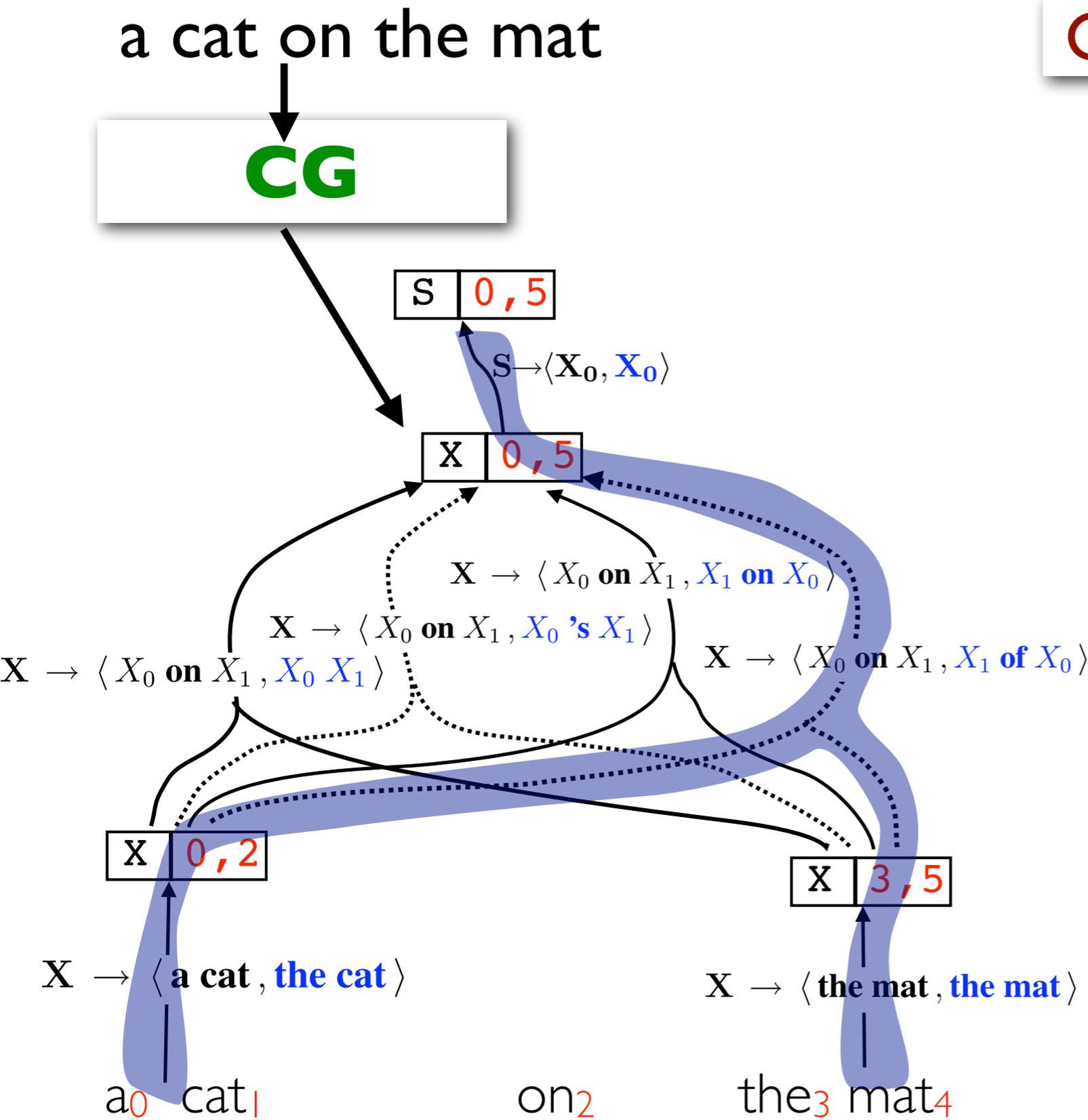
on<sub>2</sub>



# Step-2: Generating Contrastive Sets

**Contrastive set:**

the cat the mat  
 the cat 's the mat  
 the mat on the cat  
 the mat of the cat



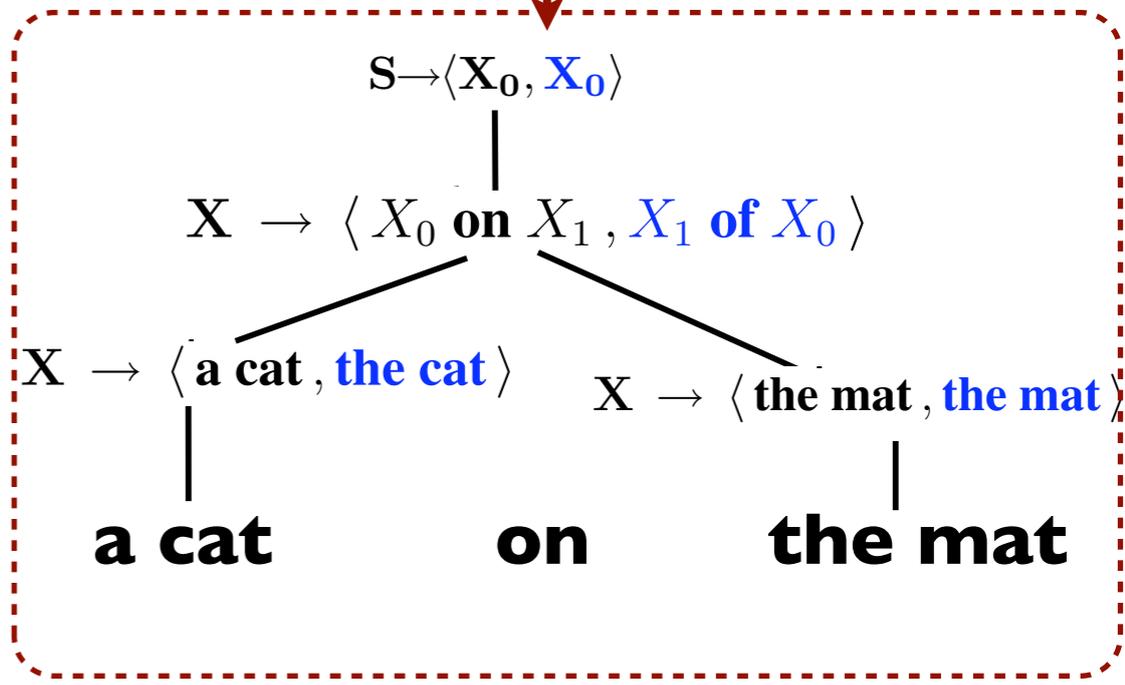
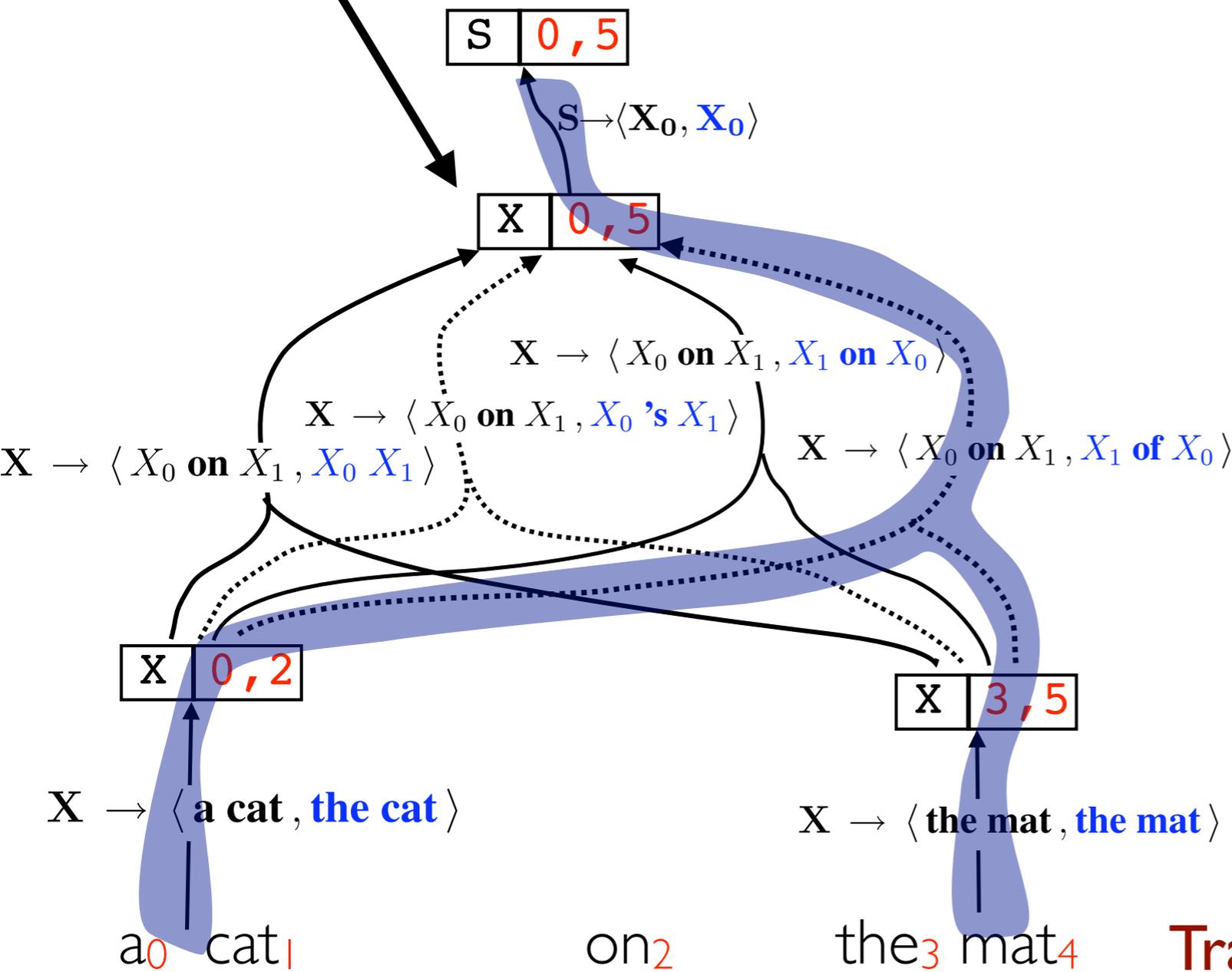
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a cat on the mat

**CG**



**Translating "dianzi shang de mao"?**

# Step-3: Discriminative Training

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- Training Objective

$$\theta^* = \arg \min_{\theta} \sum_i \sum_{y \in \mathcal{N}(\tilde{y}_i)} \mathbf{L}(y, \tilde{y}_i) p_{\theta}(y | \tilde{y}_i)$$

# Step-3: Discriminative Training

- Training Objective

$$\theta^* = \arg \min_{\theta} \sum_i \sum_{y \in \mathcal{N}(\tilde{y}_i)} L(y, \tilde{y}_i) p_{\theta}(y | \tilde{y}_i)$$

← ..... **contrastive set**

# Step-3: Discriminative Training

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$$\theta^* = \arg \min_{\theta} \sum_i \left( \sum_{y \in \mathcal{N}(\tilde{y}_i)} L(y, \tilde{y}_i) p_{\theta}(y | \tilde{y}_i) \right)$$

expected loss

contrastive set

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expected loss

contrastive set

CE maximizes the  
conditional likelihood

# Step-3: Discriminative Training

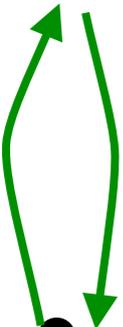
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expected loss  
↓  
contrastive set

CE maximizes the conditional likelihood

- Iterative Training

- Step-2: for each English sentence, generate a contrastive set (or neighborhood) using the CG
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# Applying the Contrastive Model

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- We can use the contrastive model as a regular language model
- We can incorporate the contrastive model into an end-to-end MT system as a feature
- We may also use the contrastive model to generate paraphrase sentences  
(if the loss function measures semantic similarity)
- the rules in CG are symmetric

# Test on Synthesized Hypergraphs of English Data

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**Monolingual  
English**

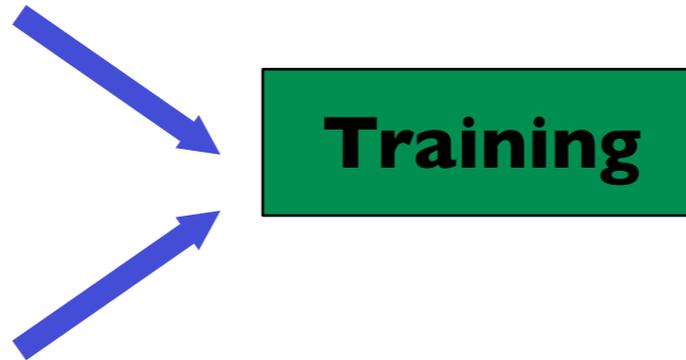
**Confusion  
grammar**

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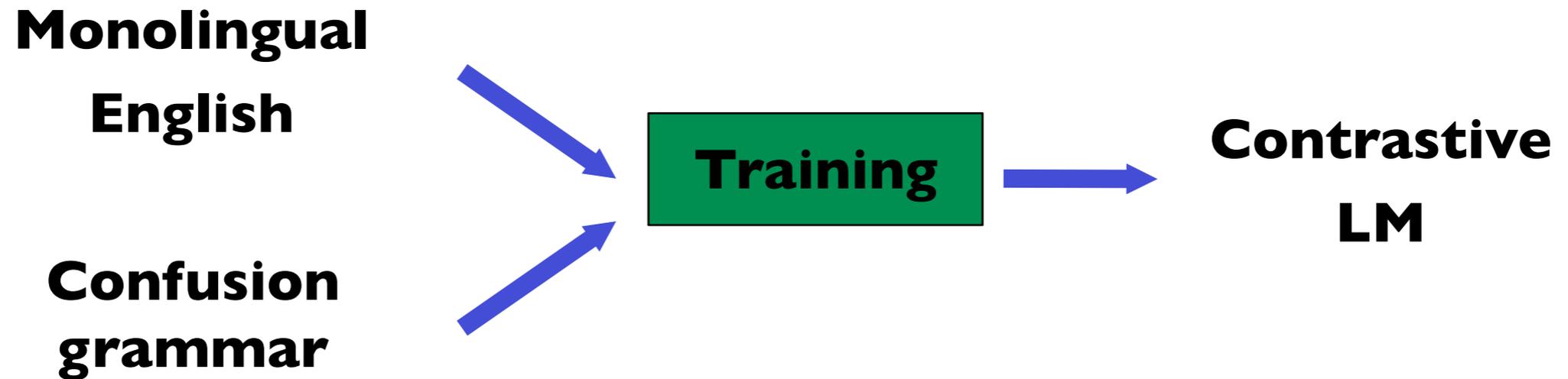
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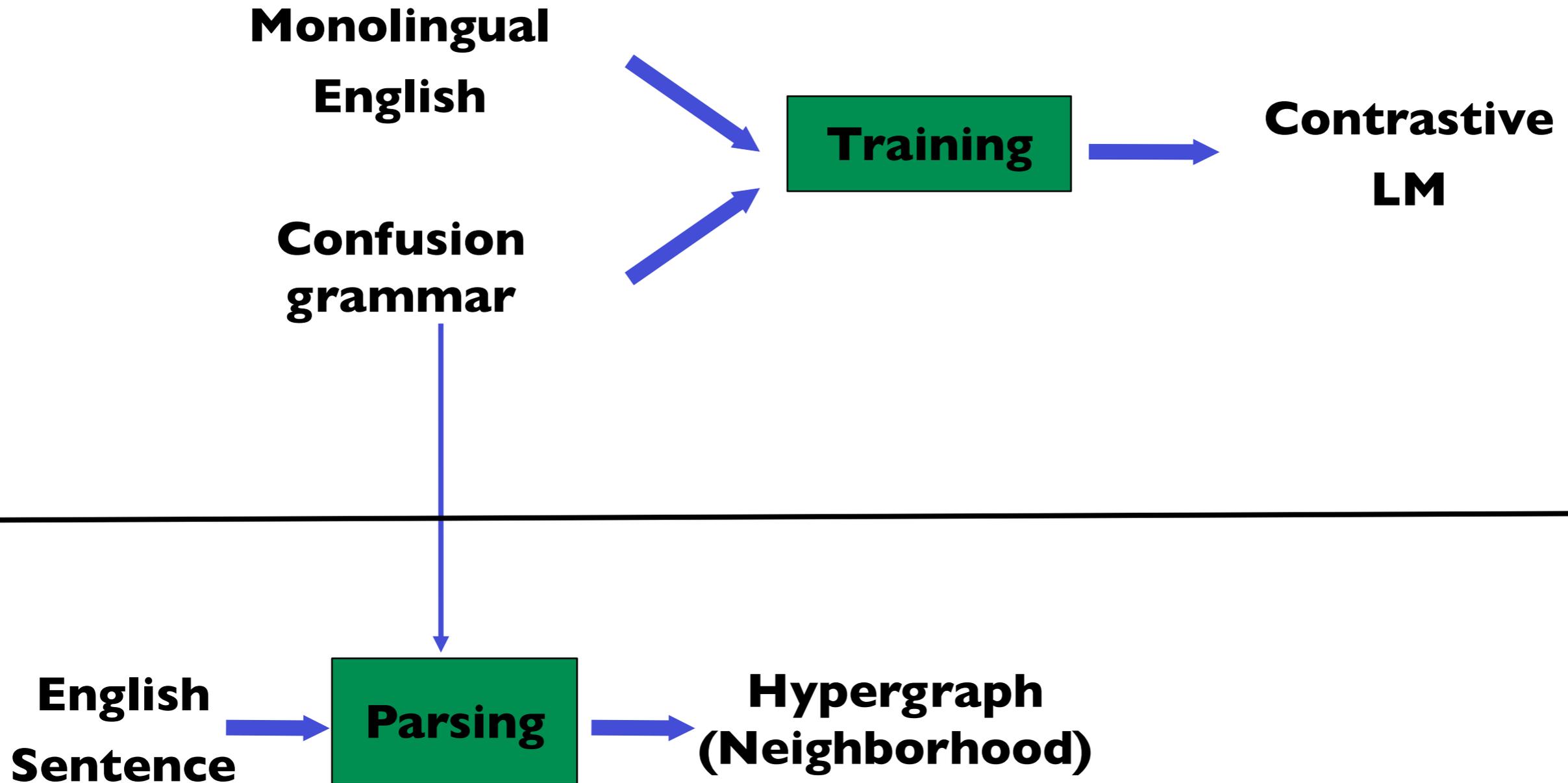
**Training**

**Contrastive  
LM**

**English  
Sentence**

**Parsing**

**Hypergraph  
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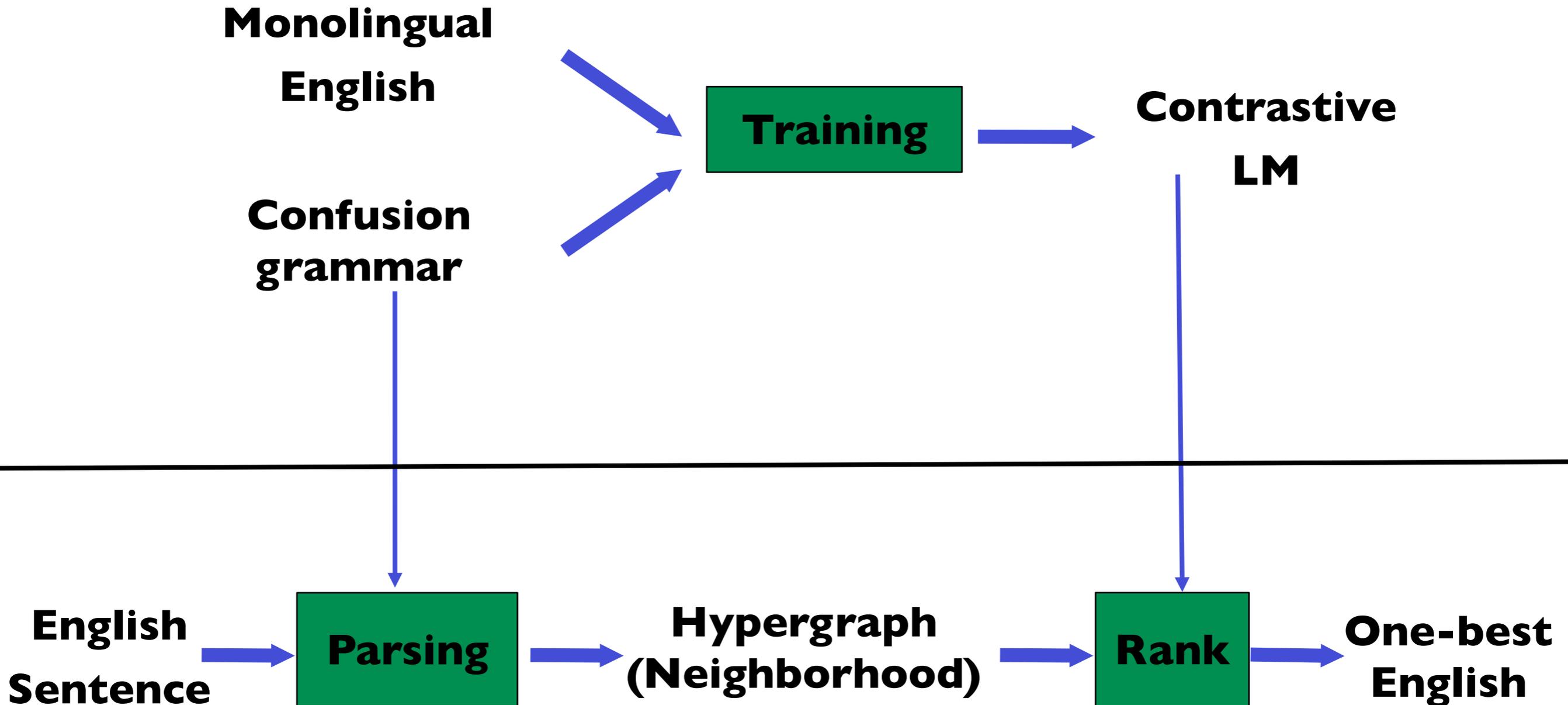
**English  
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**Rank**

**One-best  
English**



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BLEU Score?

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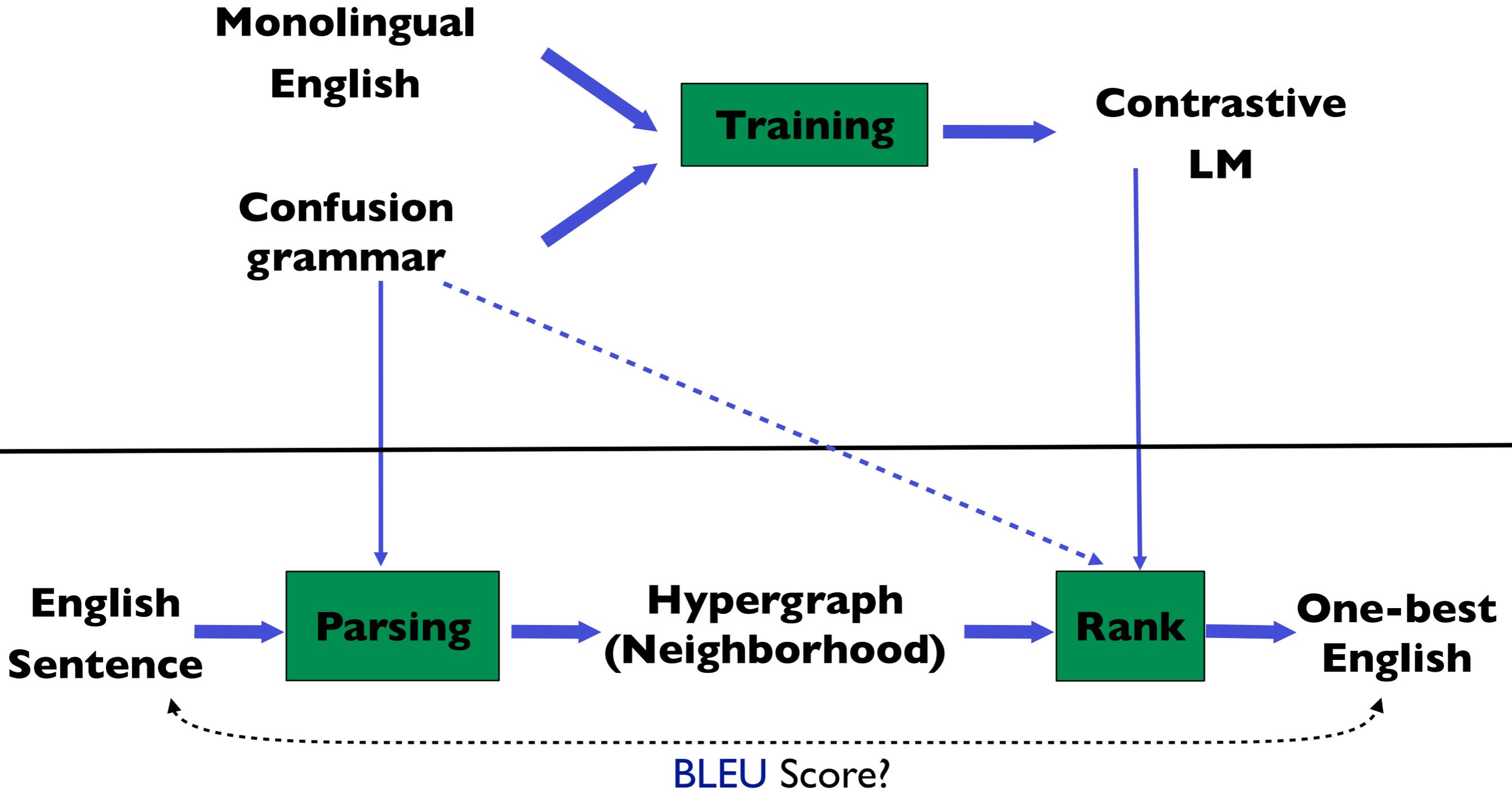
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**Rank**

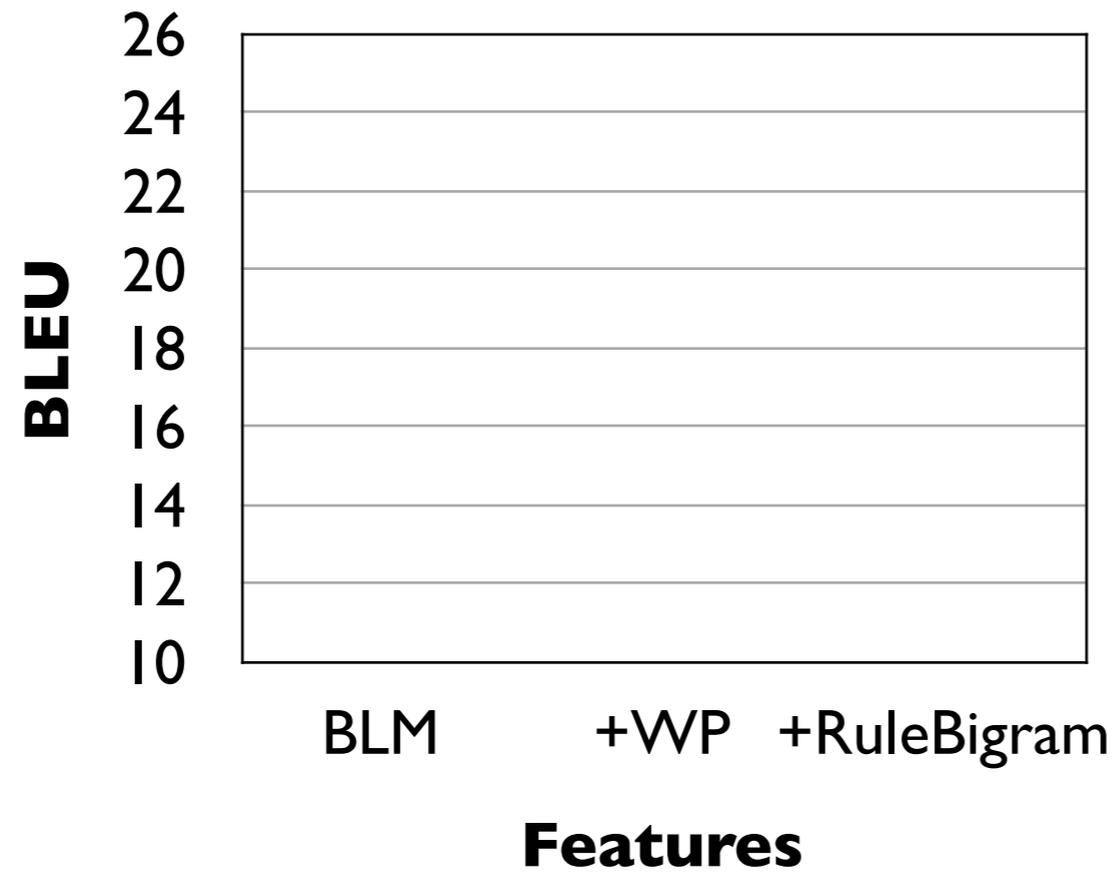
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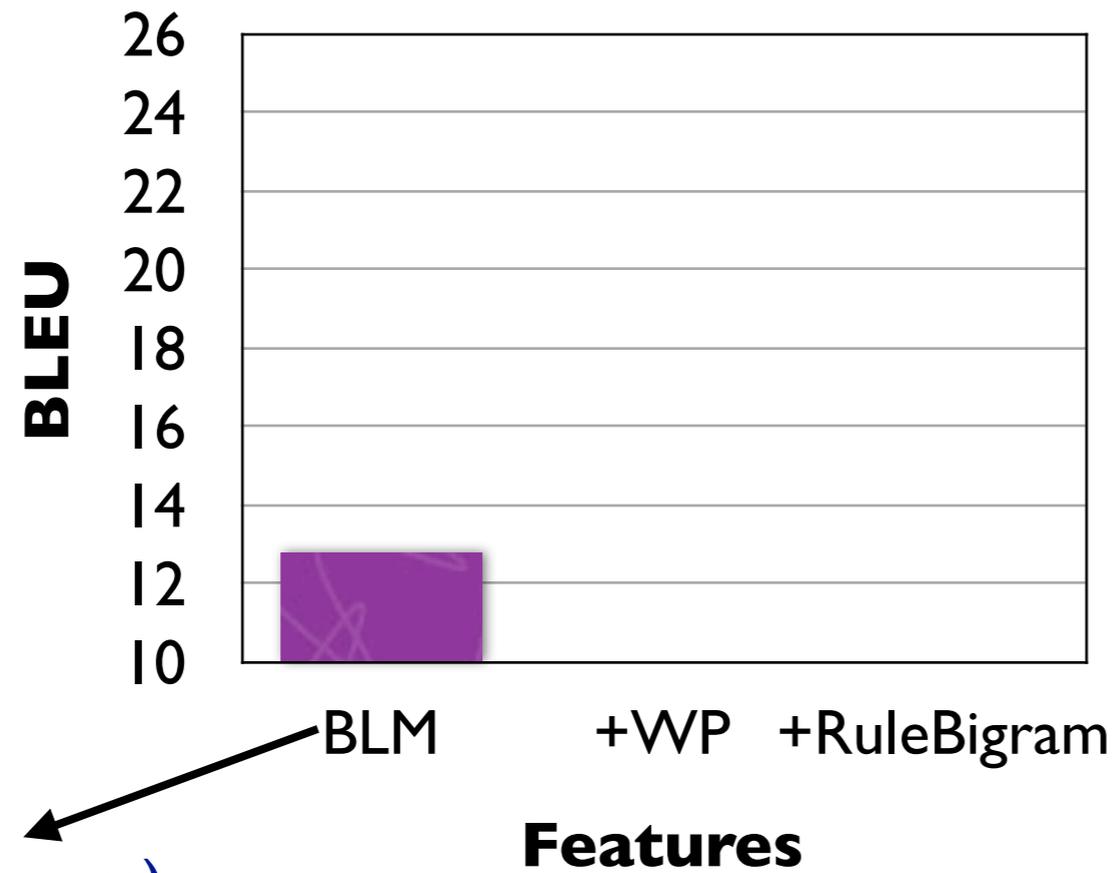


# Results on Synthesized Hypergraphs

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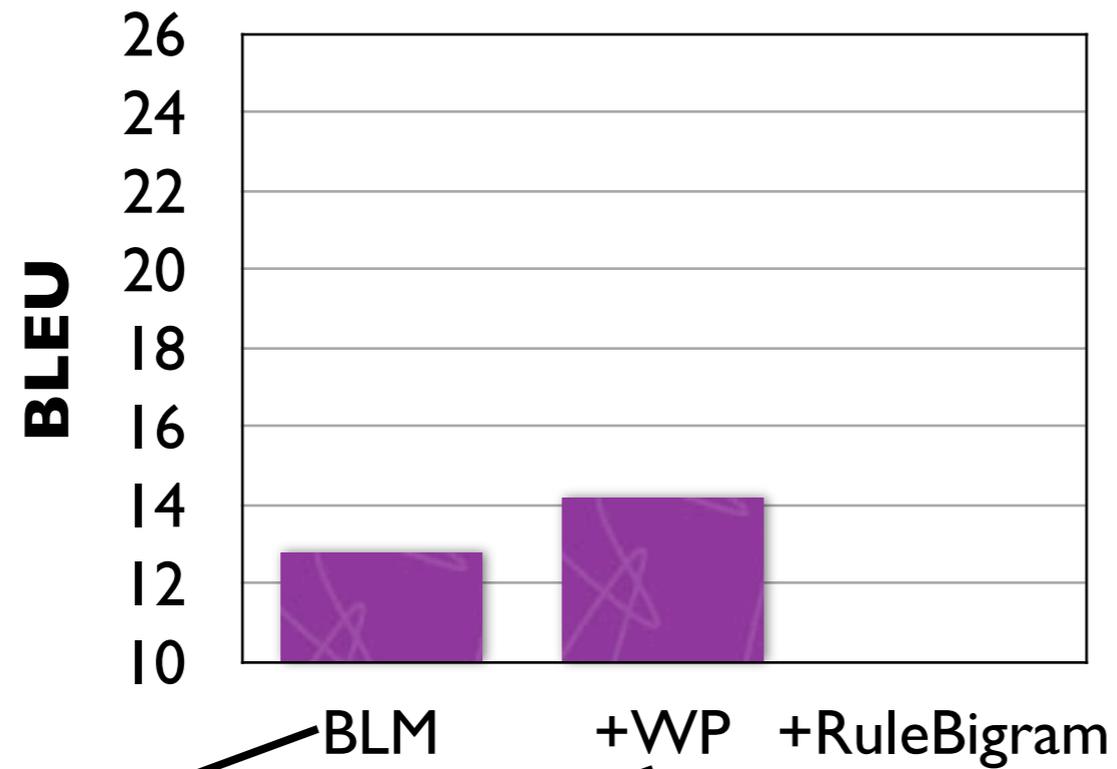


# Results on Synthesized Hypergraphs



baseline LM (5-gram)

# Results on Synthesized Hypergraphs

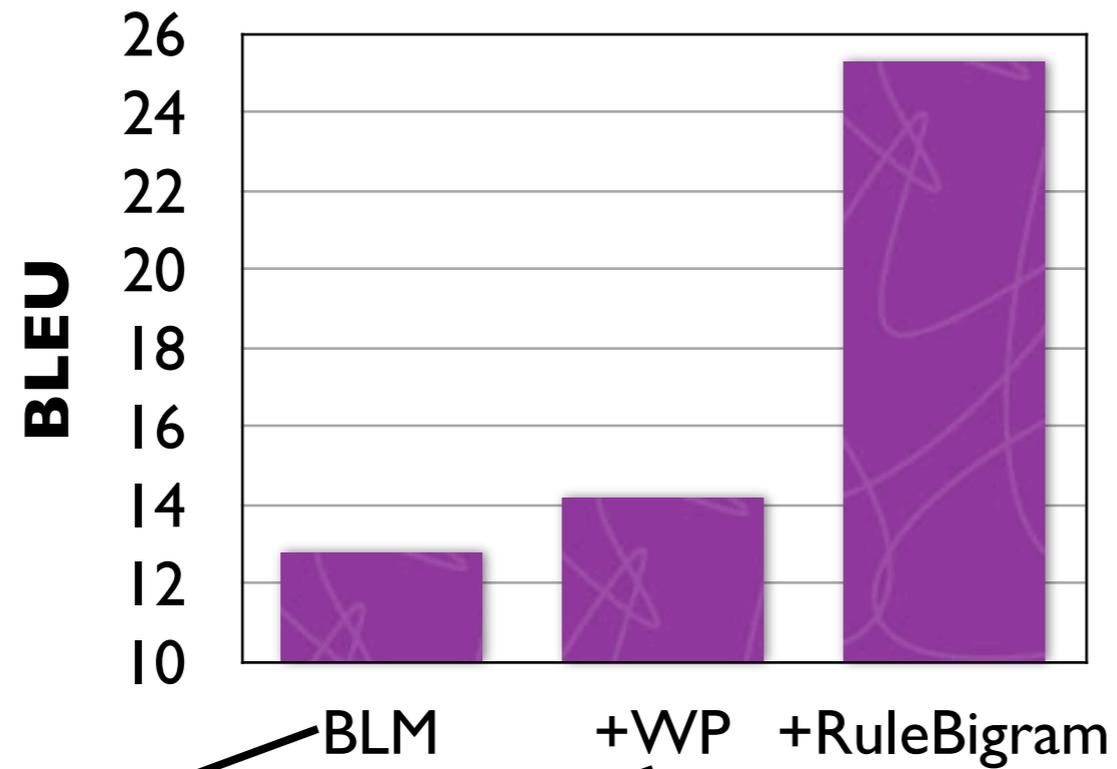


baseline LM (5-gram)

word penalty

**Features**

# Results on Synthesized Hypergraphs



BLM

+VWP

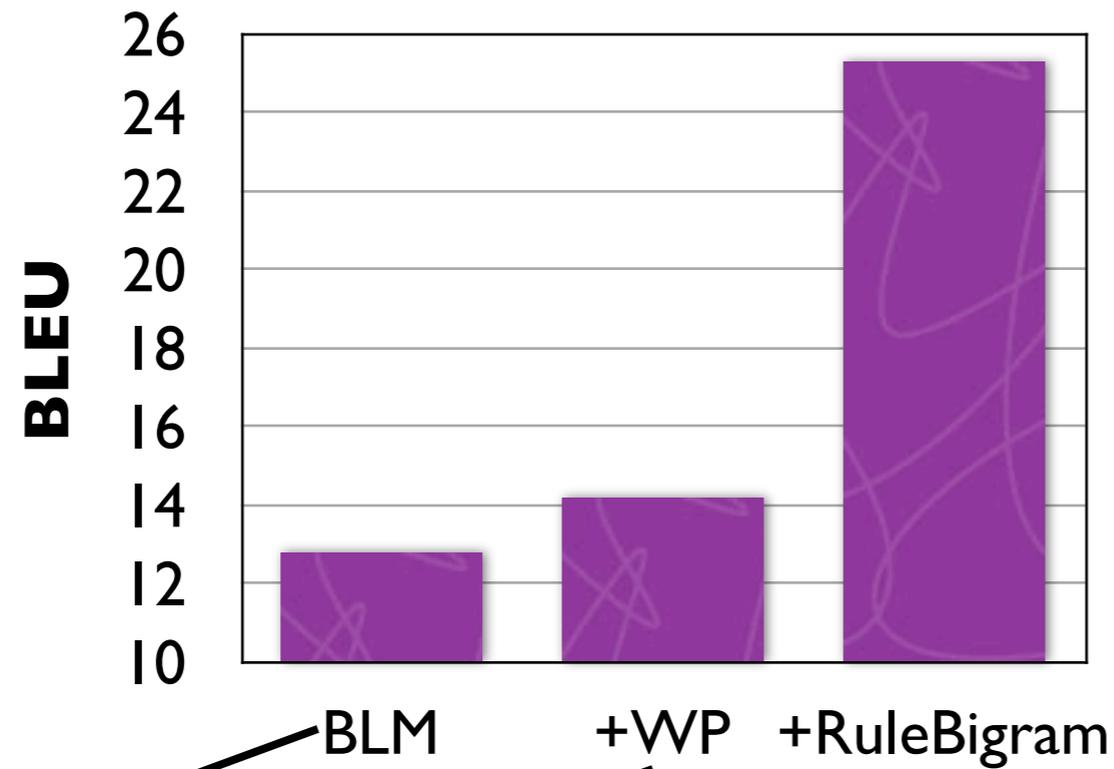
+RuleBigram

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baseline LM (5-gram)

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BLM

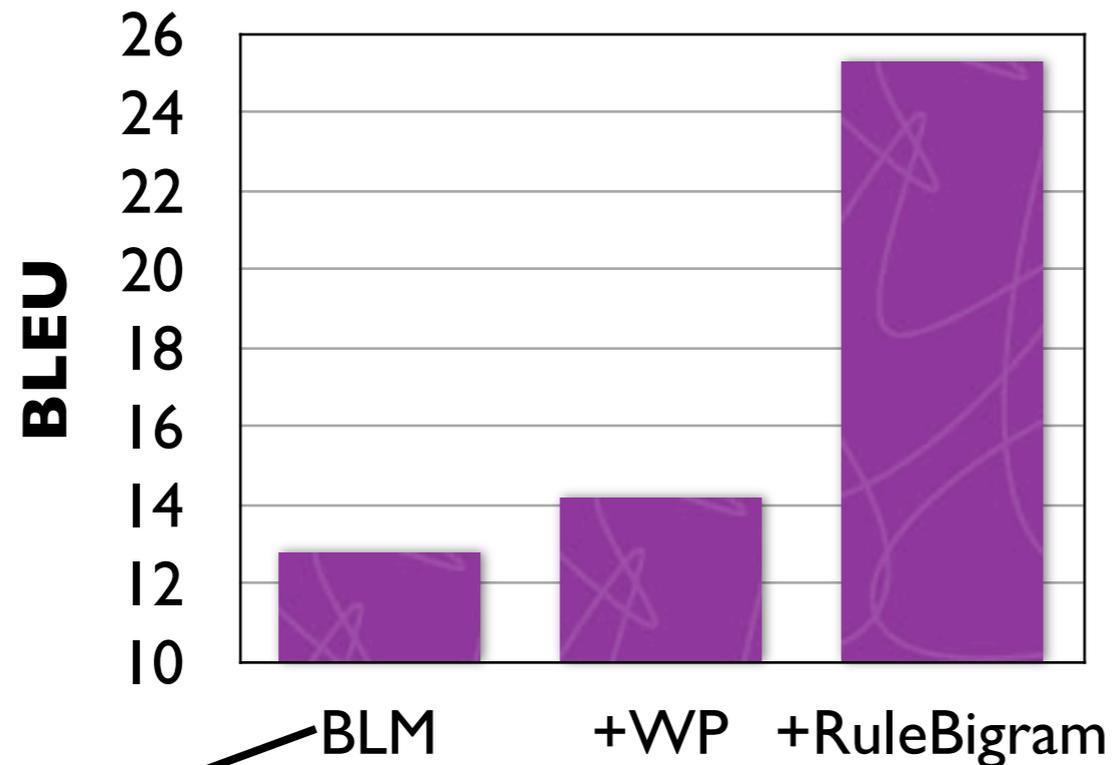
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**Features**



# Results on Synthesized Hypergraphs



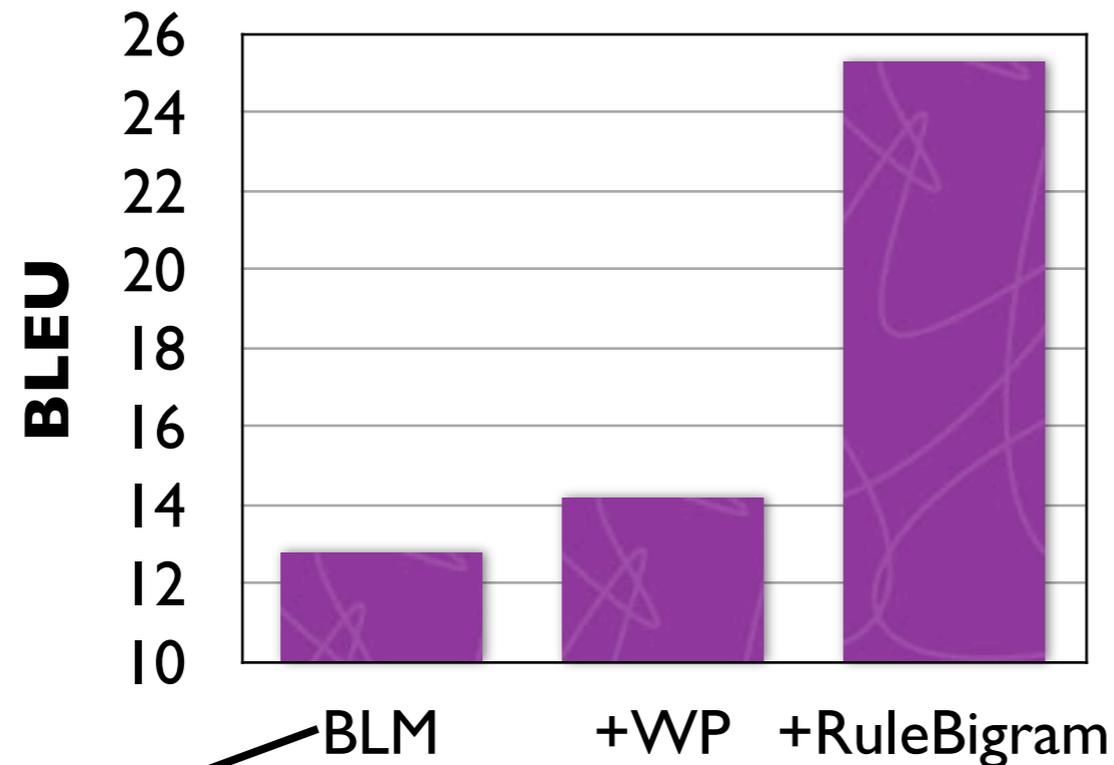
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Features

- Target side of a confusion rule  
“on the  $X_1$  issue of  $X_2$ ”

# Results on Synthesized Hypergraphs



baseline LM (5-gram)

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*“on the  $X_1$  issue of  $X_2$ ”*

- Rule bigram features

*“on the”*

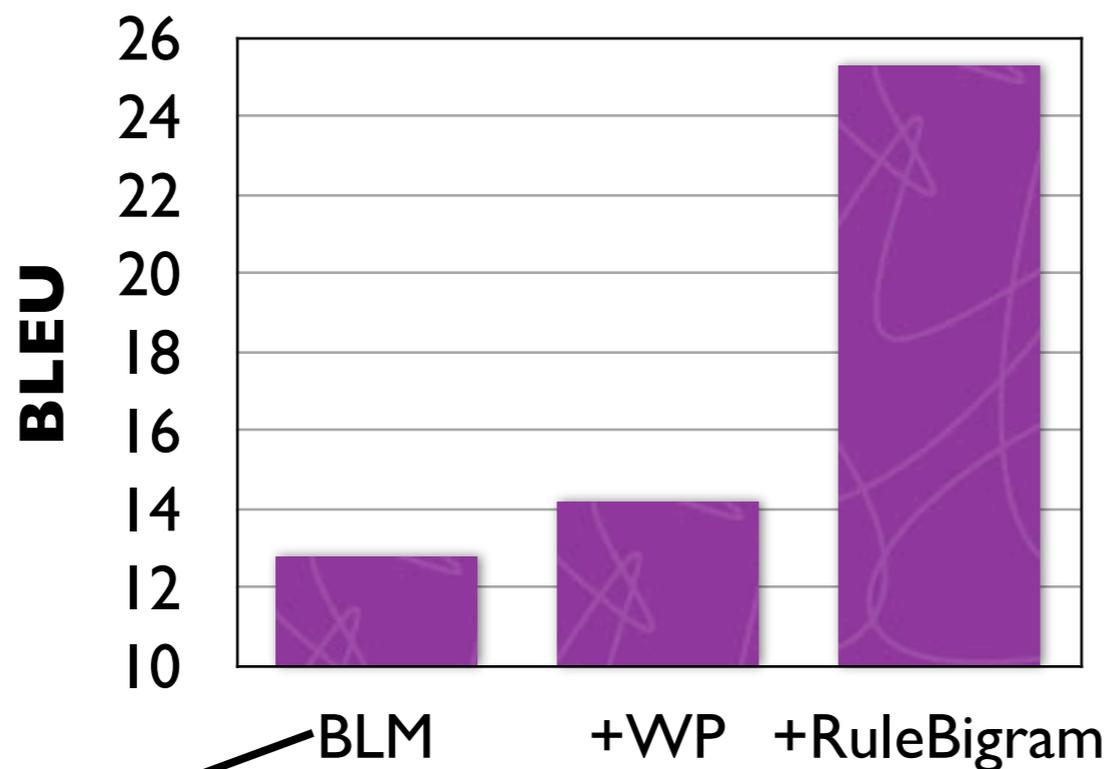
*“the  $X$ ”*

*“ $X$  issue”*

*“issue of”*

*“of  $X$ ”*

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The contrastive LM better **recovers** the original English than a regular n-gram LM.

baseline LM (5-gram)

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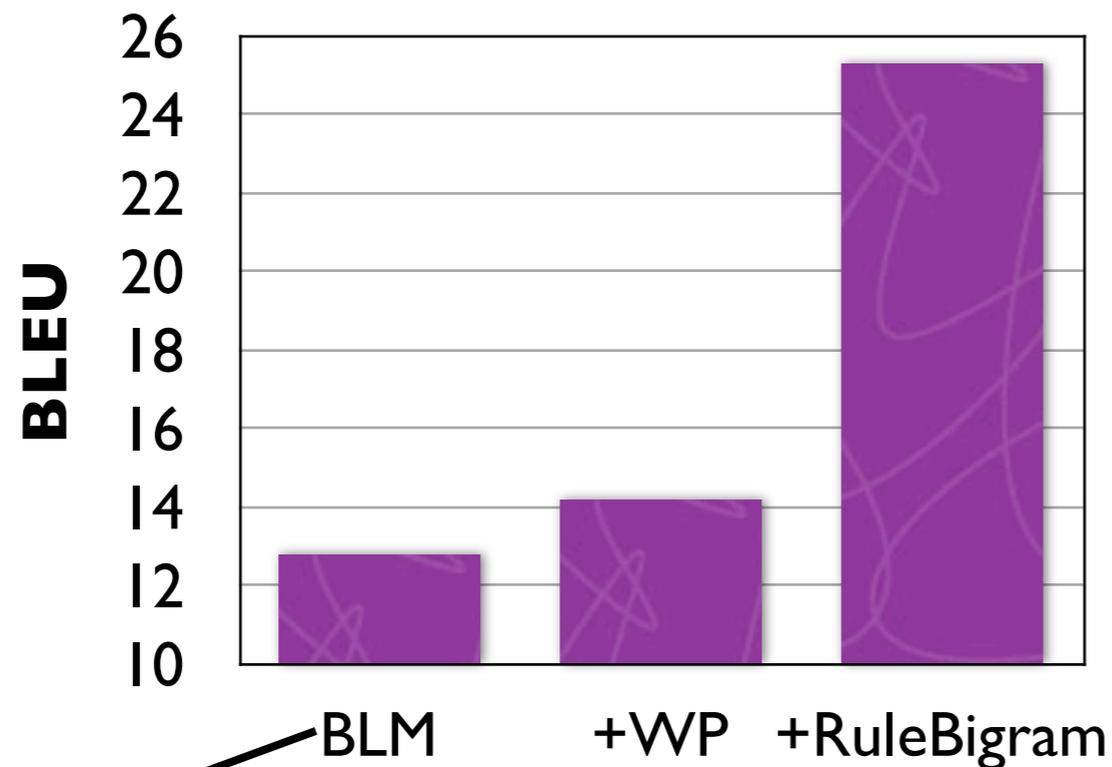
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# Results on Synthesized Hypergraphs



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All the features look at only the **target sides** of confusion rules

baseline LM (5-gram)

word penalty

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- **Target side** of a confusion rule

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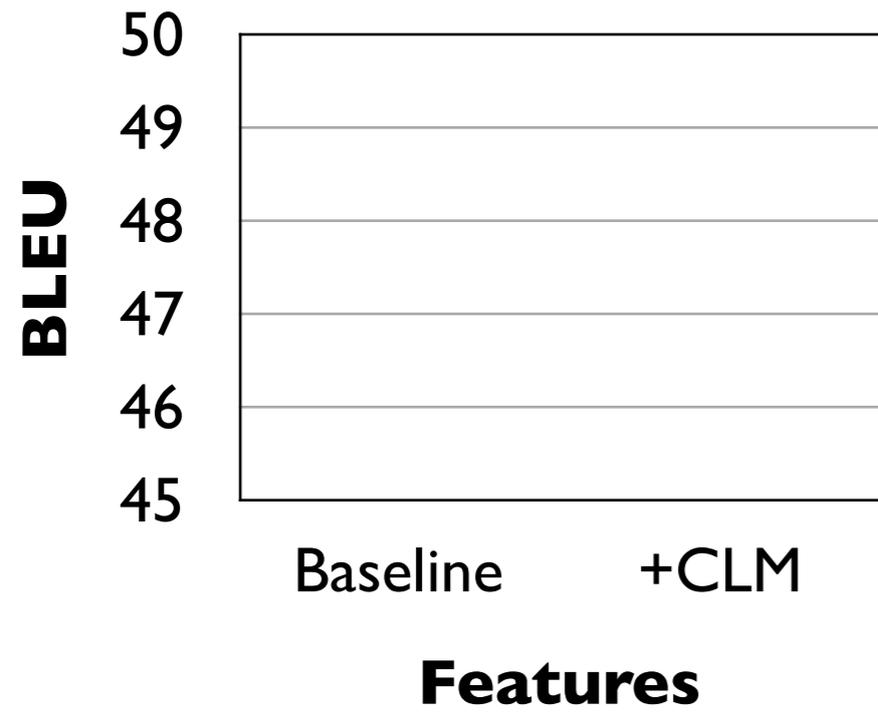
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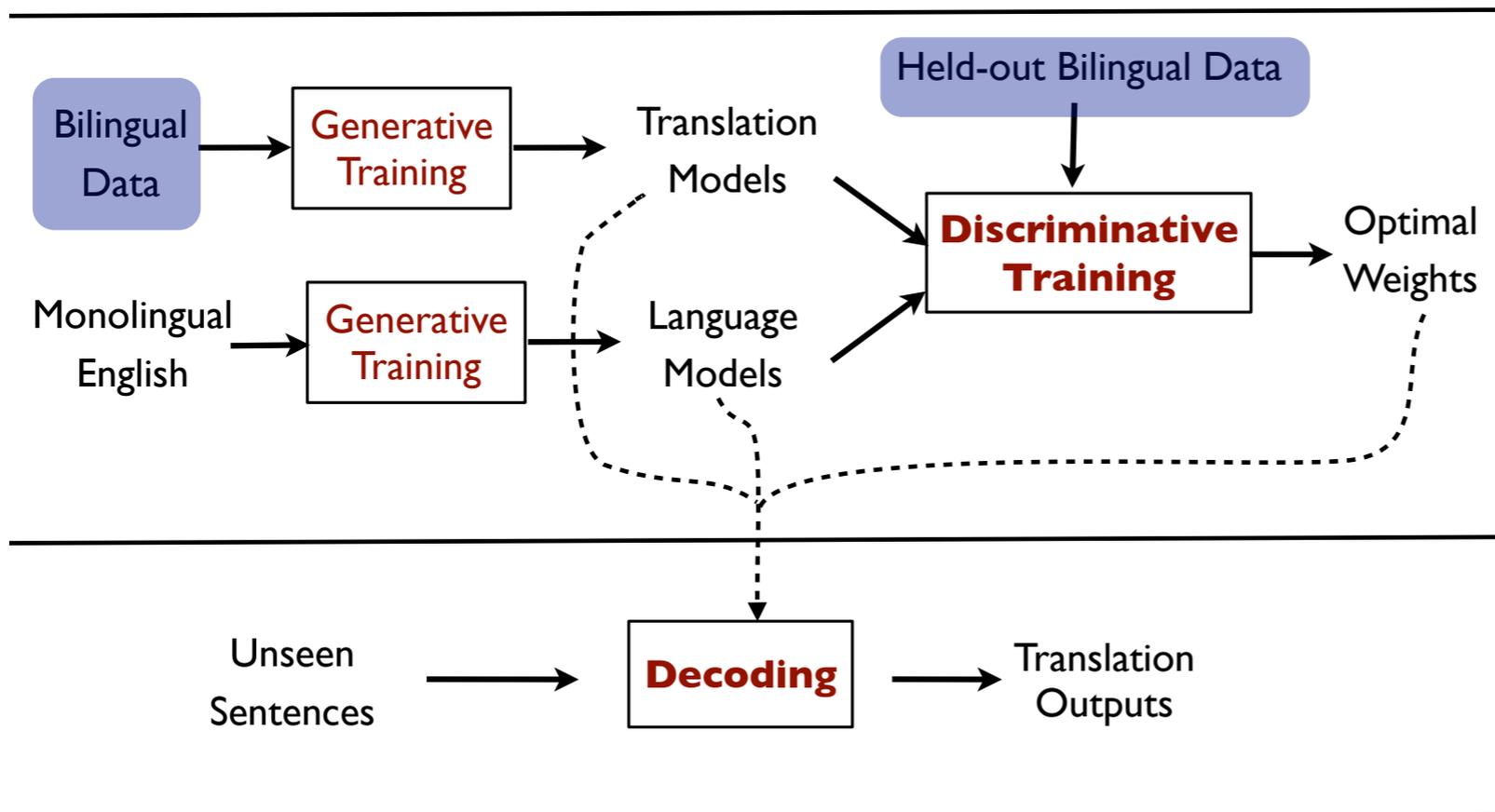
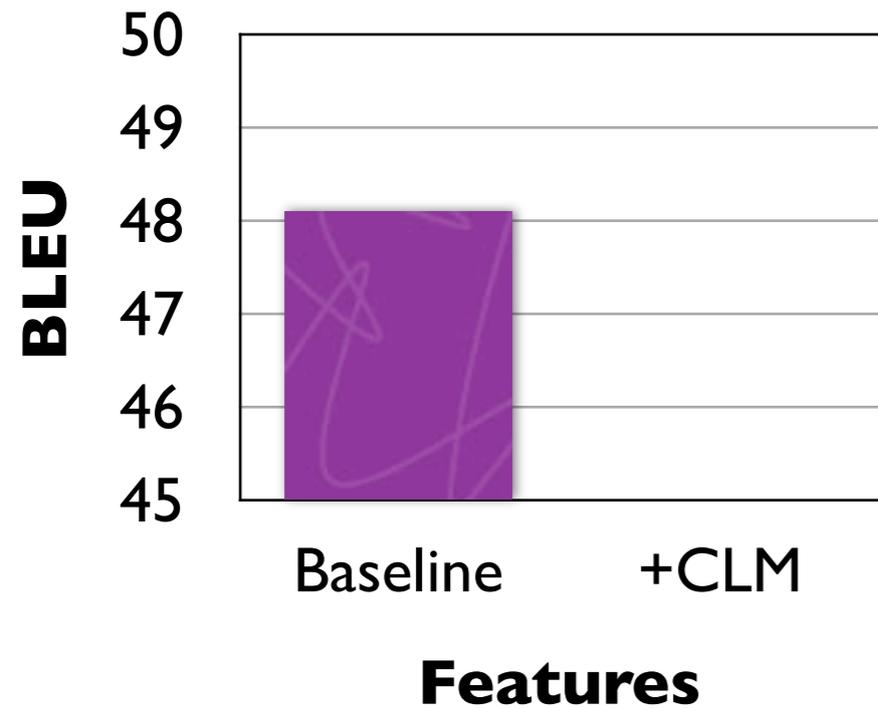
*“of  $X$ ”*

# Results on MT Test Set

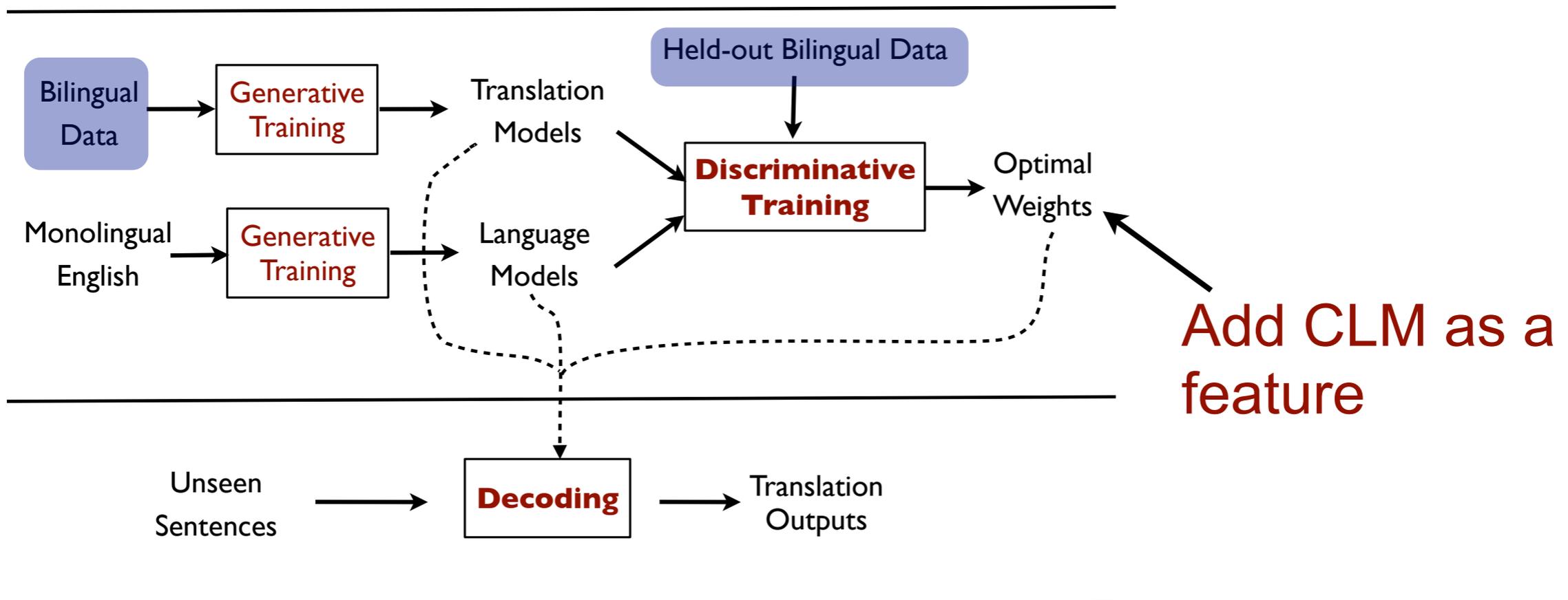
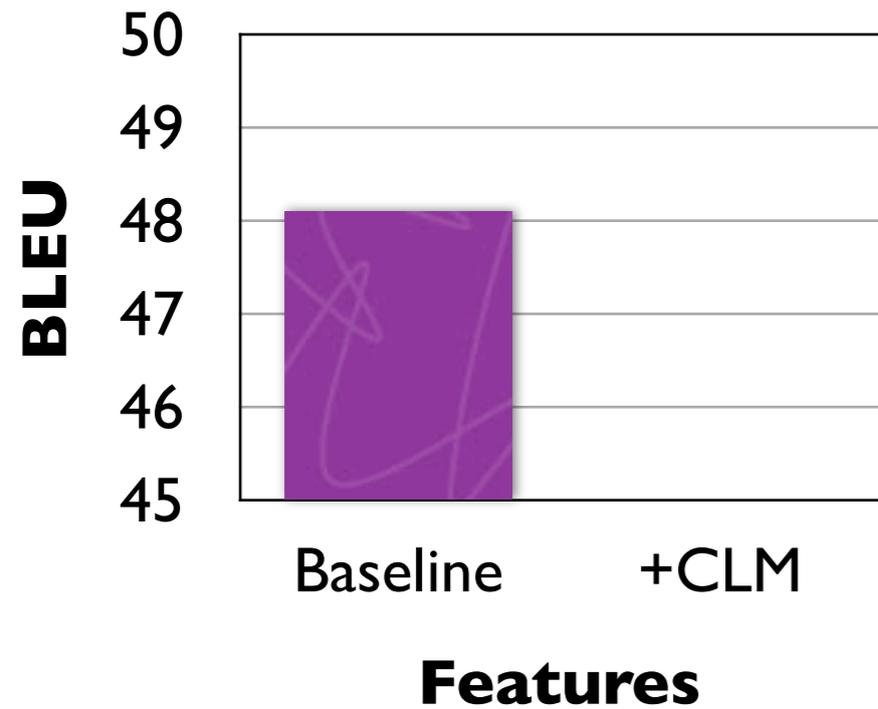
# Results on MT Test Set



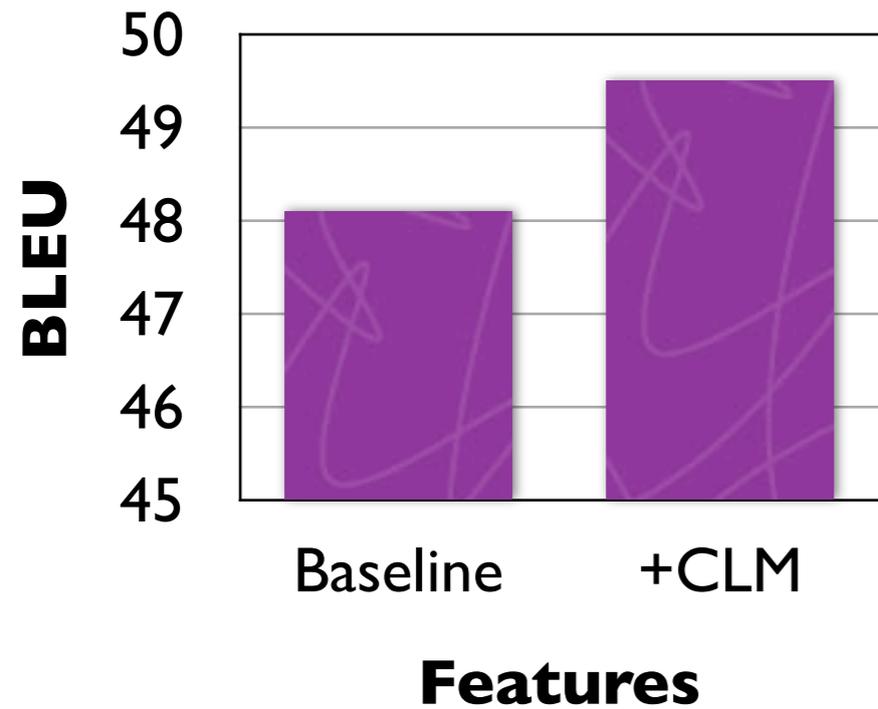
# Results on MT Test Set



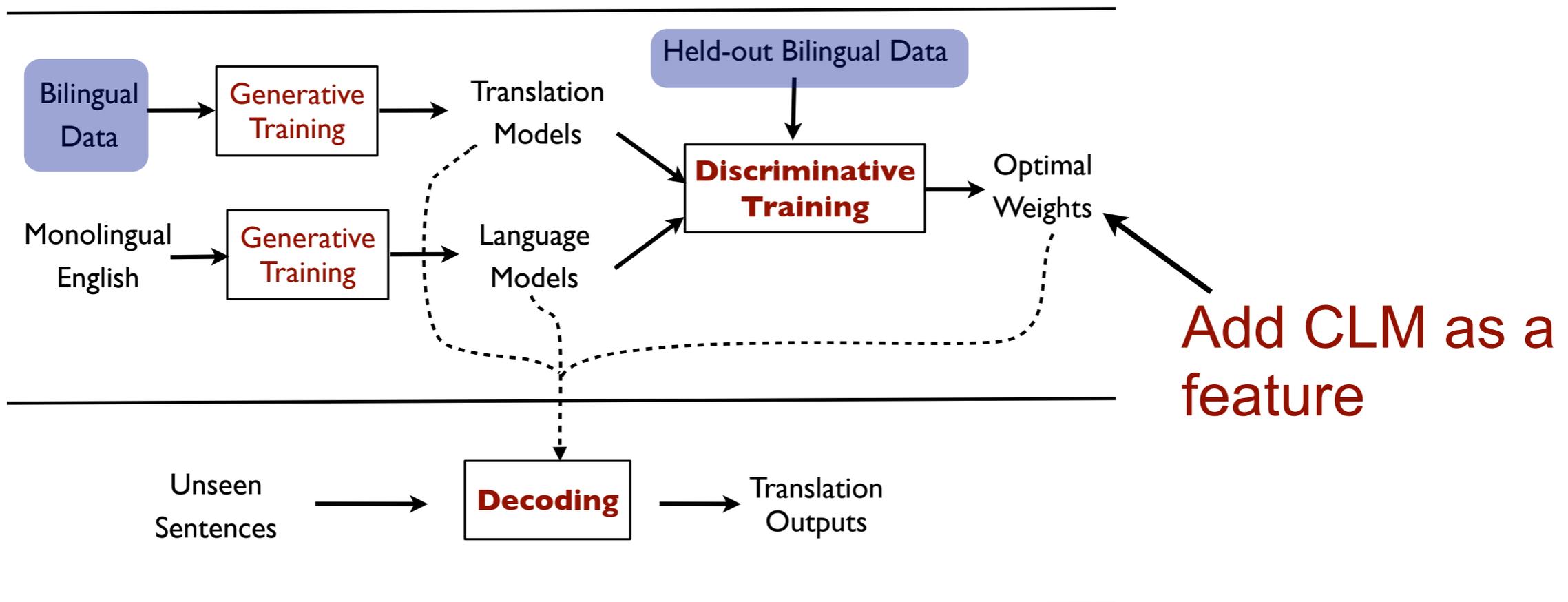
# Results on MT Test Set



# Results on MT Test Set



The contrastive LM helps to improve MT performance.



# Adding Features on the CG itself

- On English Set

- On MT Set

# Adding Features on the CG itself

- On English Set



- On MT Set

# Adding Features on the CG itself

- On English Set

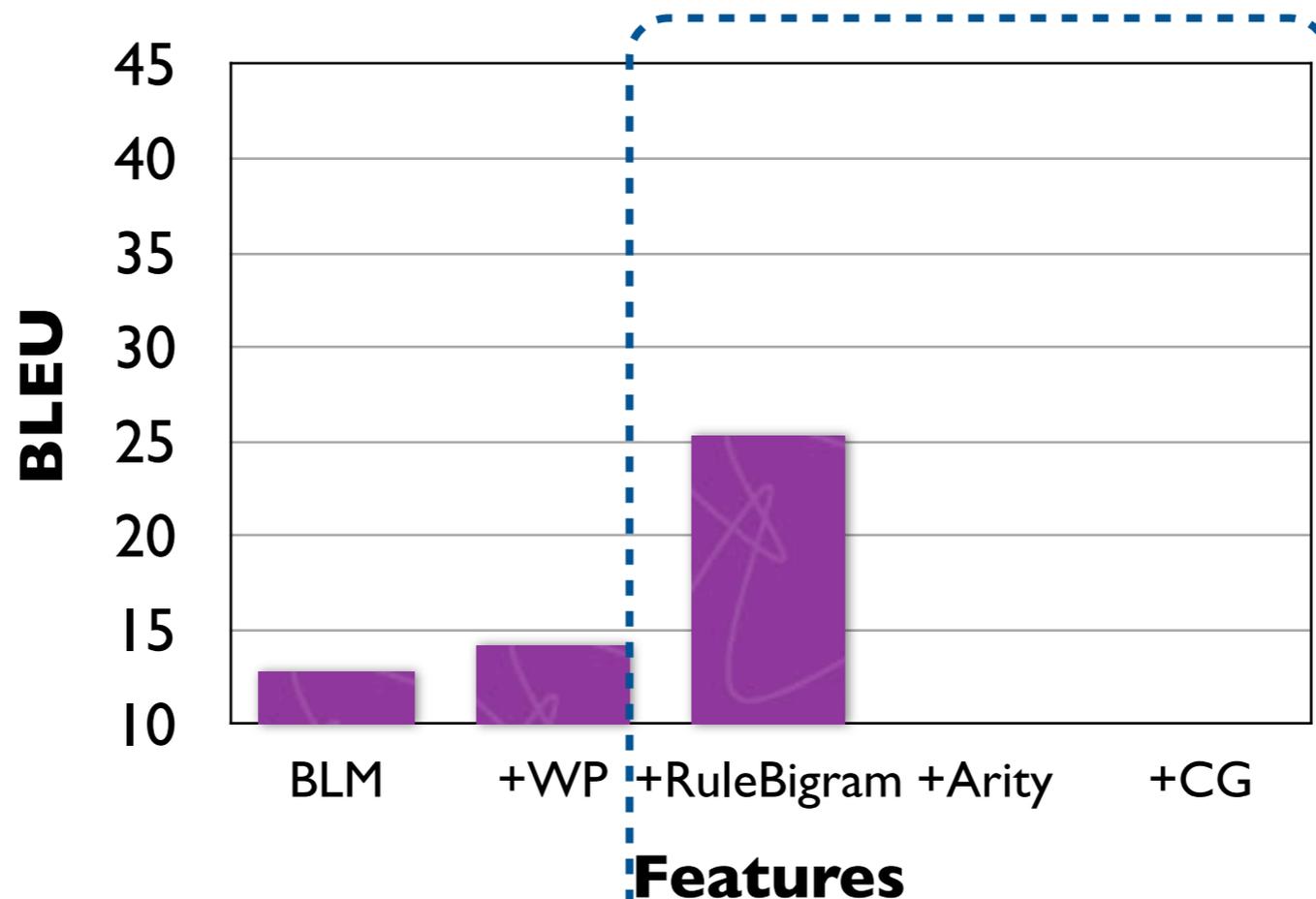


- On MT Set

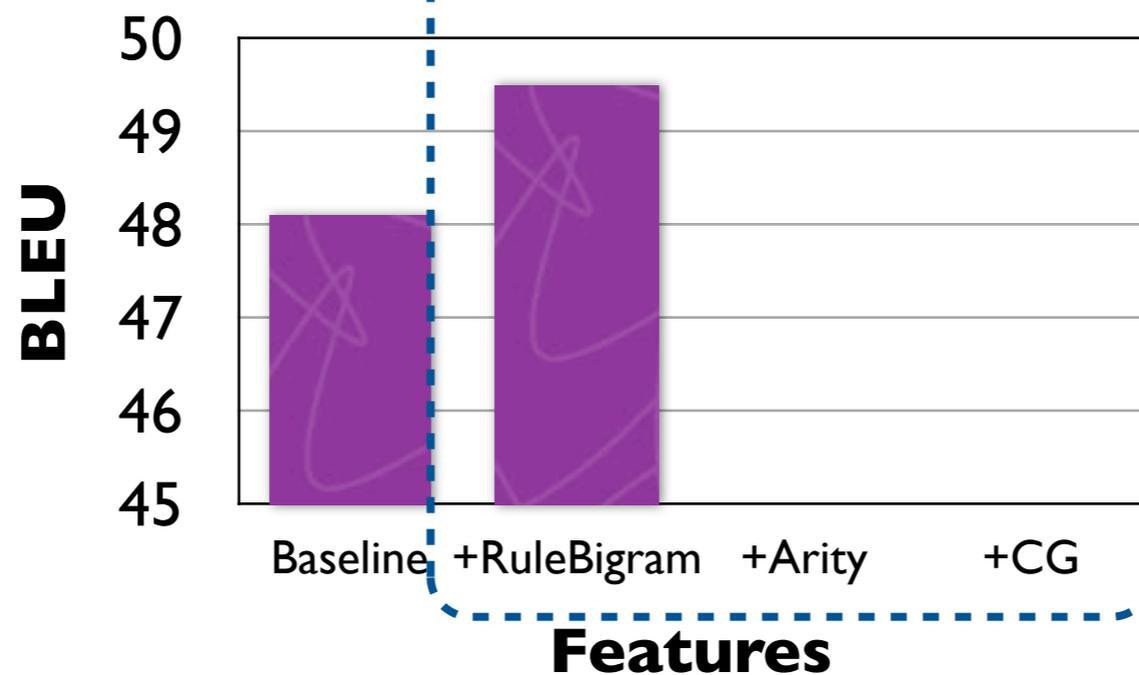


# Adding Features on the CG itself

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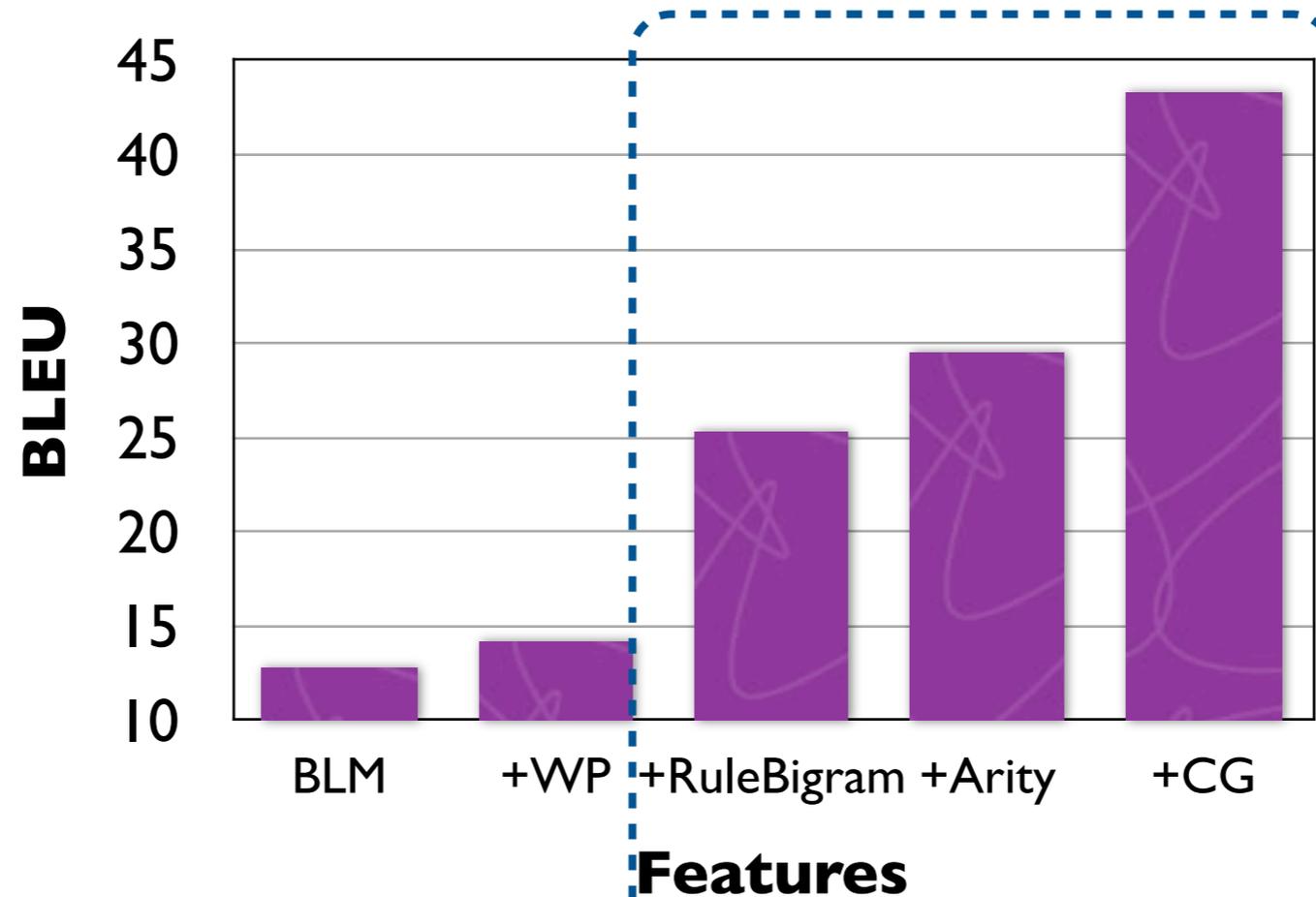


- On MT Set

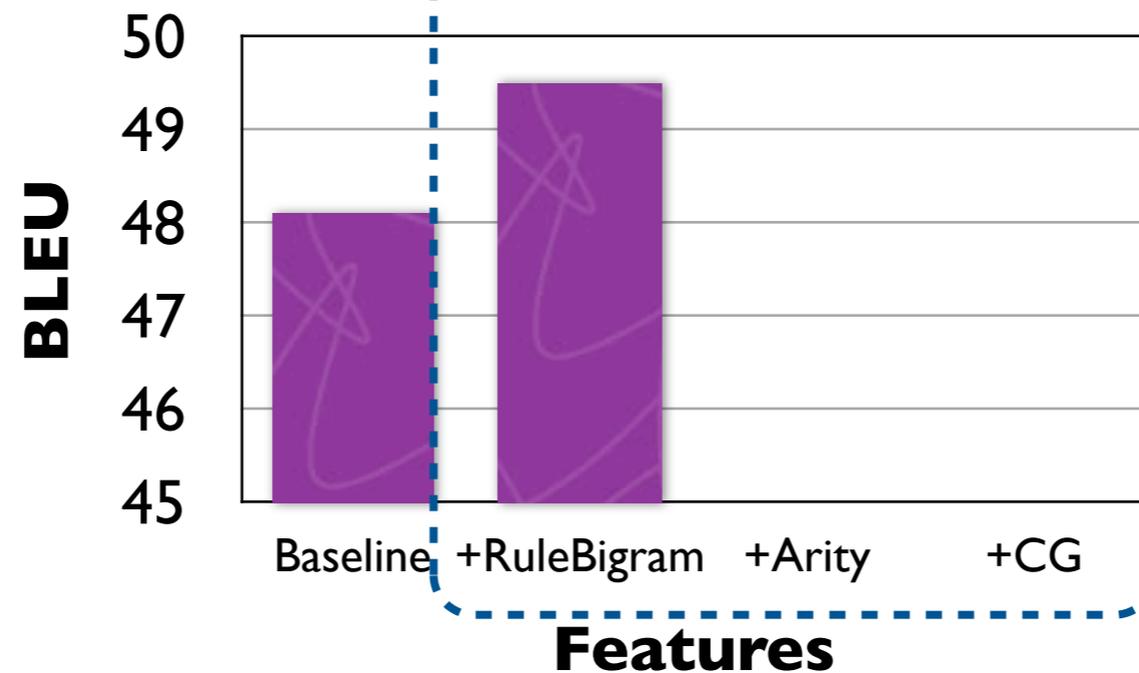


# Adding Features on the CG itself

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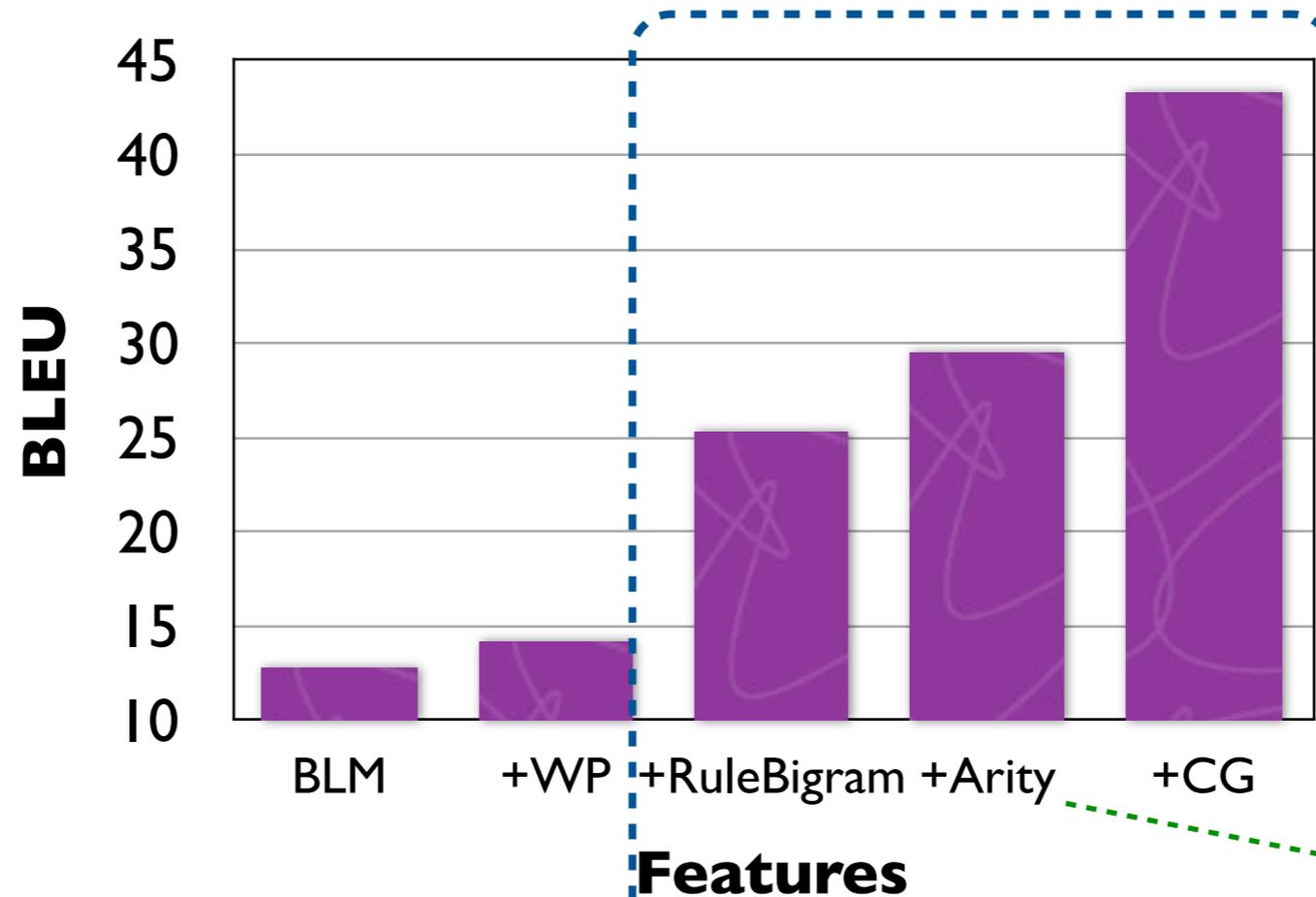


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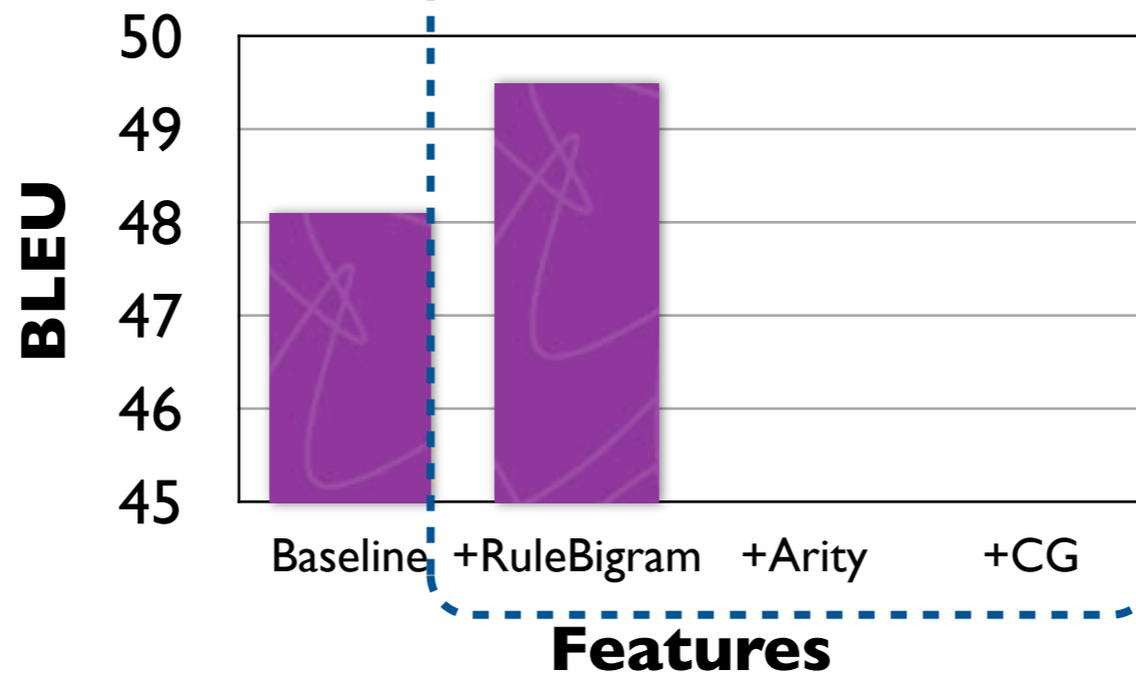


# Adding Features on the CG itself

- On English Set



- On MT Set



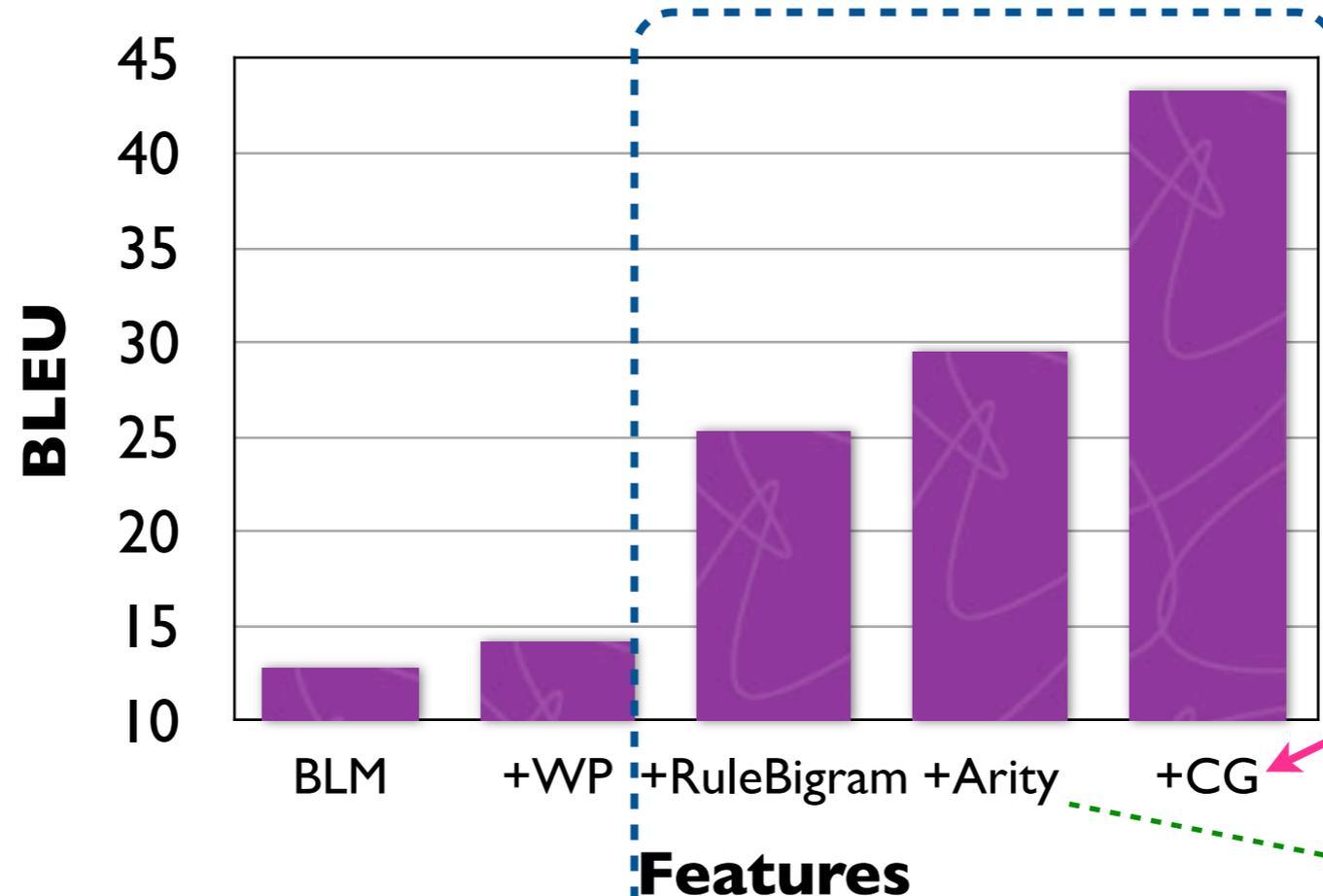
glue rules or regular confusion rules?

$$S \rightarrow \langle S_0 X_1, S_0 X_1 \rangle$$

$$S \rightarrow \langle X_0, X_0 \rangle$$

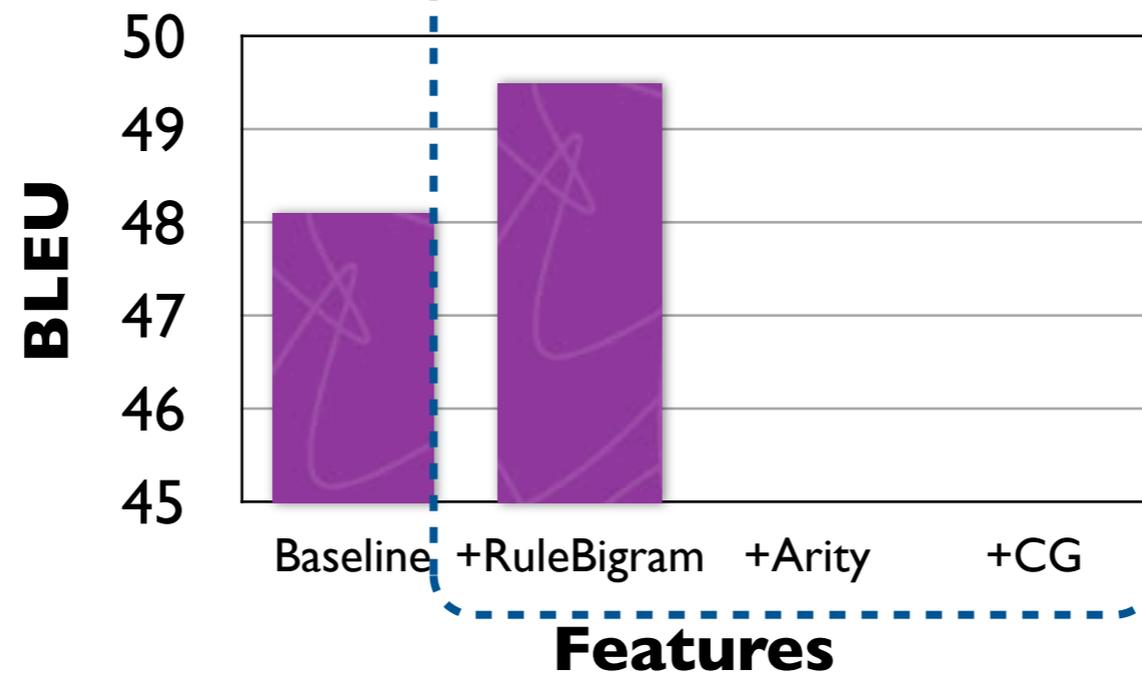
# Adding Features on the CG itself

- On English Set



one big feature

- On MT Set

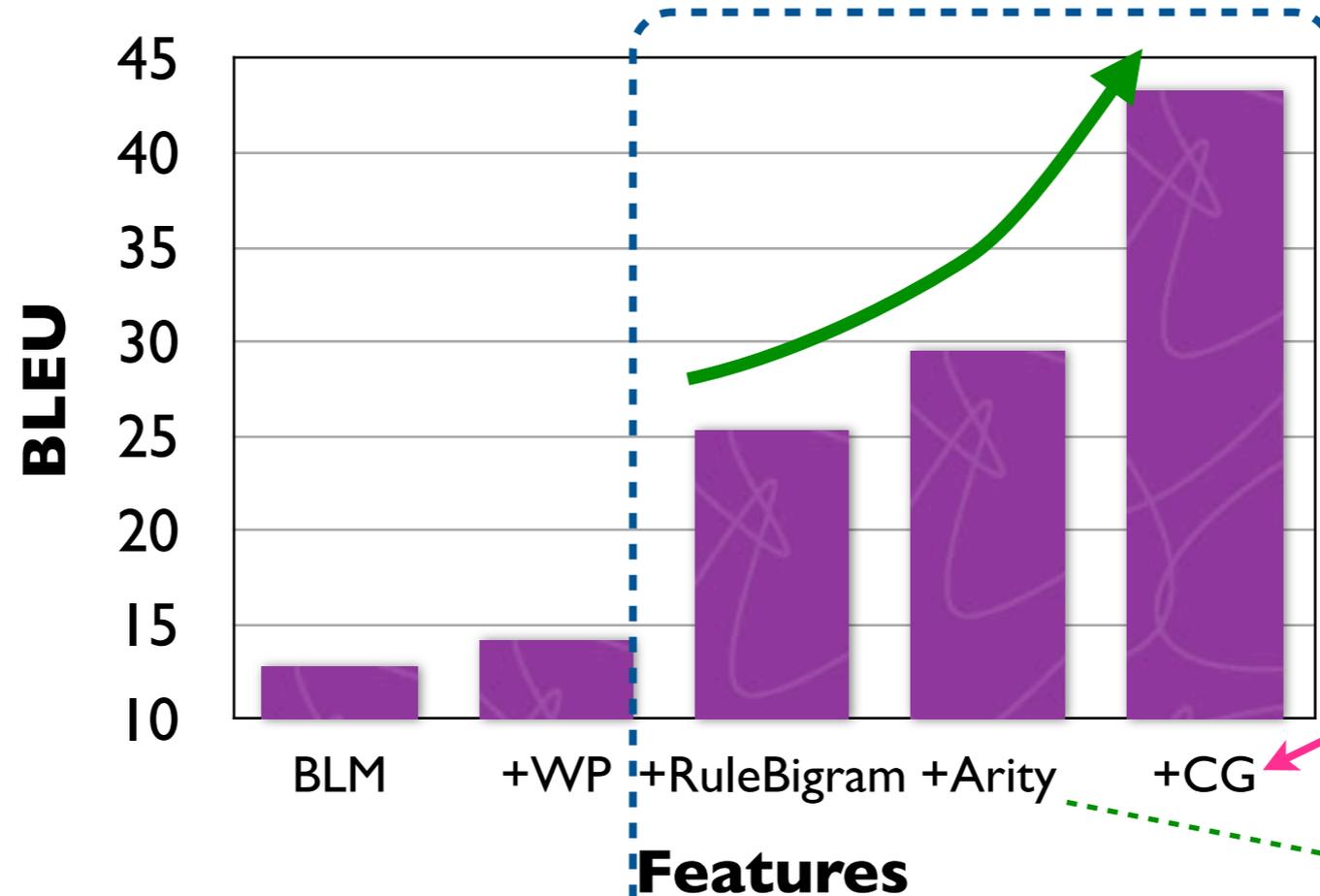


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$$S \rightarrow \langle S_0 X_1, S_0 X_1 \rangle$$
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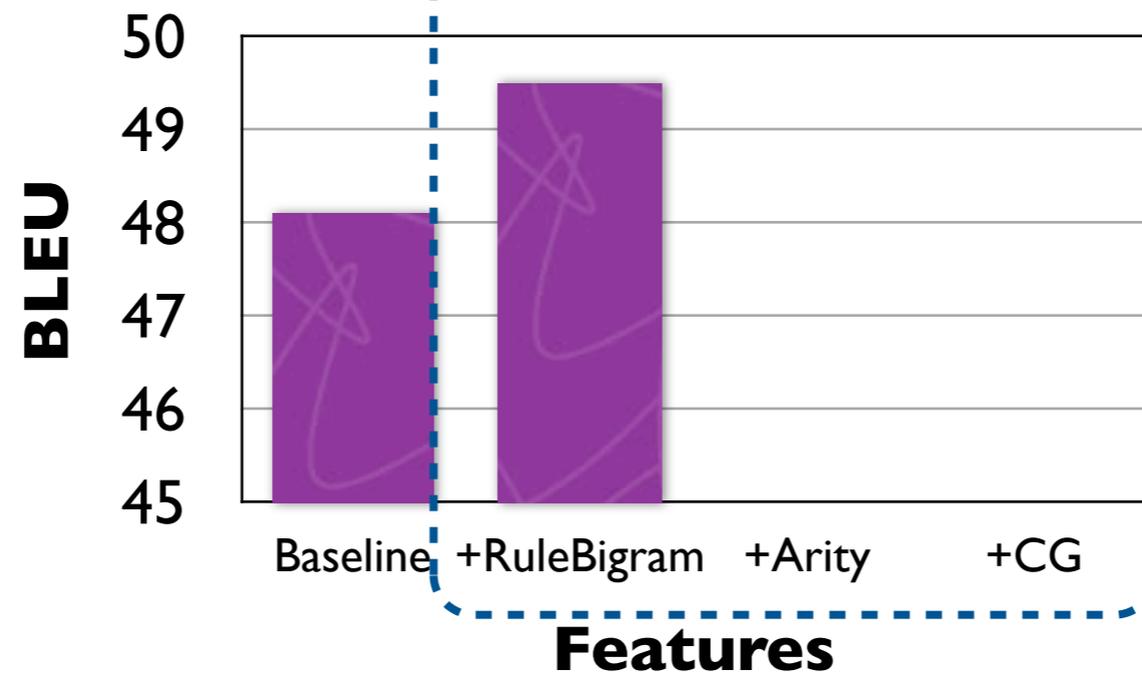
# Adding Features on the CG itself

- On English Set



one big feature

- On MT Set



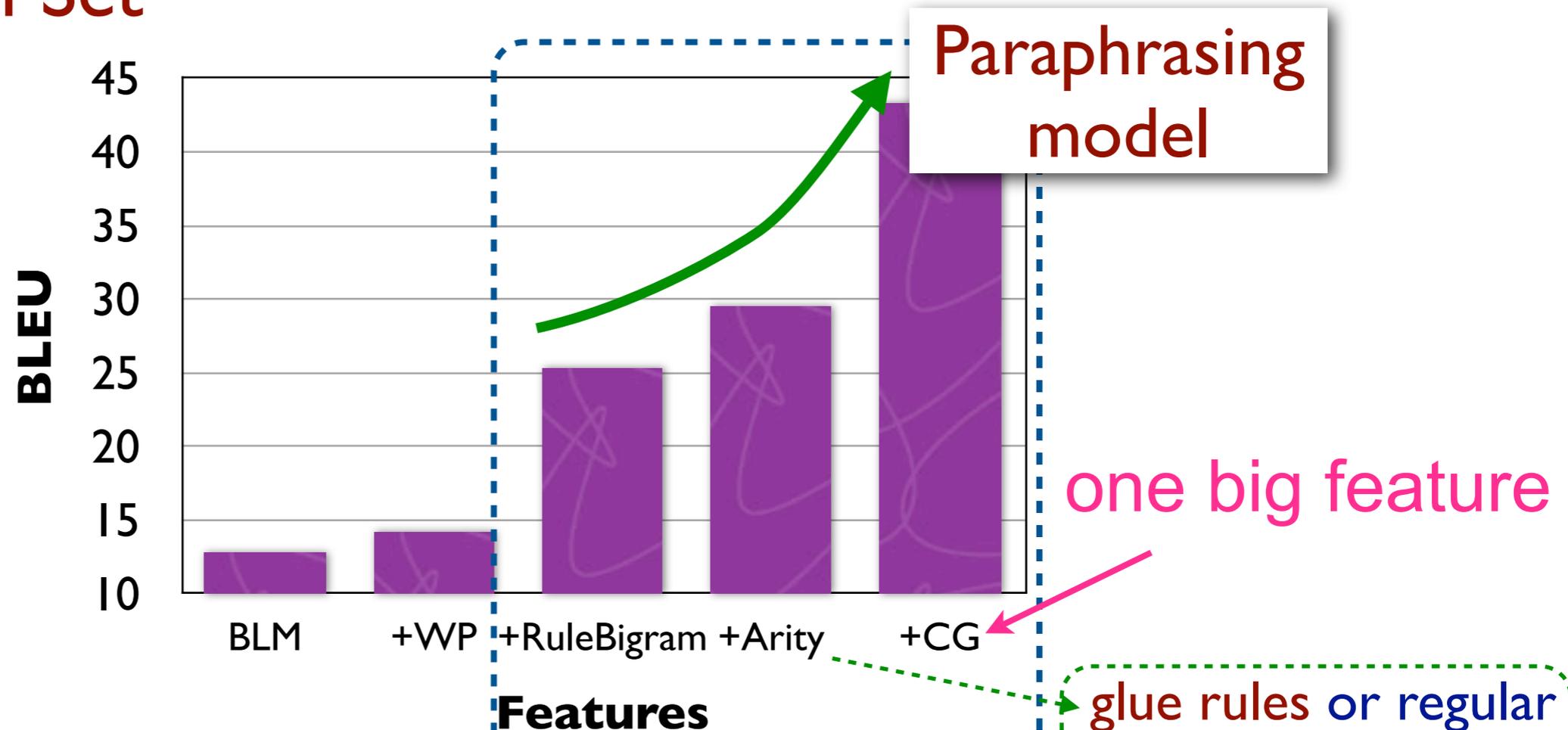
glue rules or regular confusion rules?

$$S \rightarrow \langle S_0 X_1, S_0 X_1 \rangle$$

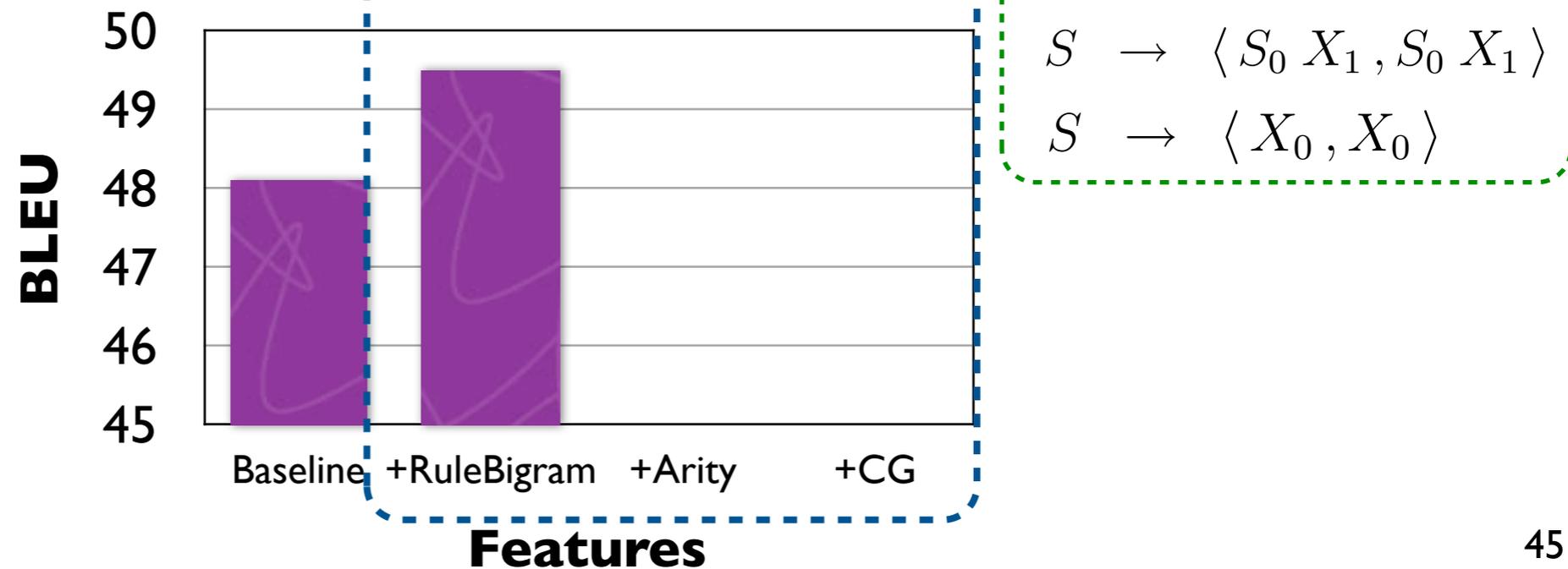
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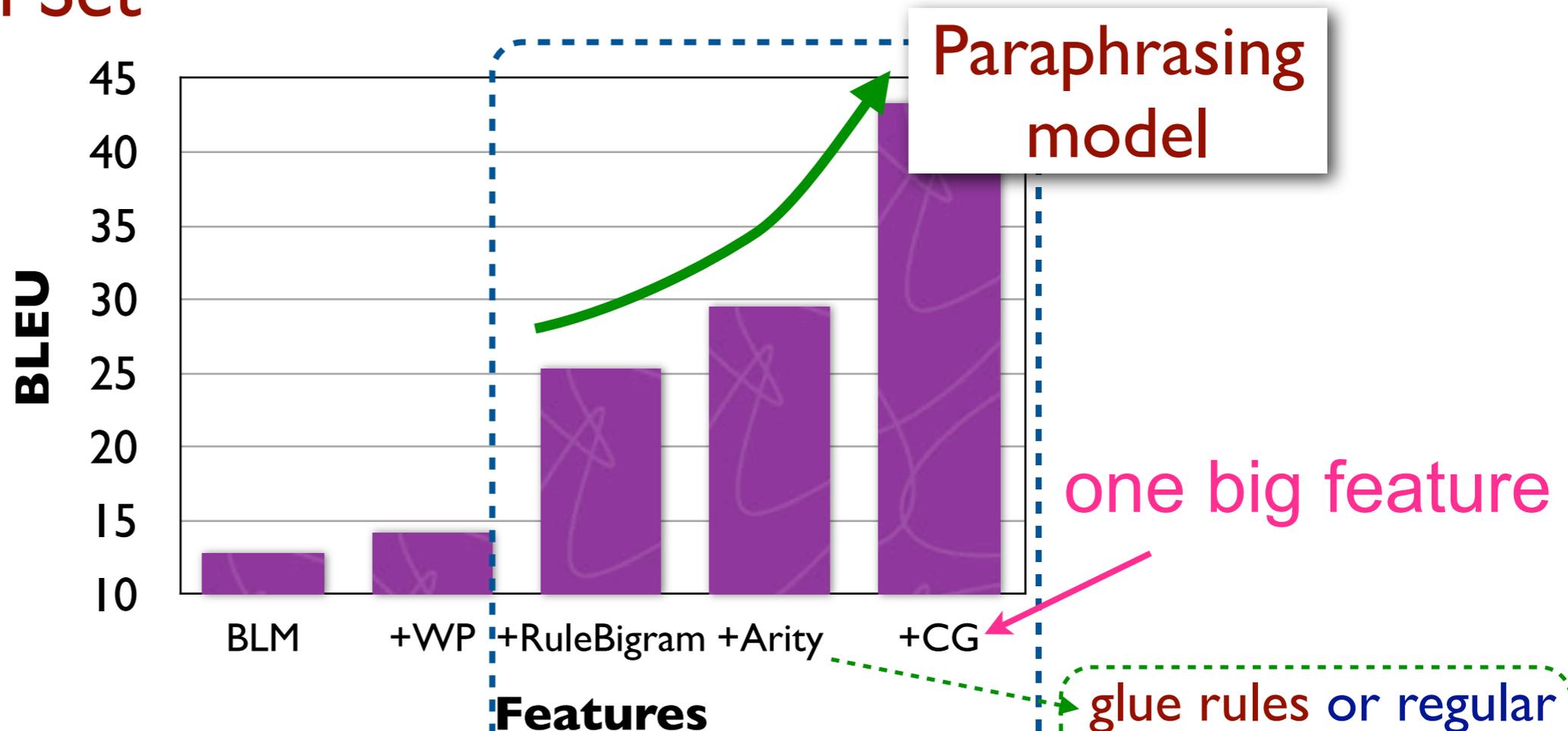


- On MT Set

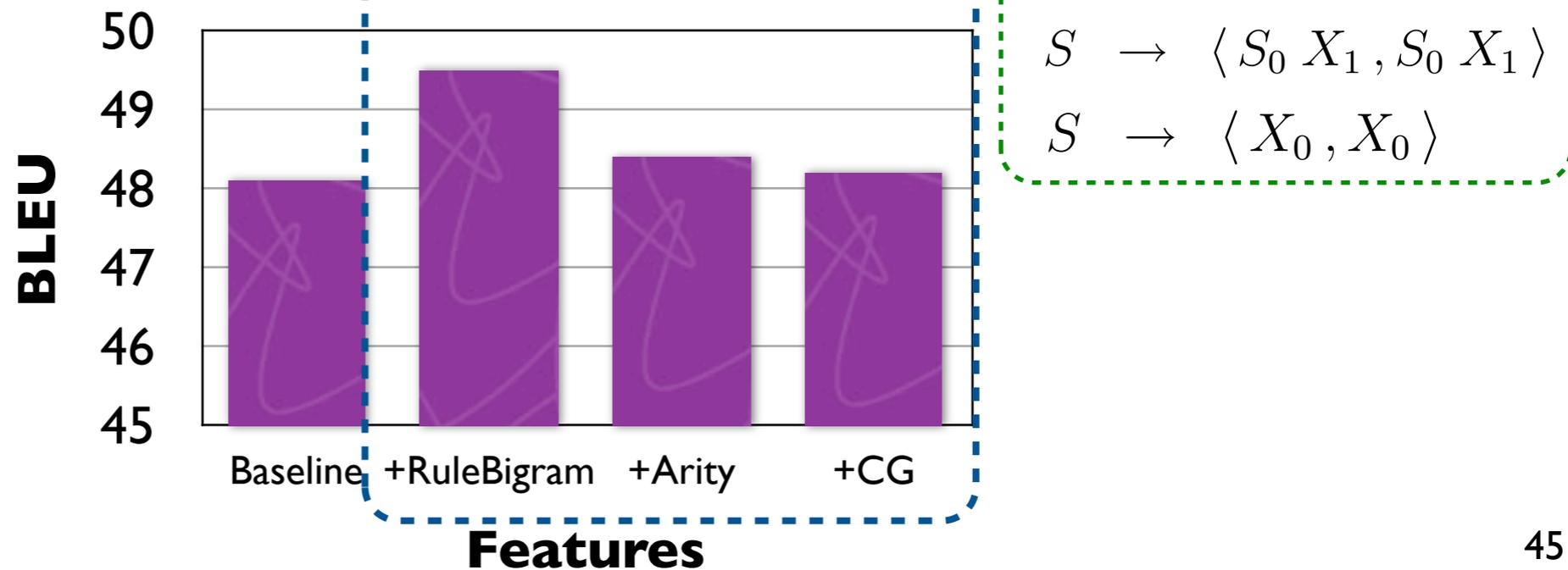


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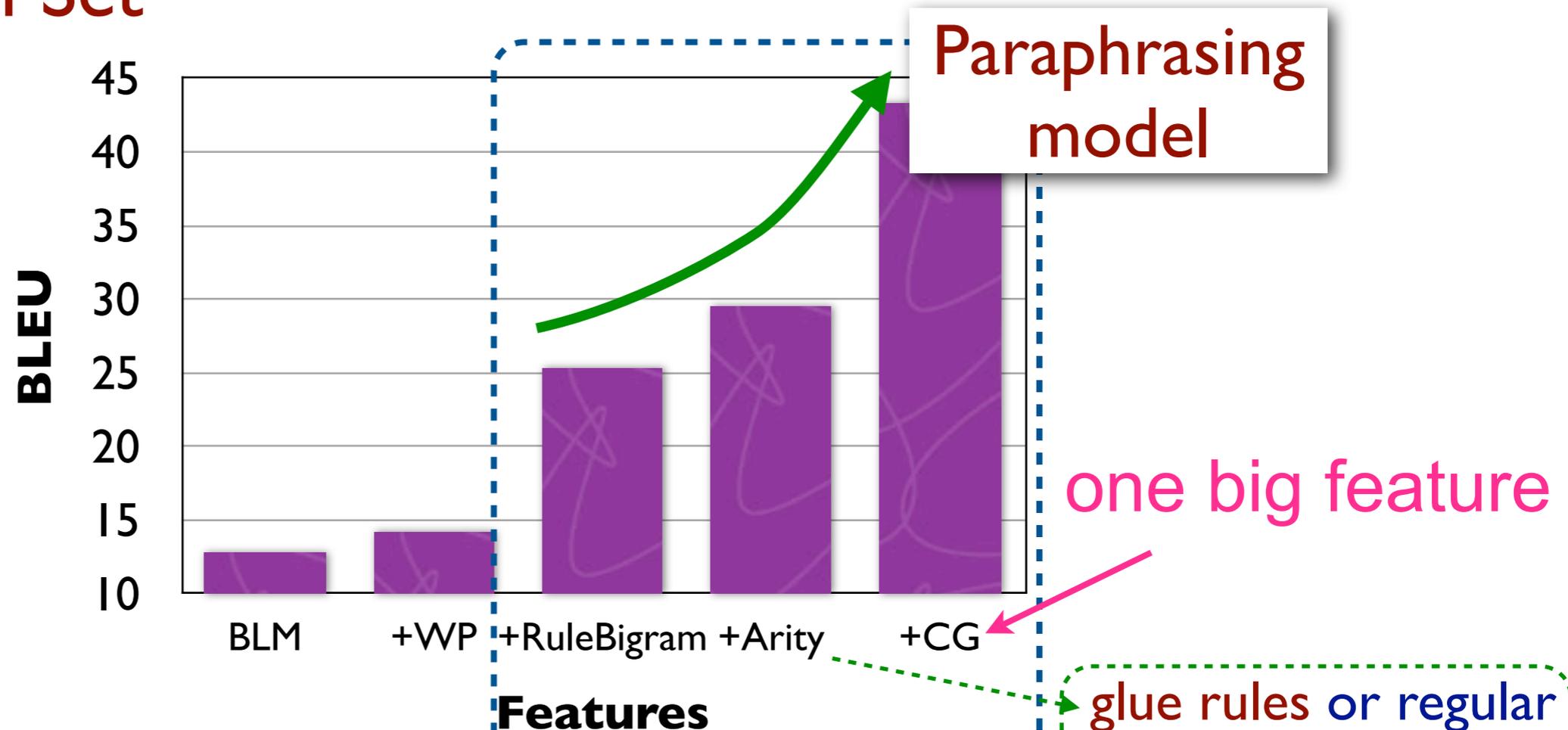


- On MT Set

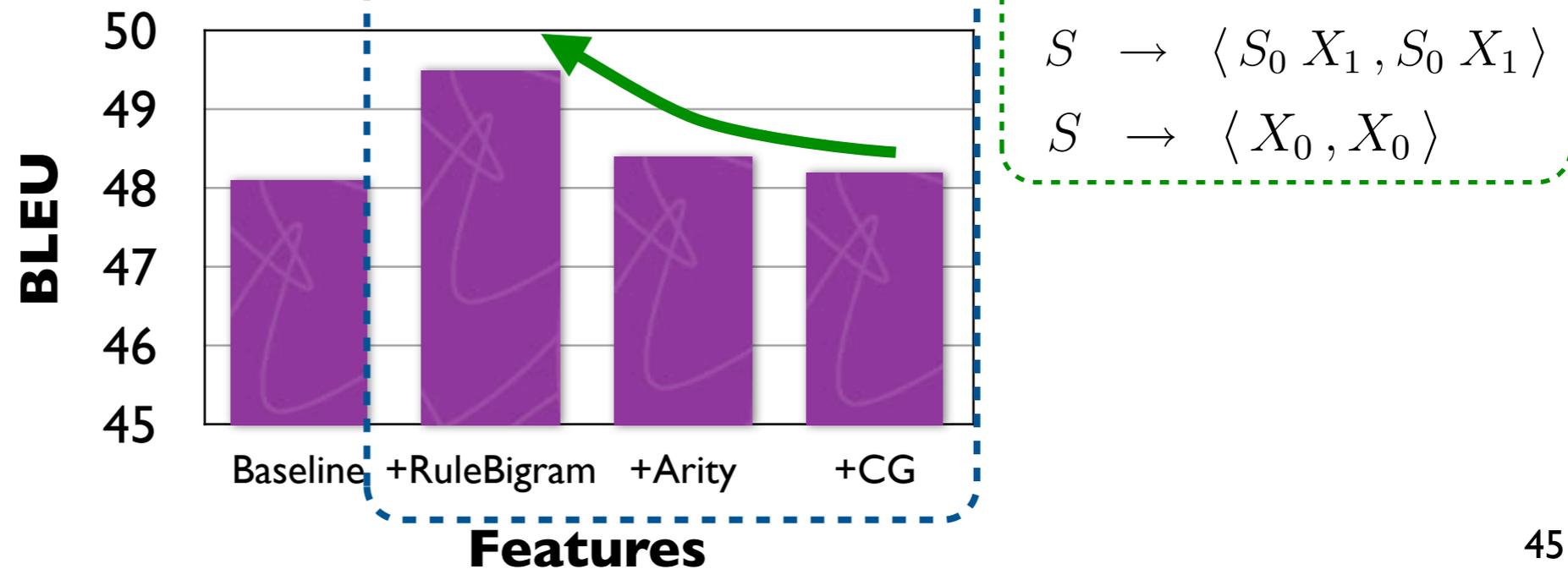


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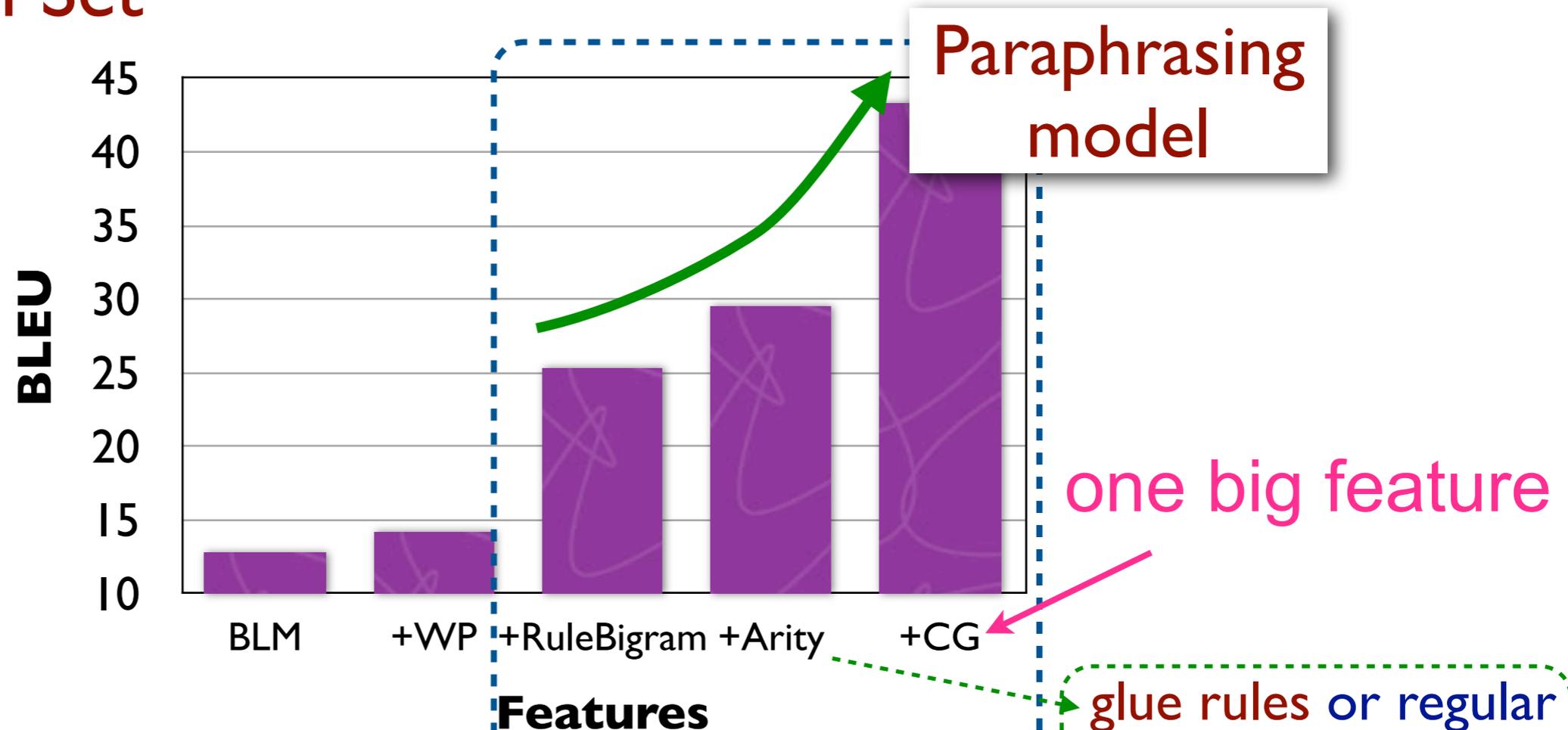


- On MT Set

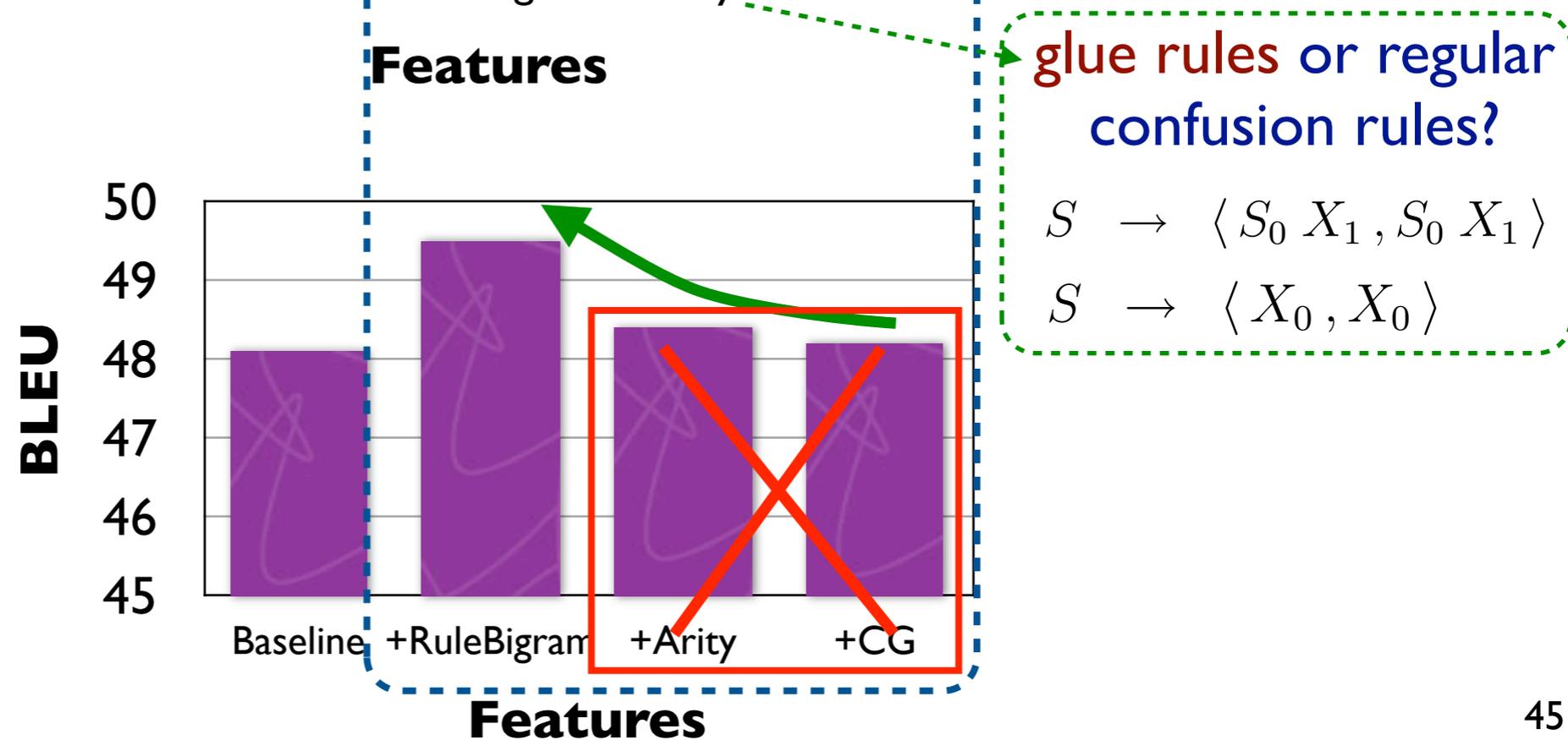


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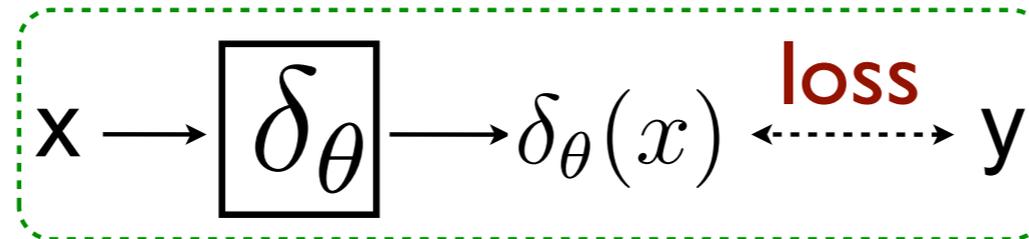
- On MT Set



# Summary for Discriminative Training

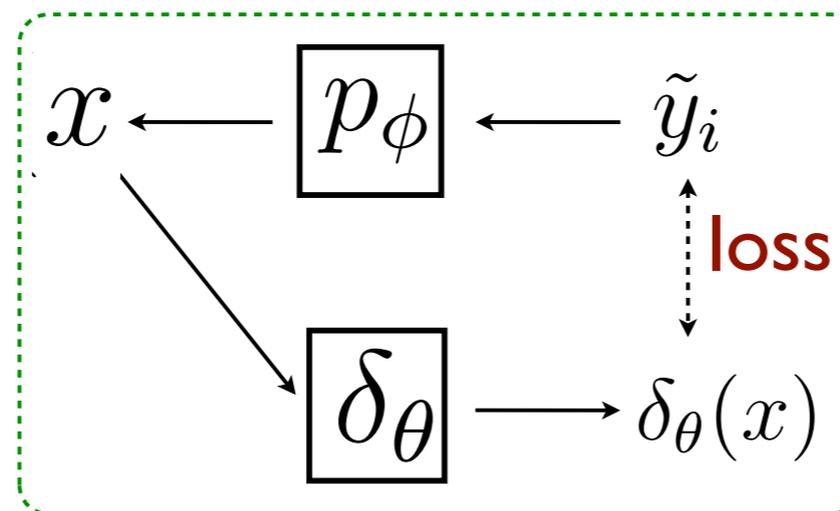
- **Supervised: Minimum Empirical Risk**

require  
bibtex



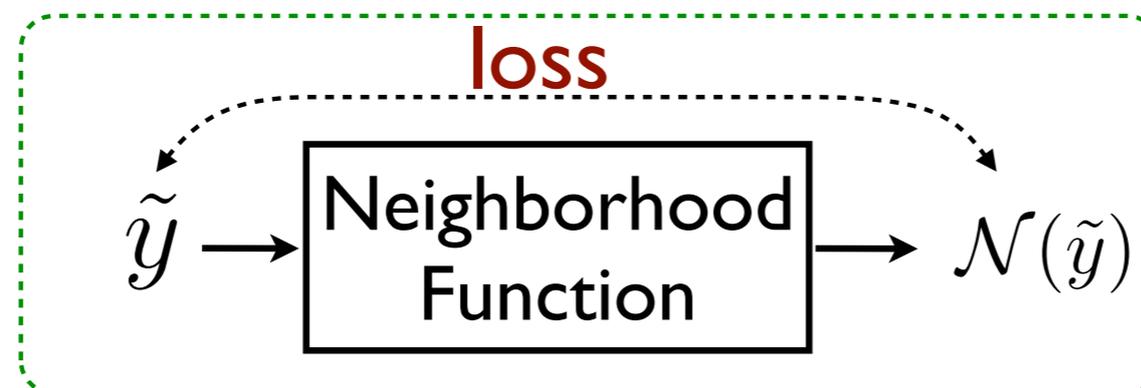
- **Unsupervised: Minimum Imputed Risk**

require  
monolingual  
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- **Unsupervised: Contrastive LM Estimation**

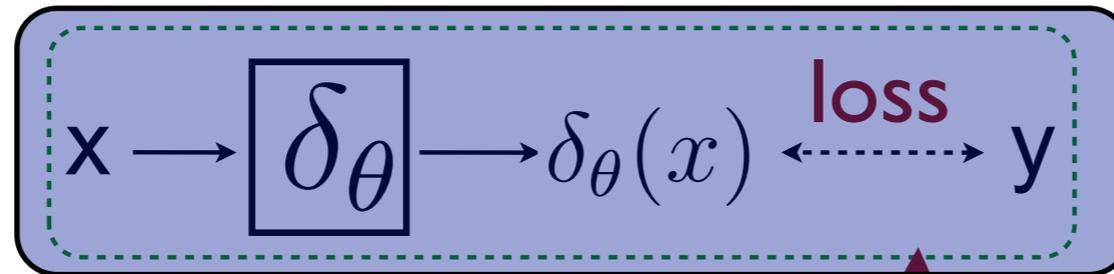
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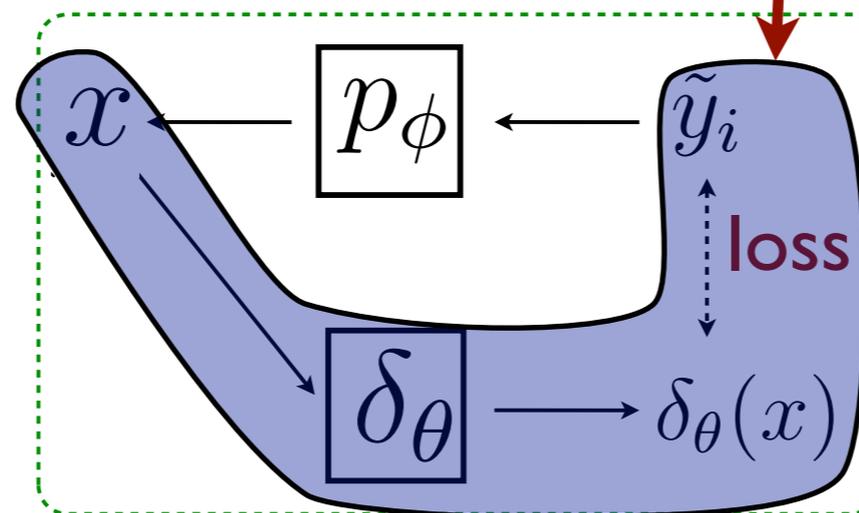
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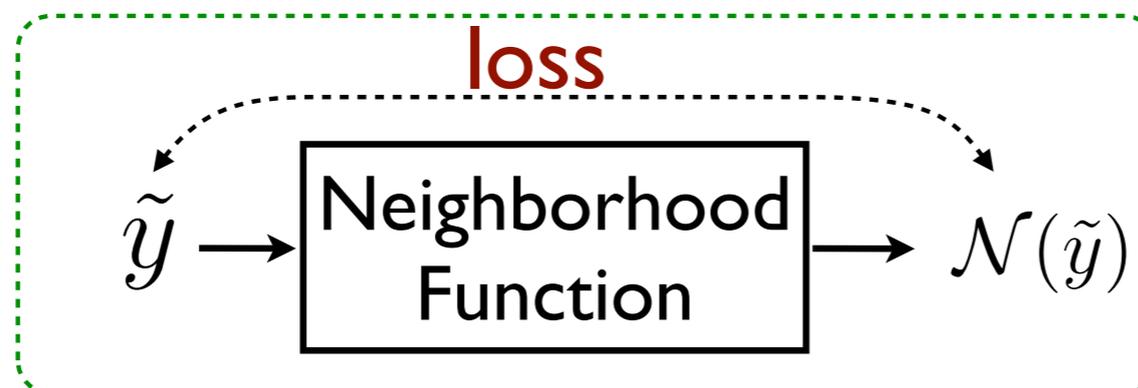
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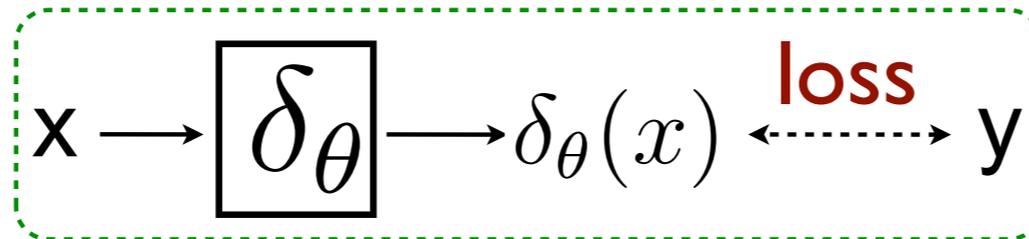
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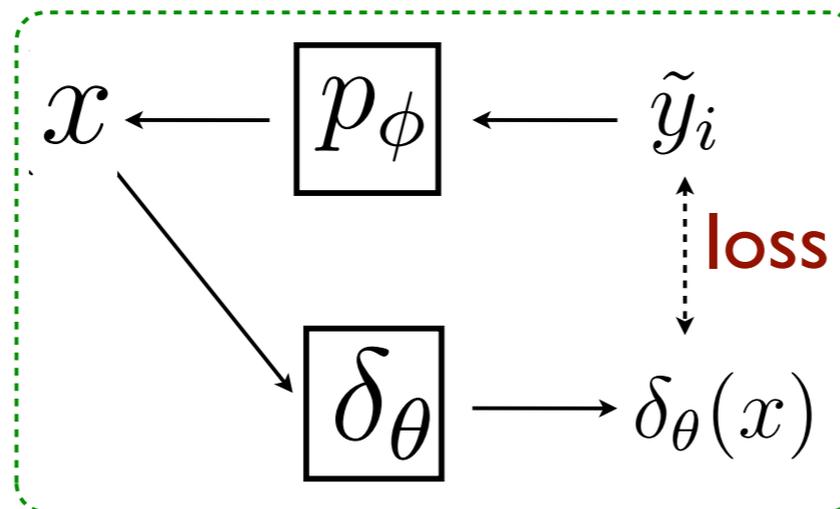
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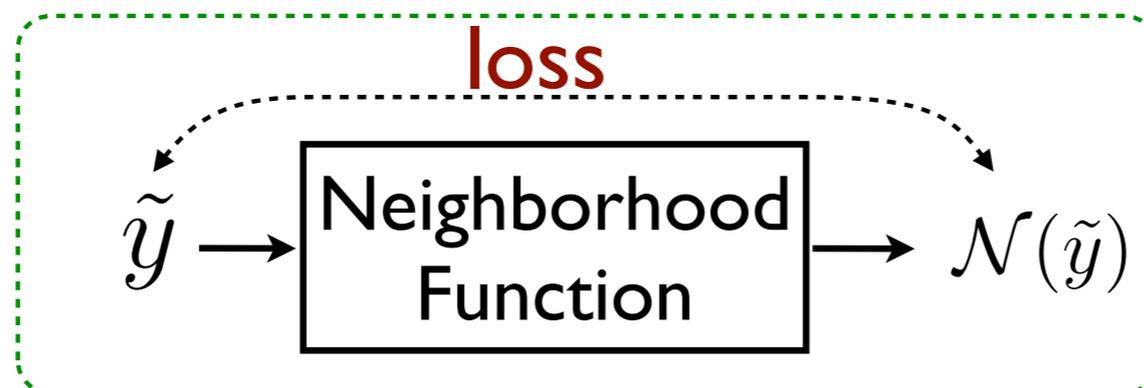
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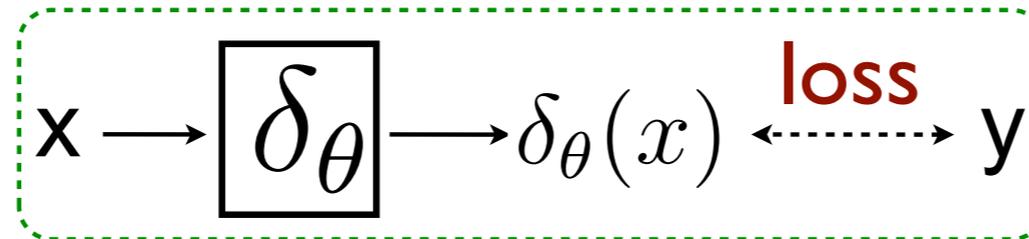
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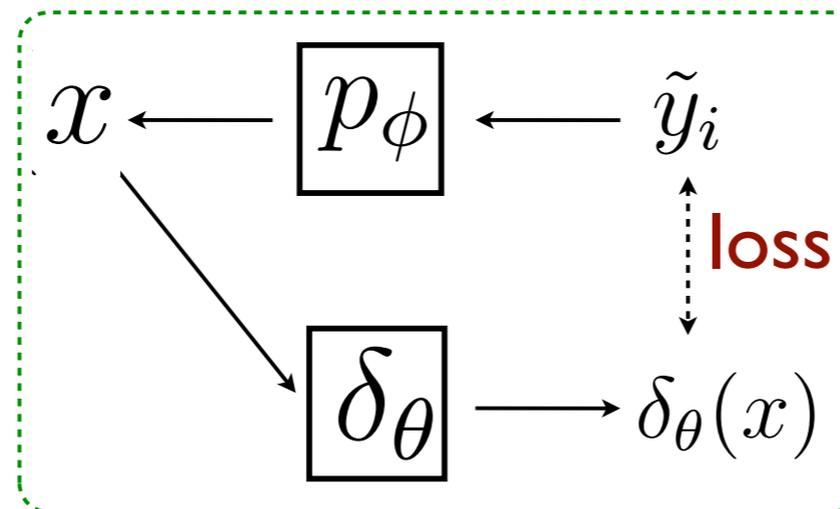
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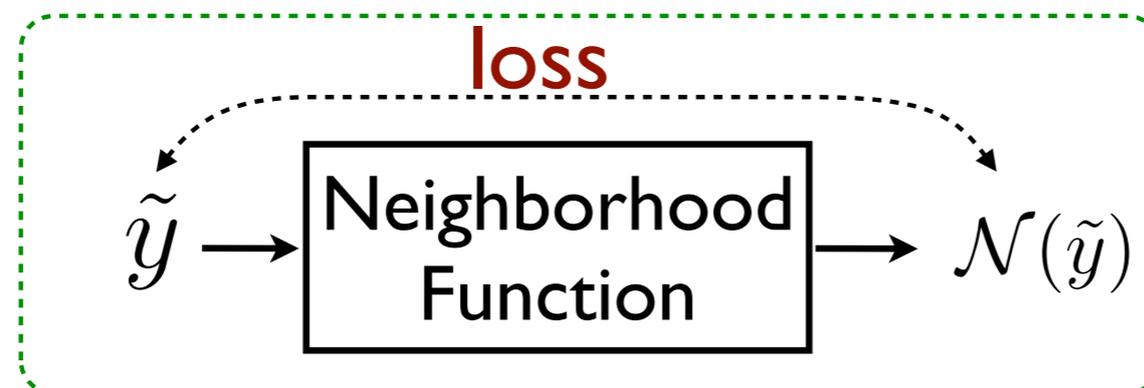
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English



require a reverse model

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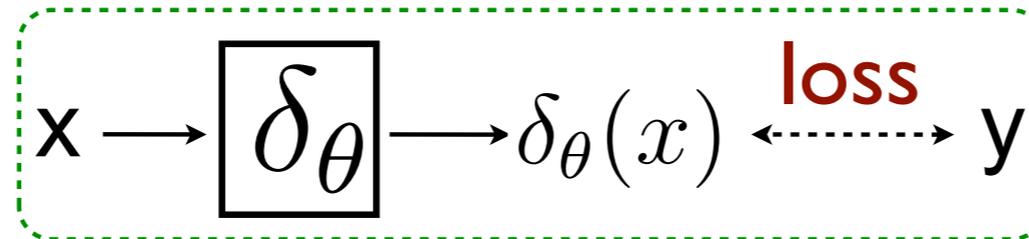
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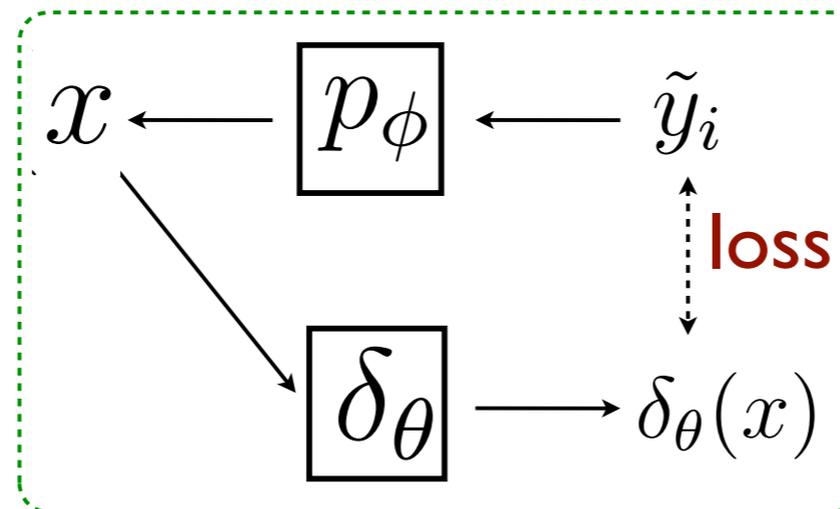
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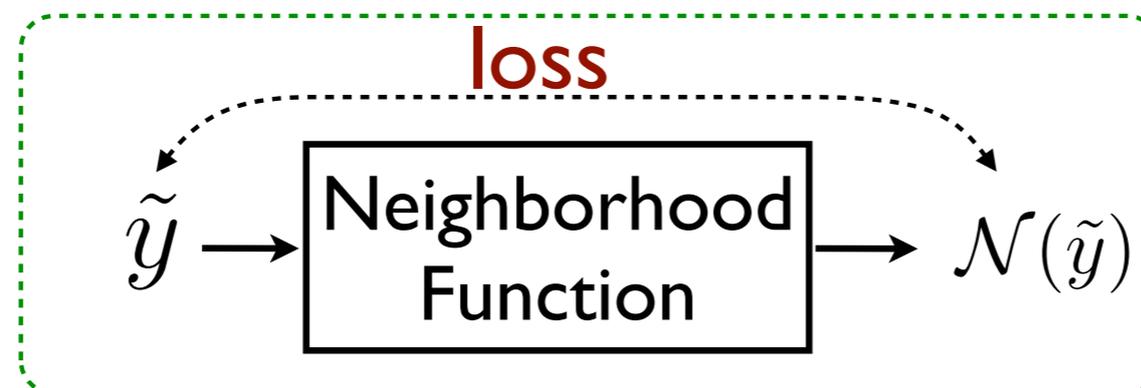


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can have both TM and  
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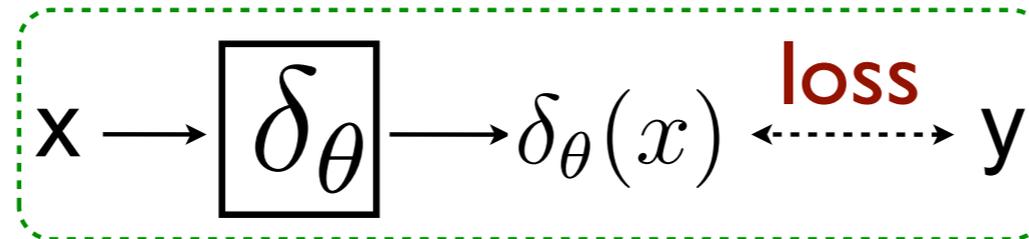
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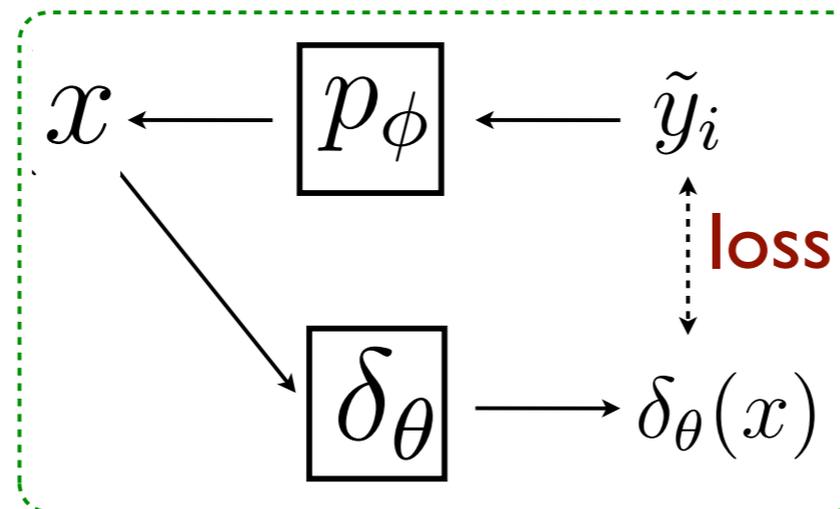
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bitext



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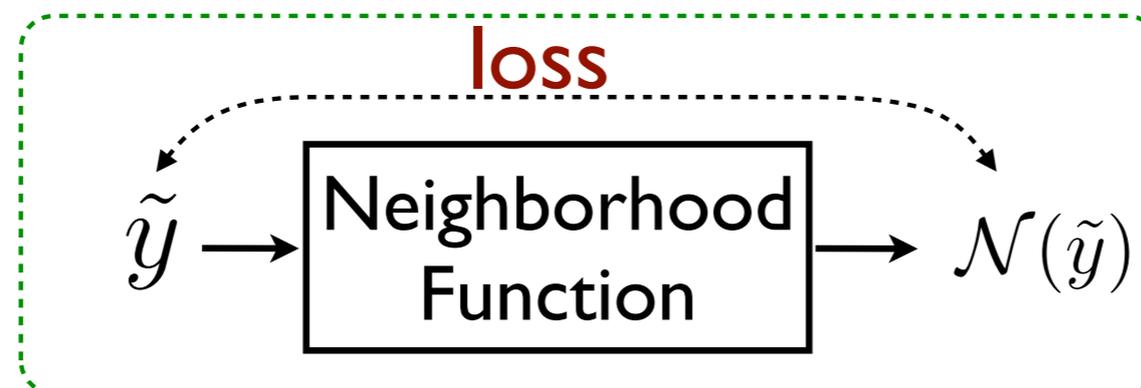


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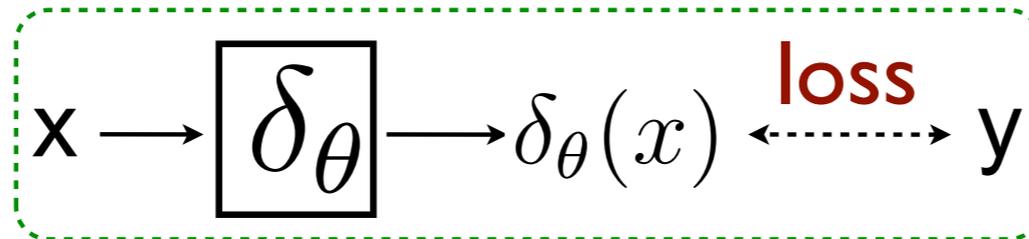


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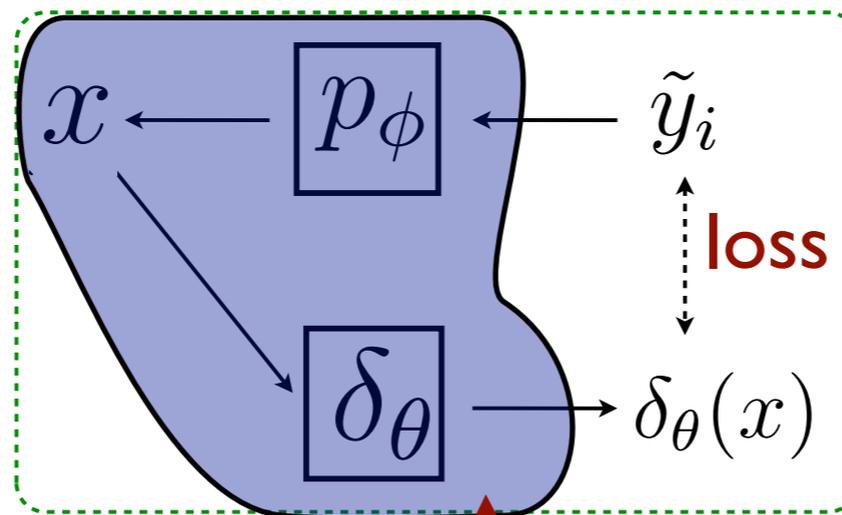
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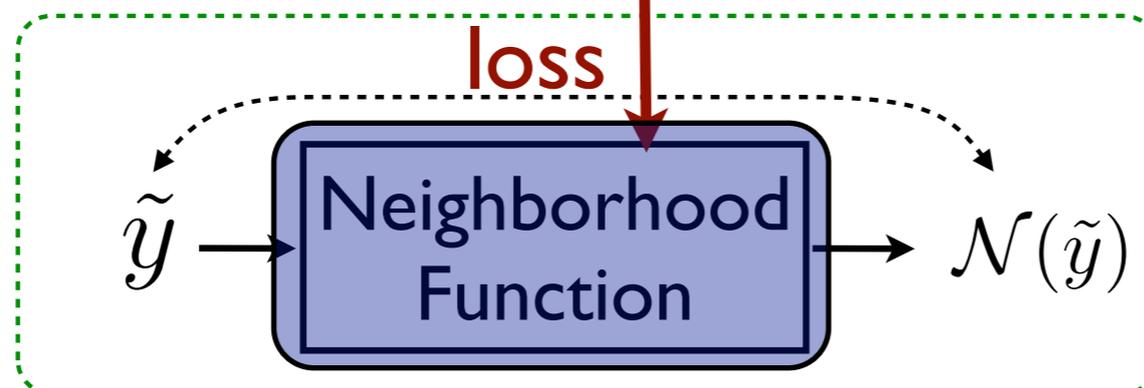


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# Outline

- Hypergraph as Hypothesis Space
- Unsupervised Discriminative Training
  - ▶ minimum imputed risk
  - ▶ contrastive language model estimation
- Variational Decoding
- First- and Second-order Expectation Semirings

<b>decoding</b> (e.g., mbr)	<b>training</b> (e.g., mert)
<b>atomic inference operations</b> (e.g., finding one-best, k-best or expectation, inference can be <i>exact</i> or <i>approximate</i> )	

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## tractable estimation

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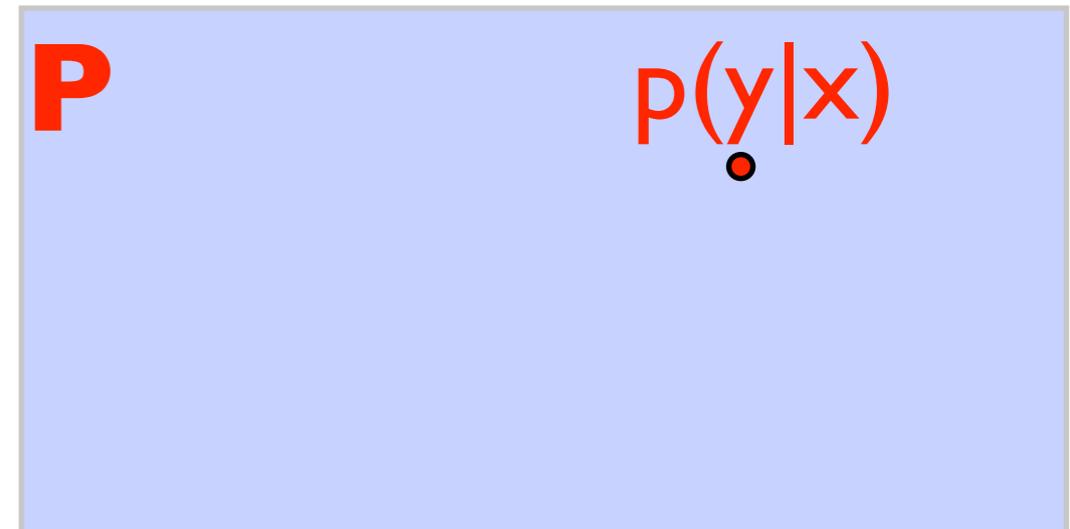
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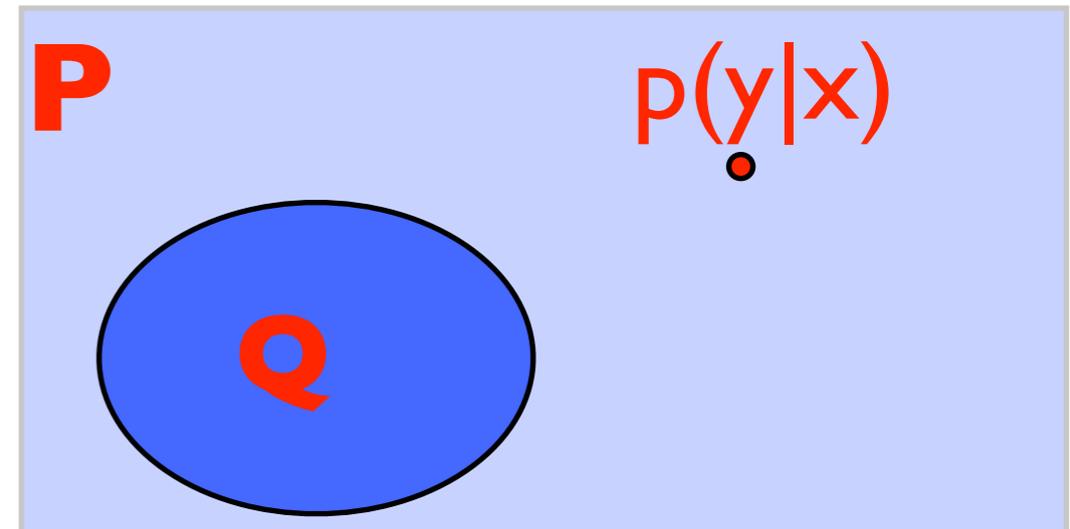
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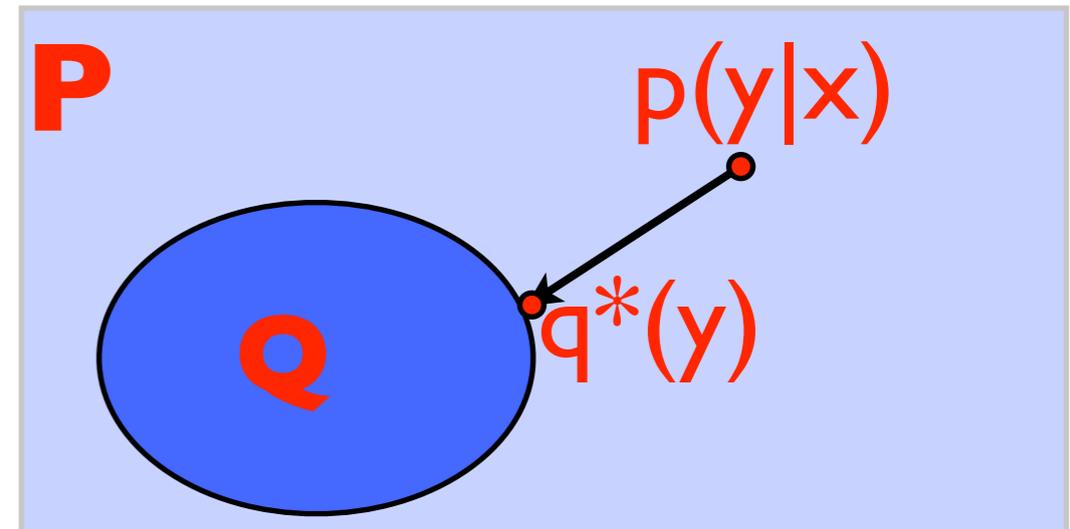
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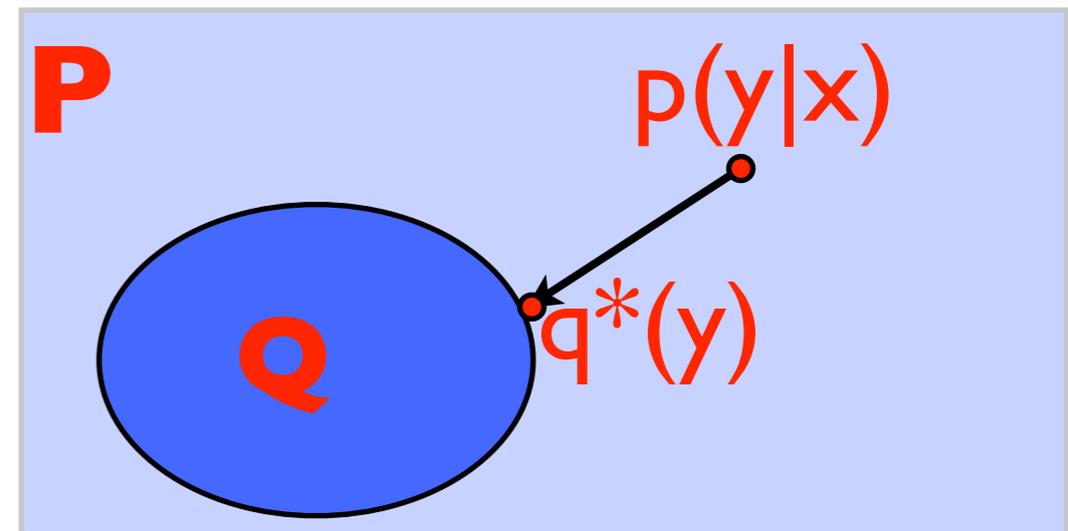
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# Variational Decoding for MT: an Overview

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Sentence-specific decoding

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Foreign  
sentence  $x$

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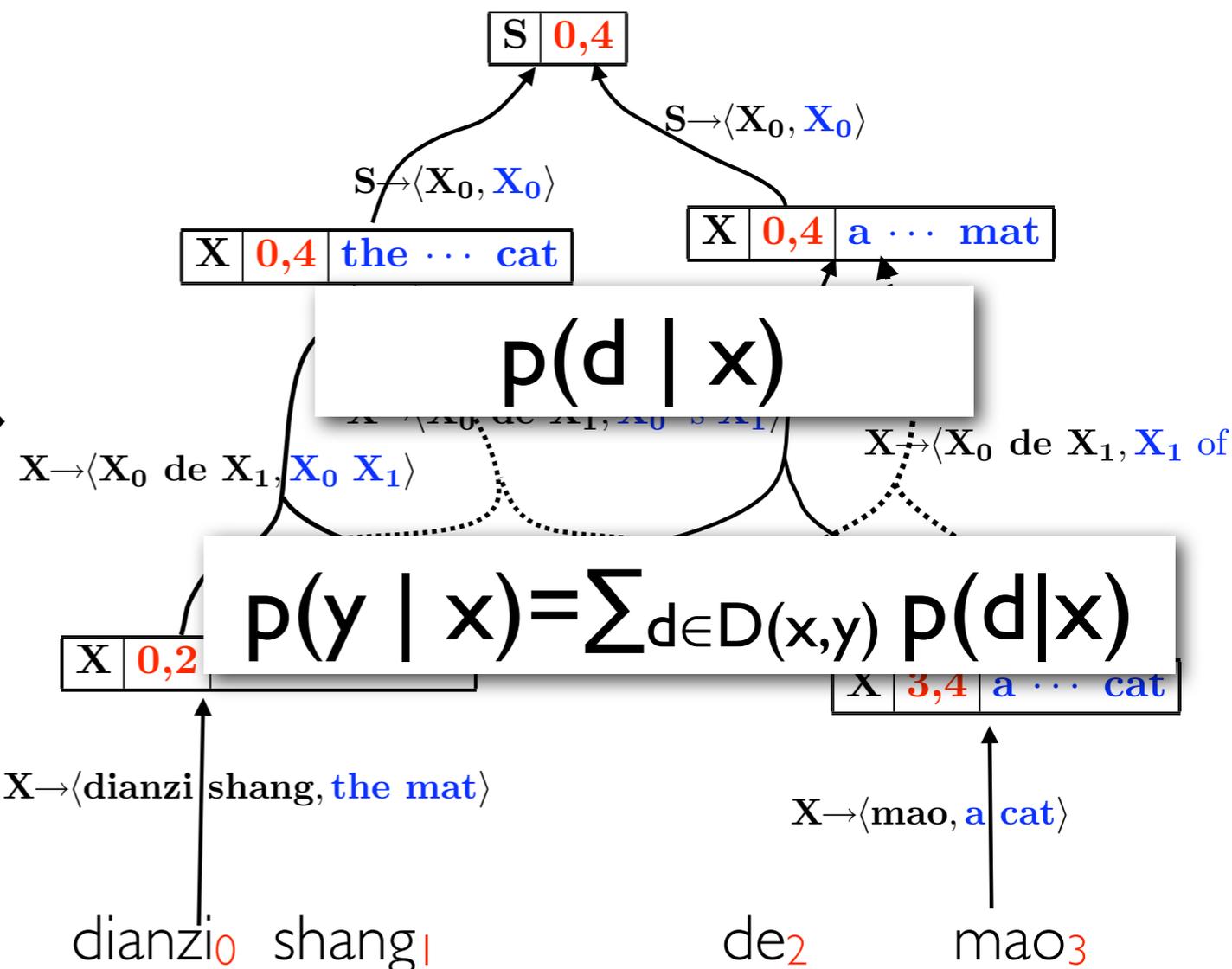
### Three steps:

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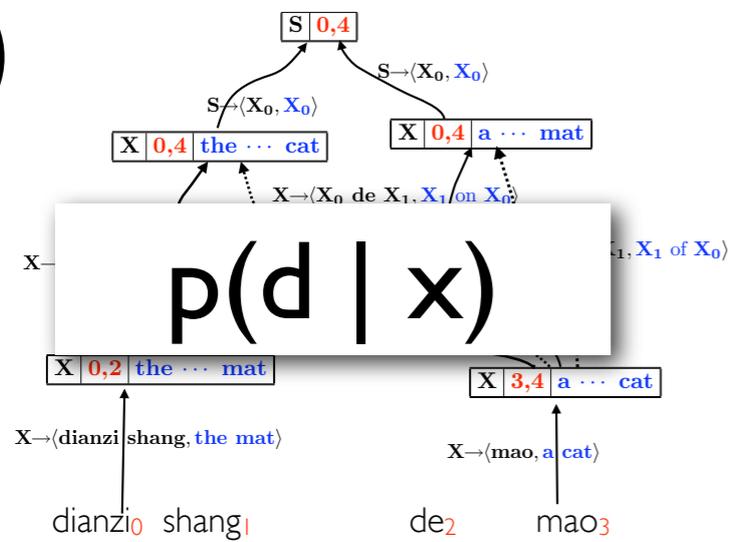
MAP decoding under P is intractable

Foreign sentence  $x$

SMT

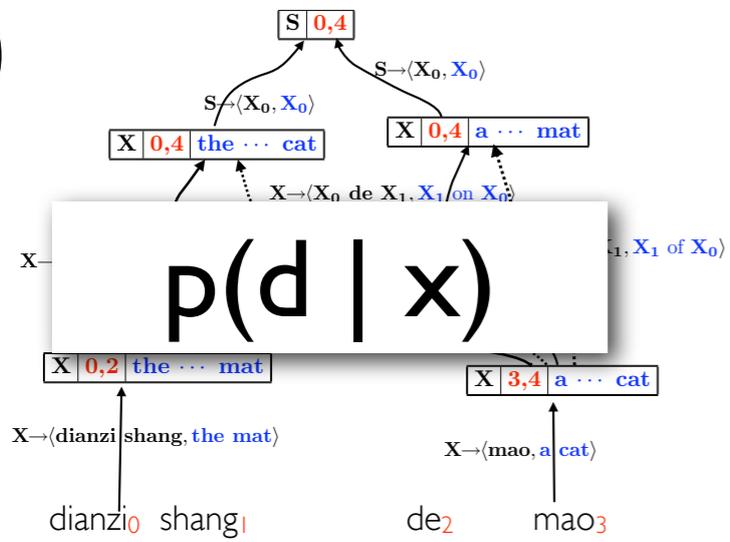


1



Generate a hypergraph

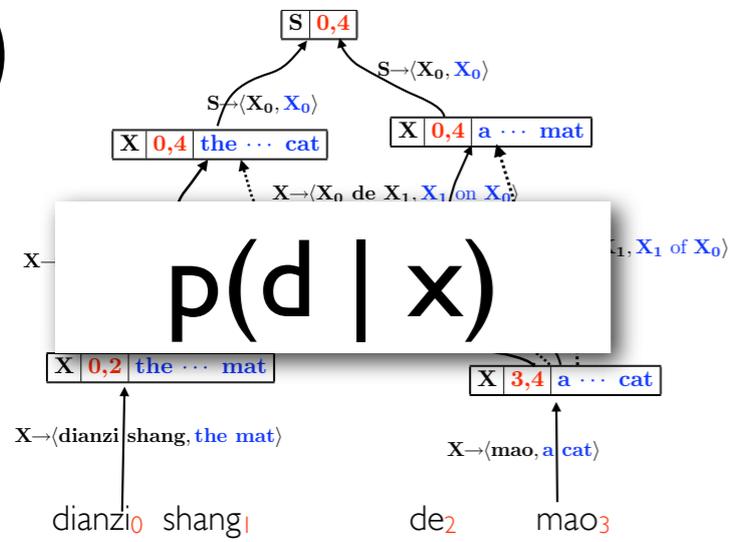
1



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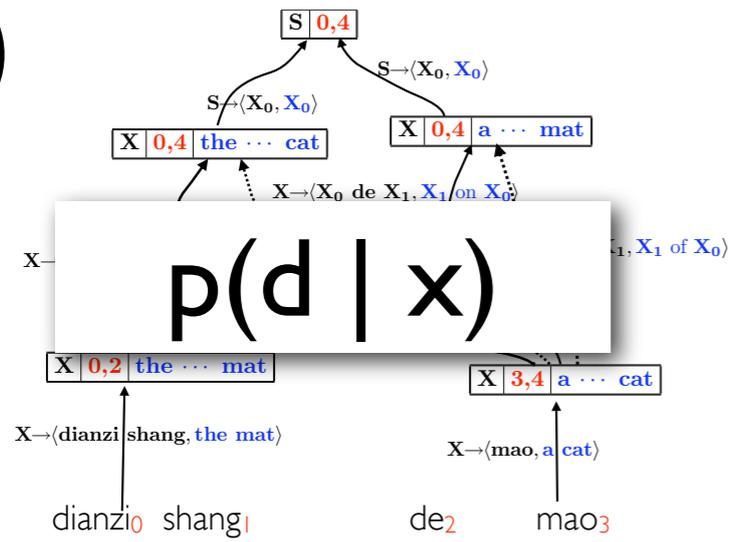


1

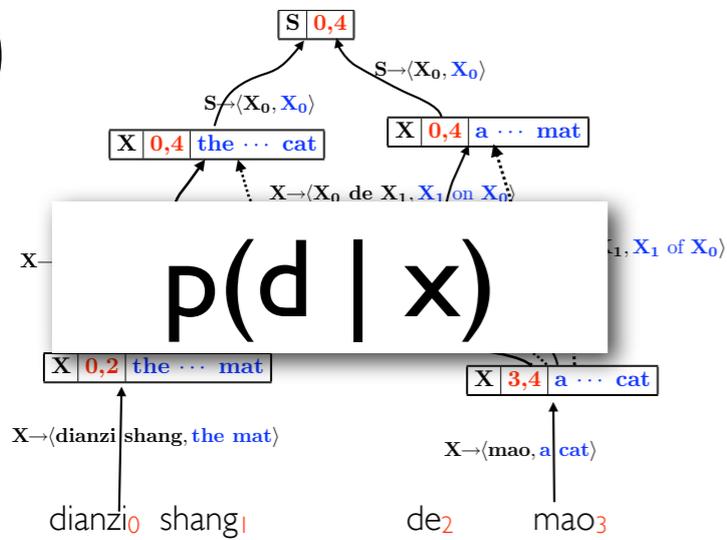


Generate a hypergraph

2

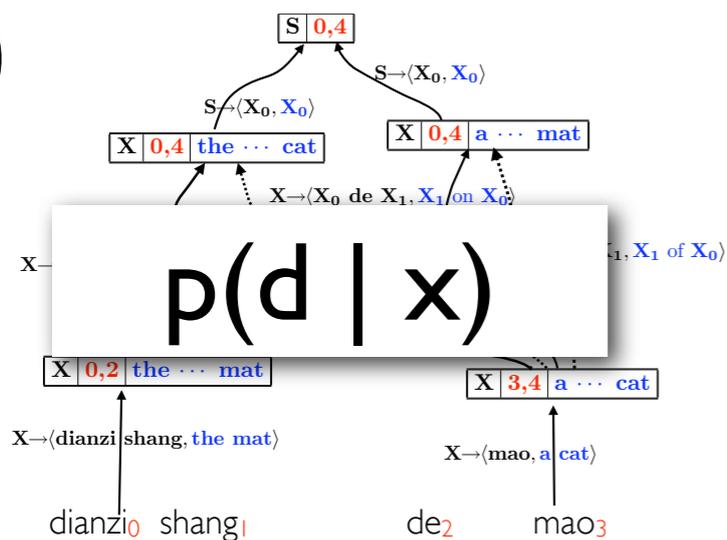


1



Generate a hypergraph

2



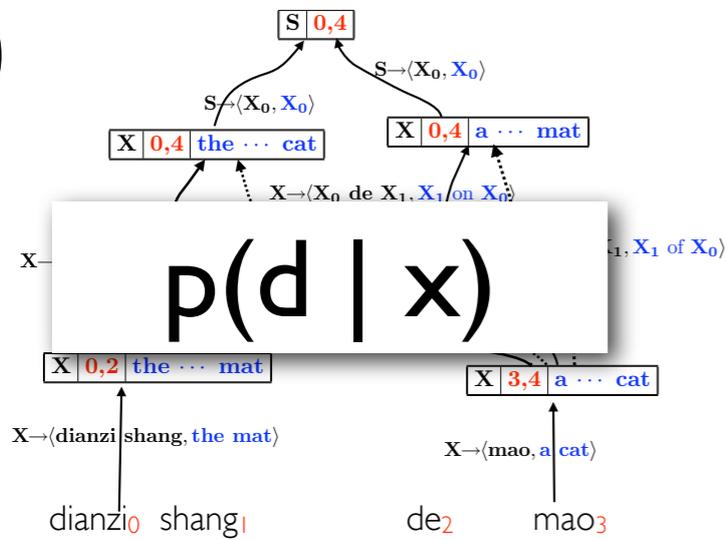
Estimate a model from the hypergraph by minimizing KL

$q^*$  is an n-gram model over output strings.



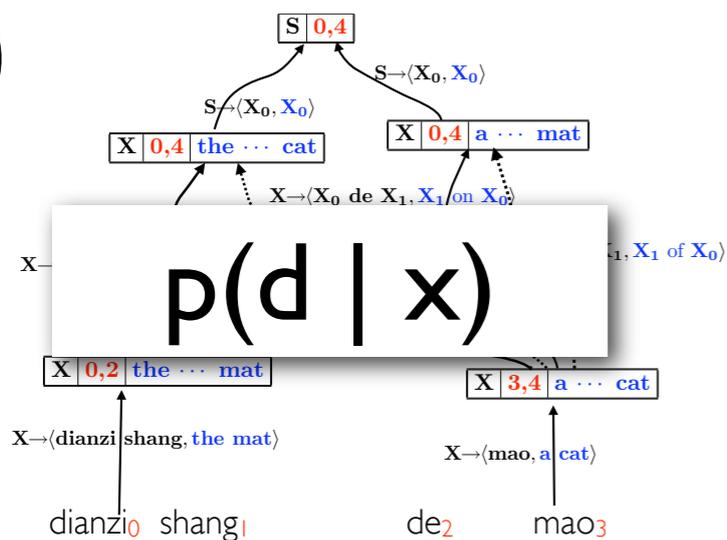
$$q^*(y | x)$$

1



Generate a hypergraph

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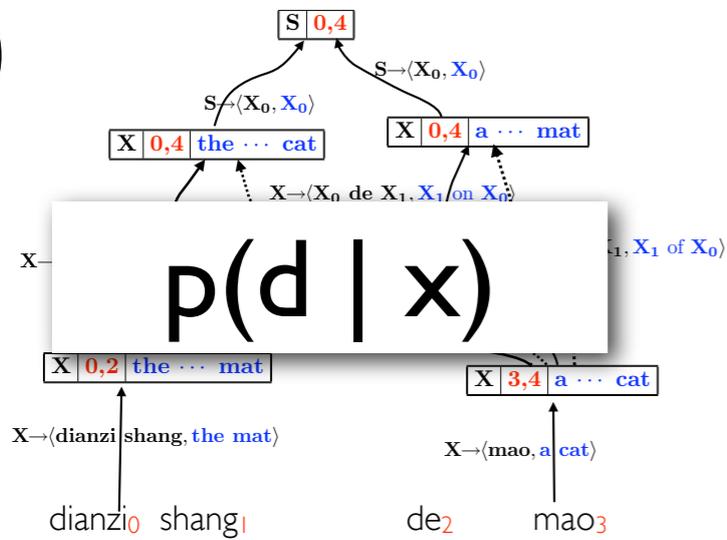
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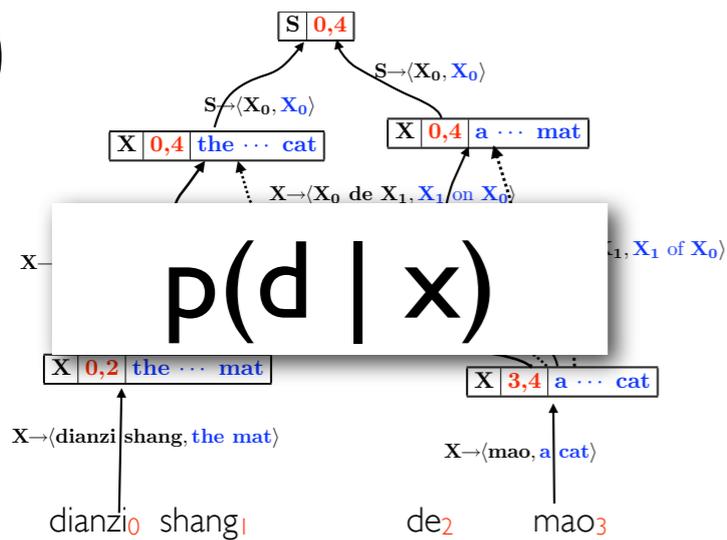
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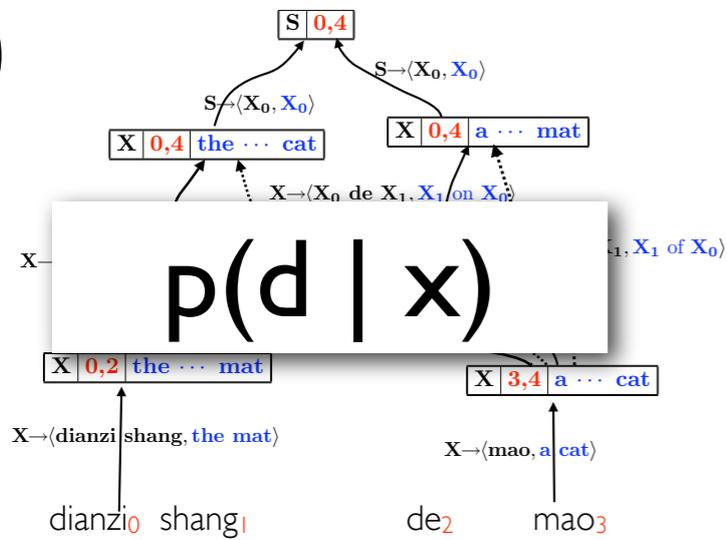


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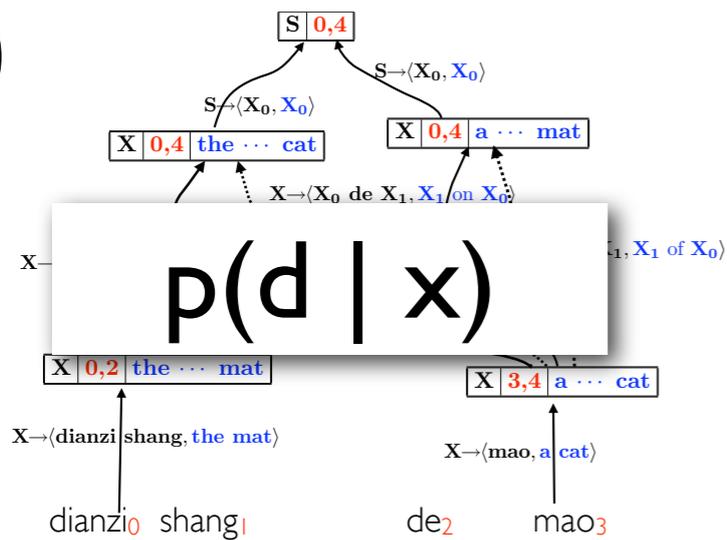
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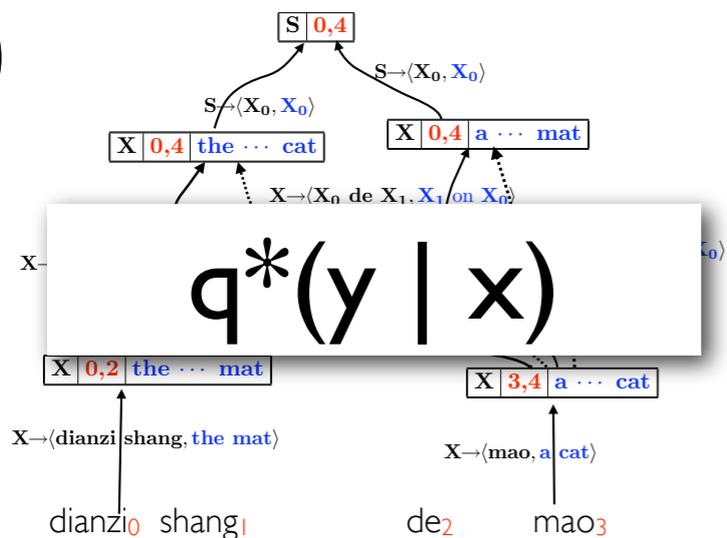
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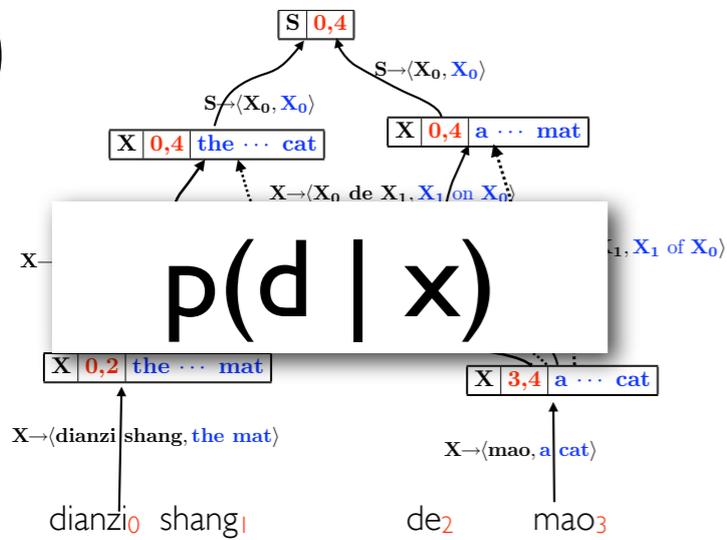
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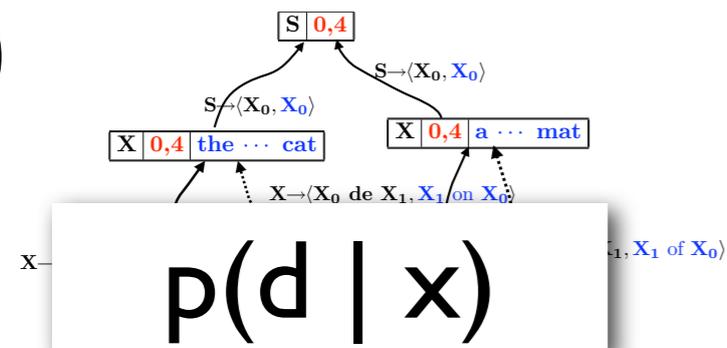
Decode using  $q^*$  on the hypergraph

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Generate a hypergraph

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Estimate a model from the hypergraph by minimizing KL

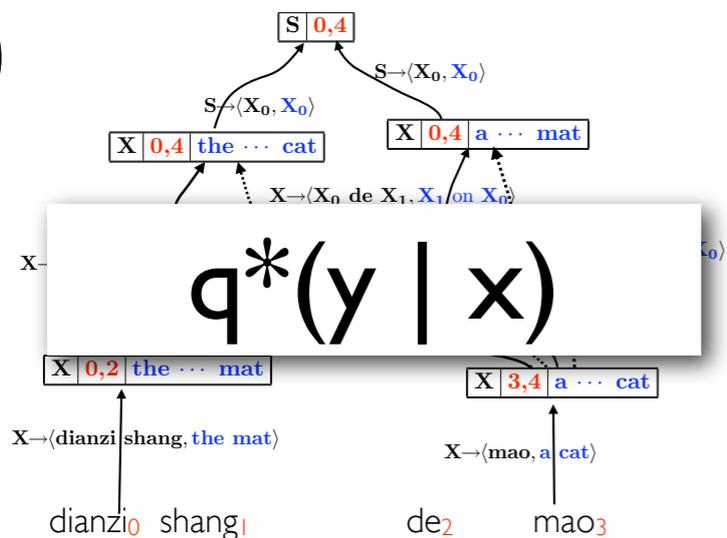
$q^*$  is an n-gram model over output strings.

Approximate a hypergraph with a lattice!

$$q^*(y | x)$$

$$\approx \sum_{d \in D(x,y)} p(d|x)$$

3



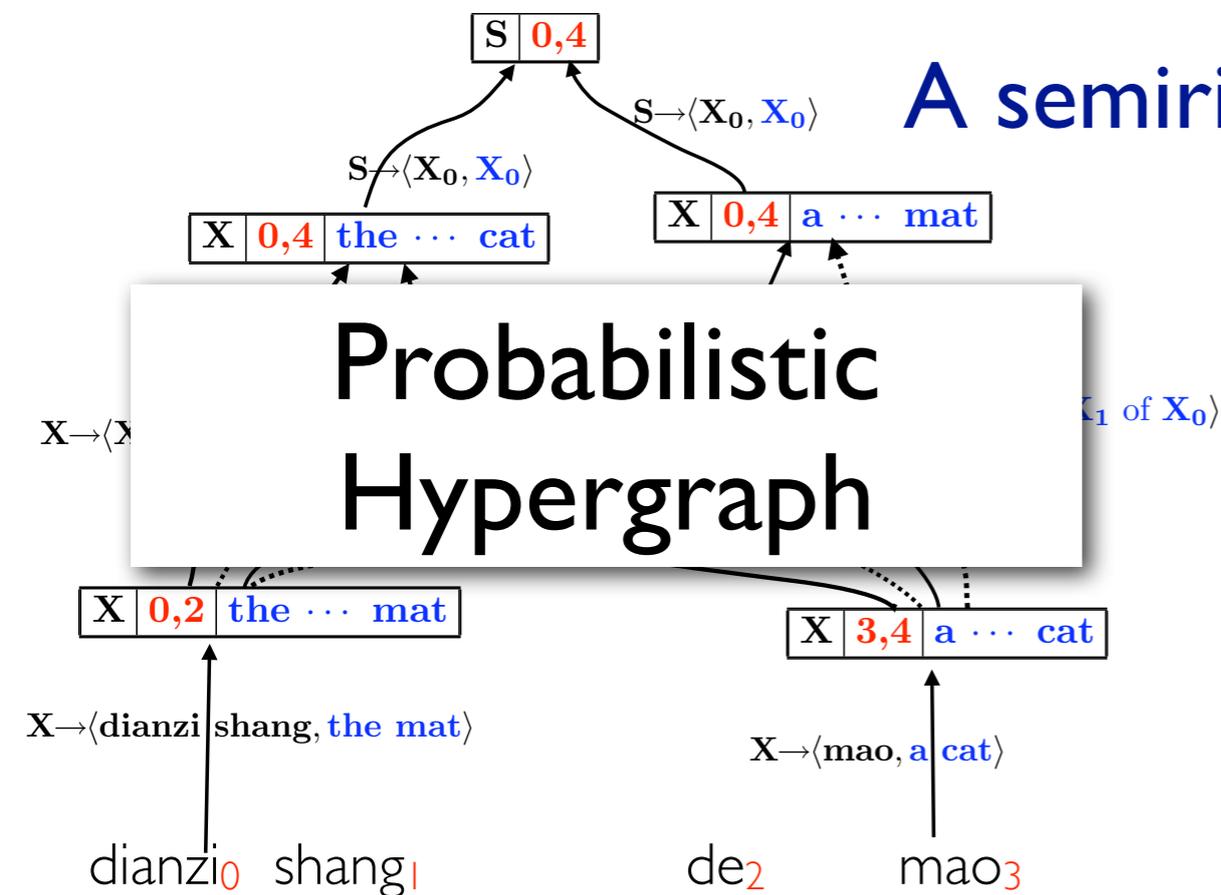
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# Outline

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  - ▶ contrastive language model estimation
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# A semiring framework to compute all of these

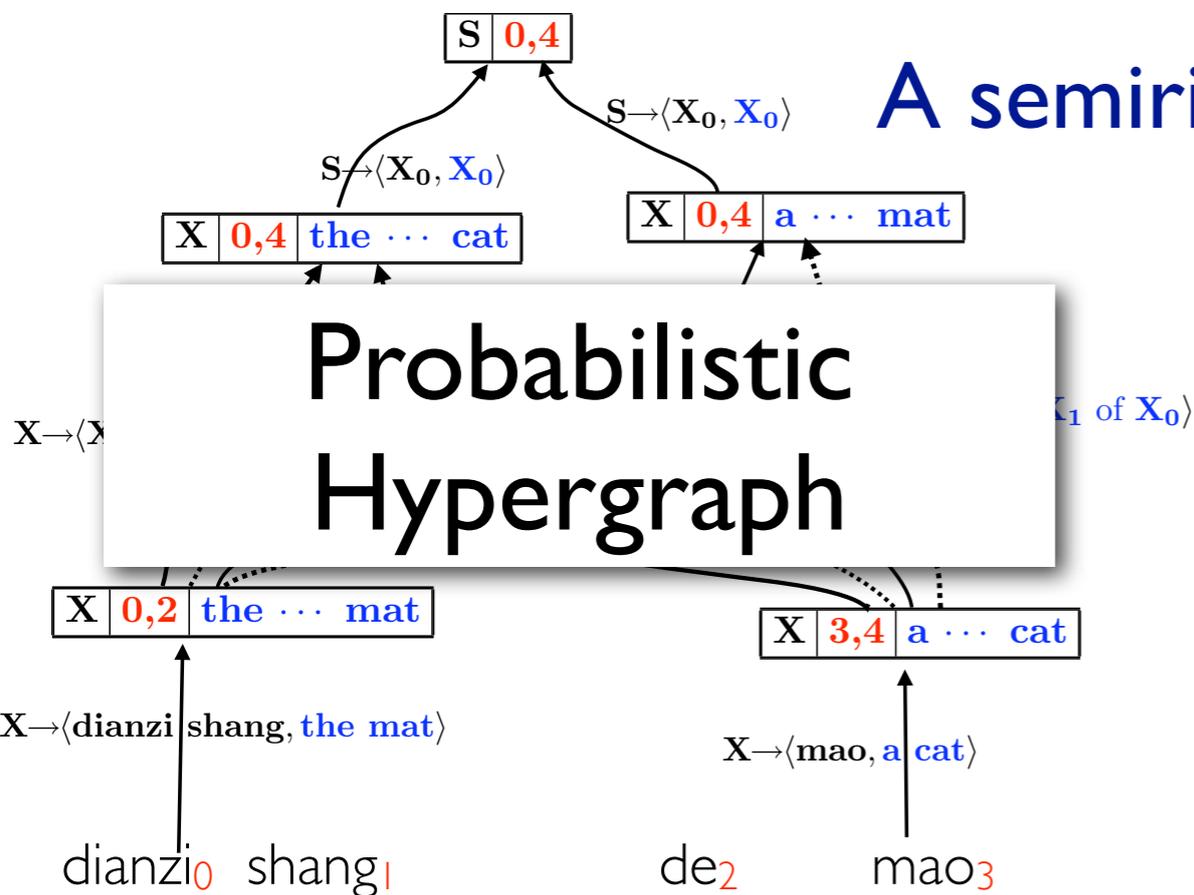


- “Decoding” quantities:
  - Viterbi
  - K-best
  - Counting
  - .....

- First-order expectations:
  - expectation
  - entropy
  - expected loss
  - cross-entropy
  - KL divergence
  - feature expectations
  - first-order gradient of  $Z$

- Second-order expectations:
  - expectation over product
  - interaction between features
  - Hessian matrix of  $Z$
  - second-order gradient descent
  - gradient of expectation
    - gradient of expected loss or entropy

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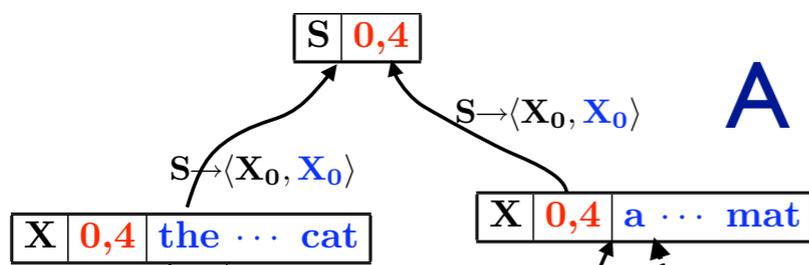


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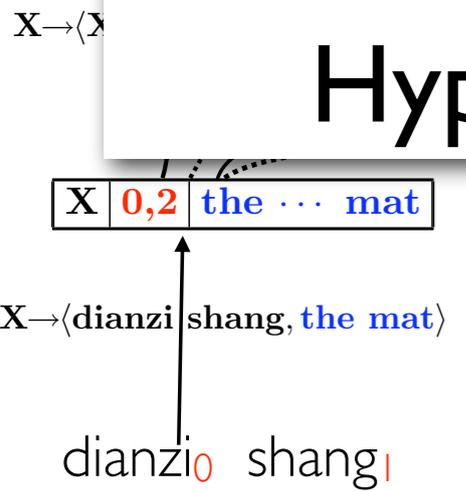


Probabilistic

Hypo

- “Decoding” quantities:

Recipe to compute a quantity:



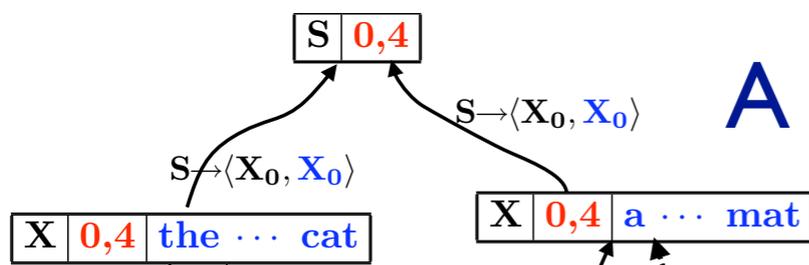
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- gradient of expected loss or entropy

# A semiring framework to compute all of these



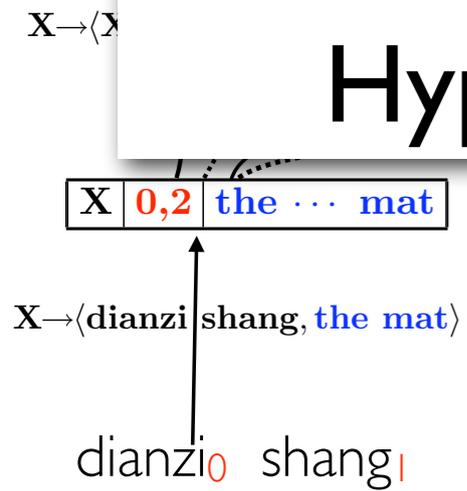
Probabilistic

Hypo

- “Decoding” quantities:

Recipe to compute a quantity:

- Choose a semiring



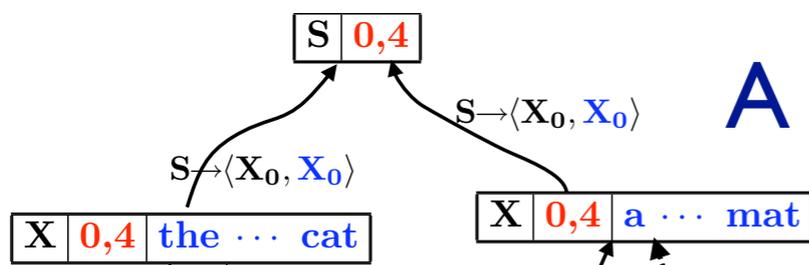
- First-order

- expectation
- entropy
- expected loss
- cross-entropy
- KL divergence
- feature expectations
- first-order gradient of  $Z$

- interaction between features

- Hessian matrix of  $Z$
- second-order gradient descent
- gradient of expectation
- gradient of expected loss or entropy

# A semiring framework to compute all of these



Probabilistic

Hypo

- “Decoding” quantities:

## Recipe to compute a quantity:

- Choose a semiring
- Specific a semiring weight for each hyperedge

X 0,2 the ... mat

X -> (dianzi shang, the mat)

dianzi shang

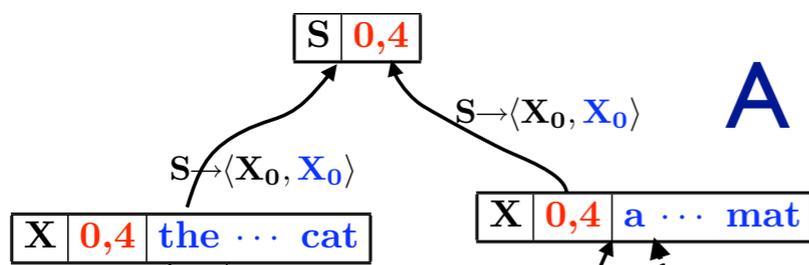
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# A semiring framework to compute all of these



Probabilistic

Hypo

- “Decoding” quantities:

## Recipe to compute a quantity:

- Choose a semiring
- Specific a semiring weight for each hyperedge
- Run the inside algorithm



$X \rightarrow \langle \text{dianzi shang, the mat} \rangle$

dianzi<sub>0</sub> shang<sub>1</sub>

- First-order

- expectation
- entropy
- **expected loss**
- cross-entropy
- KL divergence
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- first-order gradient of  $Z$

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- Hessian matrix of  $Z$
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- **gradient of expectation**
- **gradient of expected loss or entropy**

# Applications of Expectation Semirings: a Summary

Quantity	$k_e$	$k_{\text{root}}$	Final
<b>Expectation</b>	$\langle p_e, p_e r_e \rangle$	$\langle Z, \bar{r} \rangle$	$\bar{r}/Z$
Entropy	$r_e \stackrel{\text{def}}{=} \log p_e$ , so $k_e = \langle p_e, p_e \log p_e \rangle$	$\langle Z, \bar{r} \rangle$	$\log Z - \bar{r}/Z$
Cross-entropy	$\langle q_e \rangle$ $r_e \stackrel{\text{def}}{=} \log q_e$ , so $k_e = \langle p_e, p_e \log q_e \rangle$	$\langle Z_q \rangle$ $\langle Z_p, \bar{r} \rangle$	$\log Z_q - \bar{r}/Z_p$
Bayes risk	$r_e \stackrel{\text{def}}{=} L_e$ , so $k_e = \langle p_e, p_e L_e \rangle$	$\langle Z, \bar{r} \rangle$	$\bar{r}/Z$
<b>First-order gradient</b>	$\langle p_e, \nabla p_e \rangle$	$\langle Z, \nabla Z \rangle$	$\nabla Z$
<b>Covariance matrix</b>	$\langle p_e, p_e r_e, p_e s_e, p_e r_e s_e \rangle$	$\langle Z, \bar{r}, \bar{s}, \bar{t} \rangle$	$\frac{\bar{t}}{Z} - \frac{\bar{r} \bar{s}^T}{Z^2}$
<b>Hessian matrix</b>	$\langle p_e, \nabla p_e, \nabla p_e, \nabla^2 p_e \rangle$	$\langle Z, \nabla Z, \nabla Z, \nabla^2 Z \rangle$	$\nabla^2 Z$
<b>Gradient of expectation</b>	$\langle p_e, p_e r_e, \nabla p_e, (\nabla p_e) r_e + p_e (\nabla r_e) \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$
Gradient of entropy	$\langle p_e, p_e \log p_e, \nabla p_e, (1 + \log p_e) \nabla p_e \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{\nabla Z}{Z} - \frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$
Gradient of risk	$\langle p_e, p_e L_e, \nabla p_e, L_e \nabla p_e \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$

# Inference, Training and Decoding on Hypergraphs

- Unsupervised Discriminative Training
  - ▶ minimum imputed risk (In Preparation)
  - ▶ contrastive language model estimation (In Preparation)
- Variational Decoding  
(Li et al., ACL 2009)
- First- and Second-order Expectation Semirings  
(Li and Eisner, EMNLP 2009)

# My Other MT Research

- **Training methods (supervised)**

- Discriminative forest reranking with Perceptron  
(Li and Khudanpur, GALE book chapter 2009)
- Discriminative n-gram language models  
(Li and Khudanpur, AMTA 2008)

- **Algorithms**

- Oracle extraction from hypergraphs  
(Li and Khudanpur, NAACL 2009)
- Efficient intersection between n-gram LM and CFG  
(Li and Khudanpur, ACL SSST 2008)

- **Others**

- System combination (Smith et al., GALE book chapter 2009)
- Unsupervised translation induction for Chinese abbreviations (Li and Yarowsky, ACL 2008)

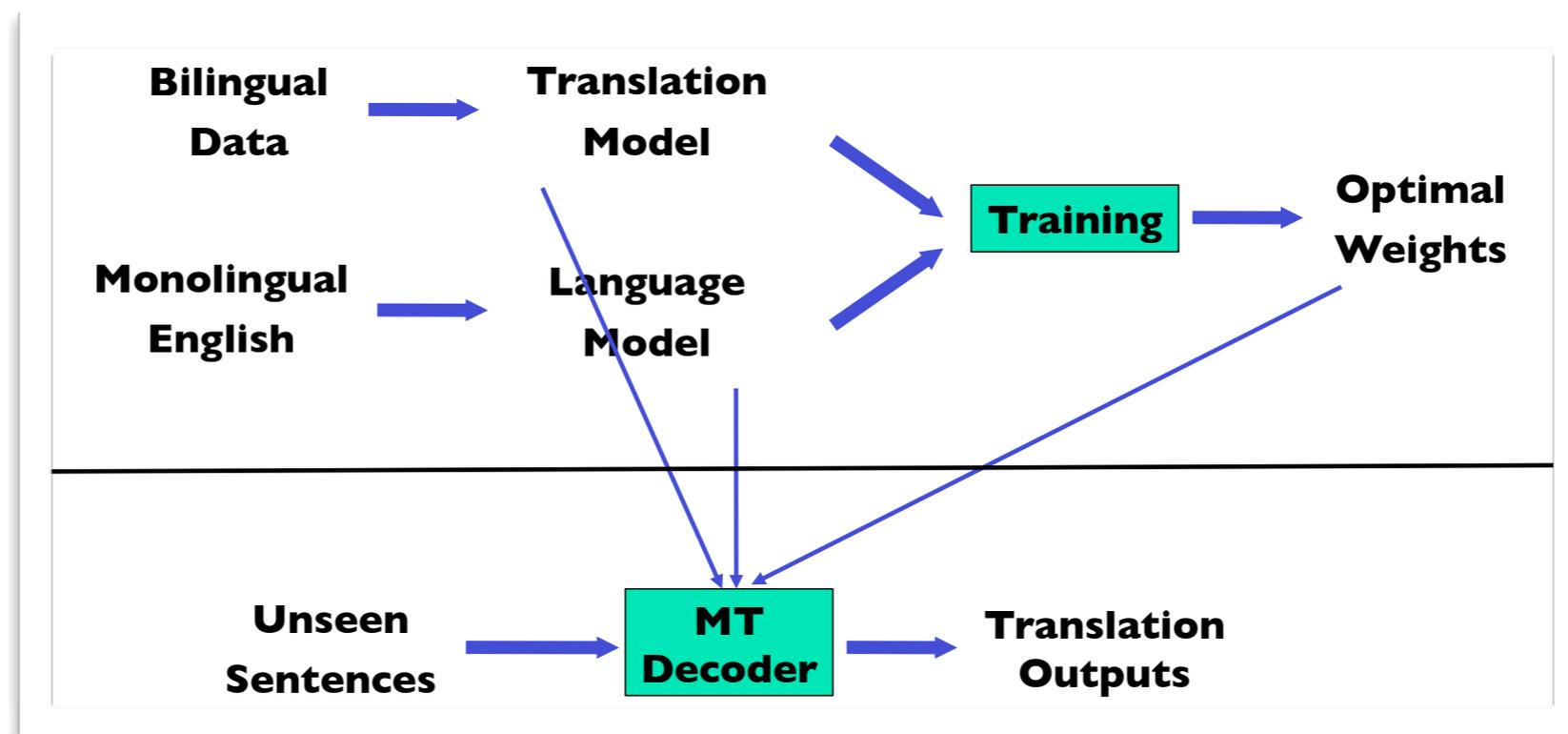
# Research other than MT

- **Information extraction**
  - Relation extraction between formal and informal phrases (Li and Yarowsky, EMNLP 2008)
- **Spoken dialog management**
  - Optimal dialog in consumer-rating systems using a POMDP (Li et al., SIGDial 2008)



# Joshua project

- An open-source parsing-based MT toolkit (Li et al. 2009)
- support Hiero (Chiang, 2007) and SAMT (Venugopal et al., 2007)
- Team members
  - **Zhifei Li**, Chris Callison-Burch, Chris Dyer, Sanjeev Khudanpur, Wren Thornton, Jonathan Weese, Juri Ganitkevitch, Lane Schwartz, and Omar Zaidan



Only rely on word-aligner and SRI LM!

All the methods presented have been implemented in Joshua!

Thank you!

XieXie!

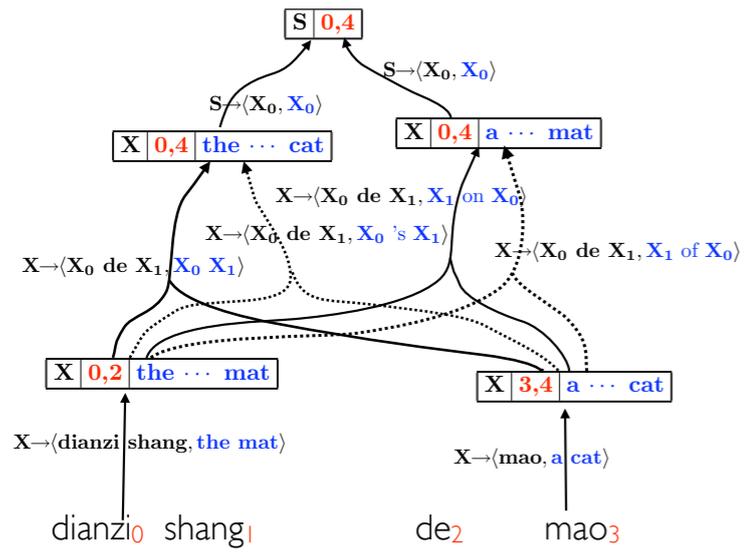
谢谢!



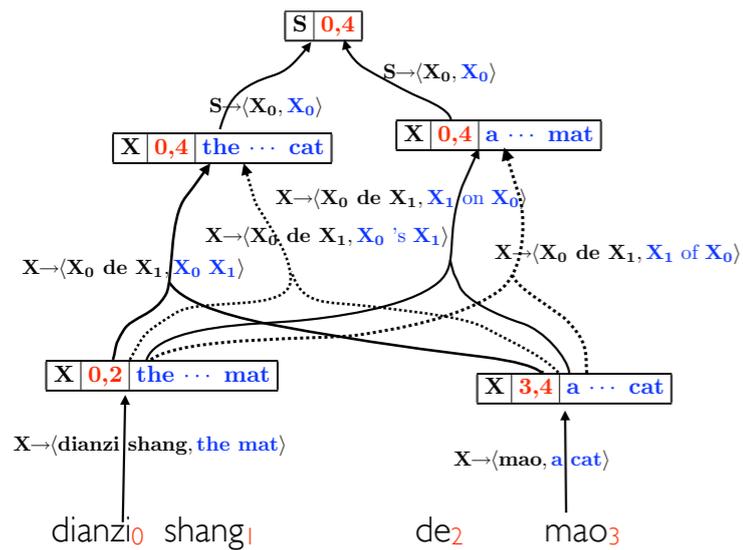




# Decoding over a hypergraph

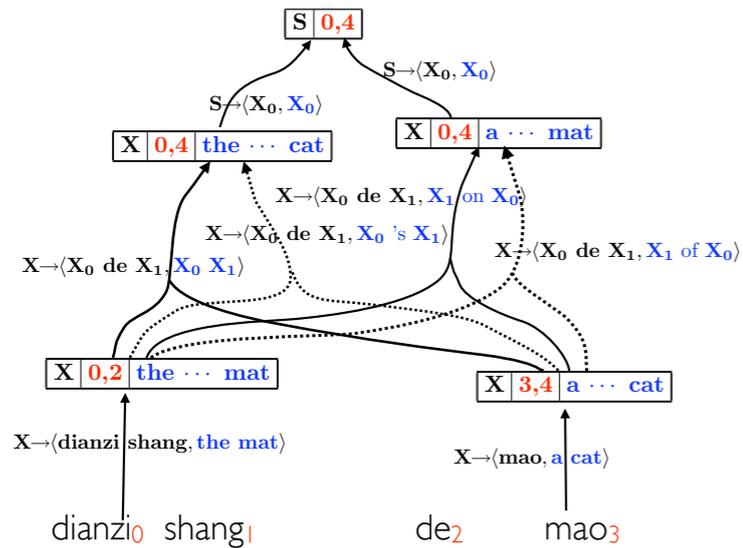


# Decoding over a hypergraph



Given a hypergraph of possible translations  
(generated for a given foreign sentence by already-trained model)

# Decoding over a hypergraph



Given a hypergraph of possible translations  
(generated for a given foreign sentence by already-trained model)

Pick a single translation to output  
(why not just pick the tree with the highest weight?)

# Spurious Ambiguity

- Statistical models in MT exhibit **spurious ambiguity**
  - Many **different derivations** (e.g., trees or segmentations) generate the **same translation string**
- Tree-based MT systems
  - **derivation tree** ambiguity
- Regular phrase-based MT systems
  - **phrase segmentation** ambiguity

# Spurious Ambiguity in Derivation Trees

# Spurious Ambiguity in Derivation Trees

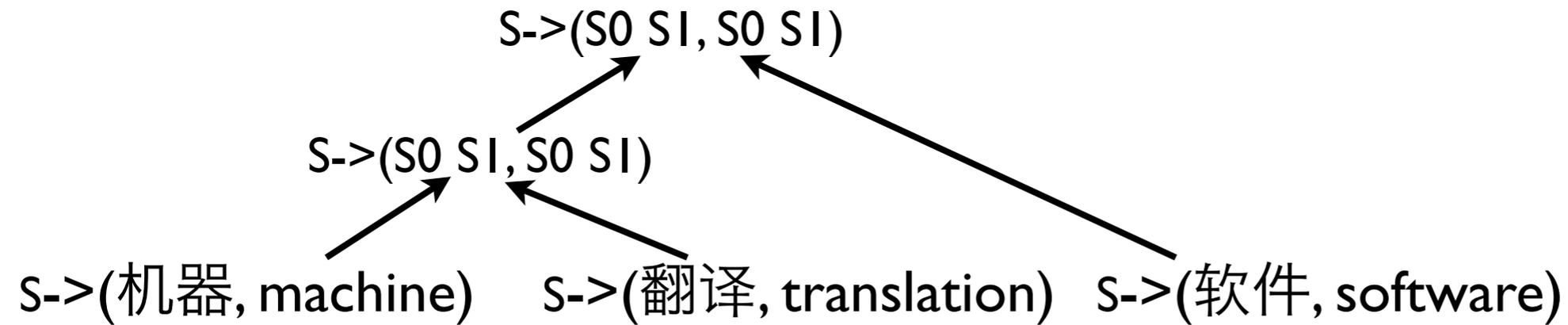
机器 翻译 软件 machine translation software

jìqī fānyì yuǎnjiàn

# Spurious Ambiguity in Derivation Trees

机器 翻译 软件 machine translation software

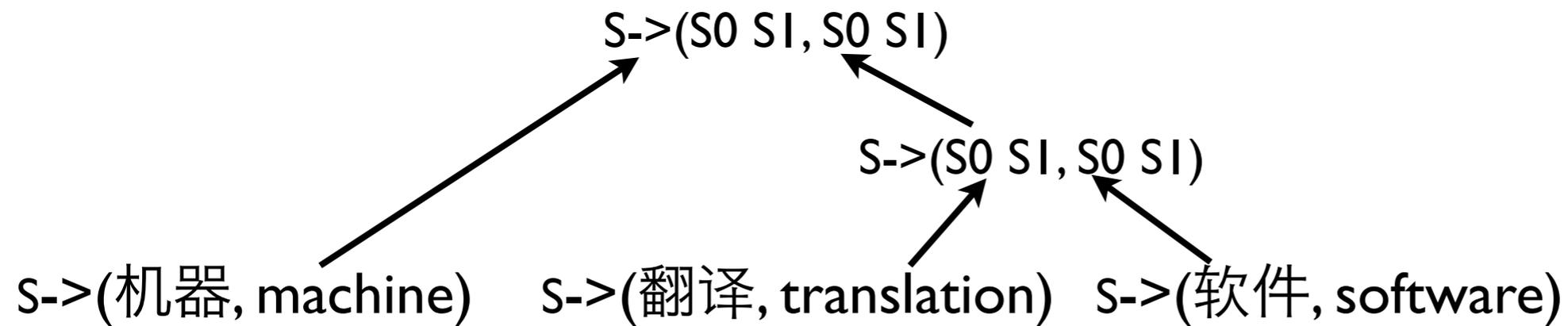
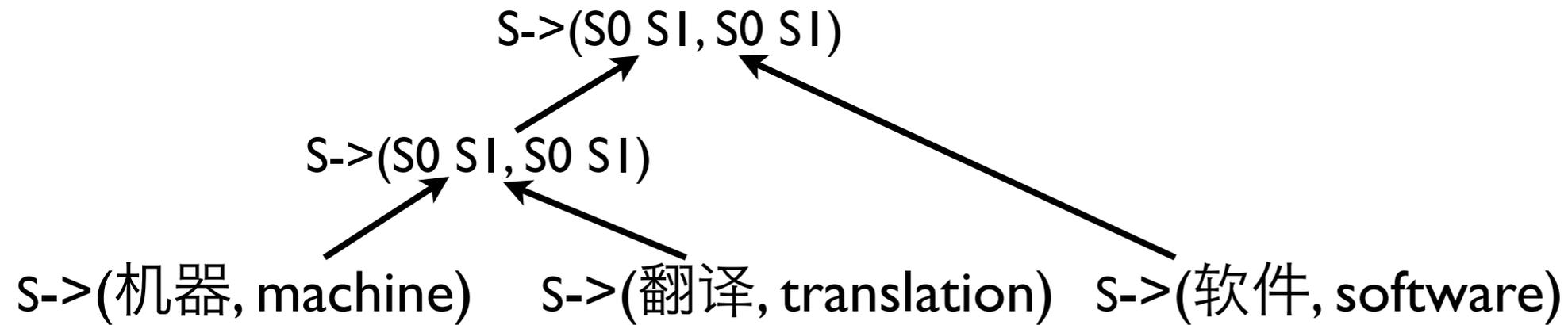
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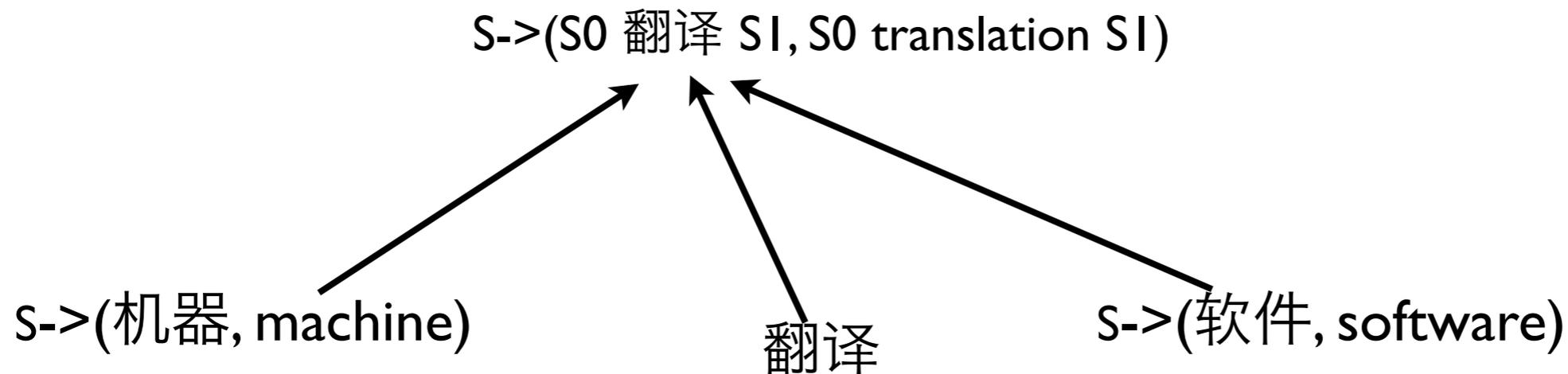
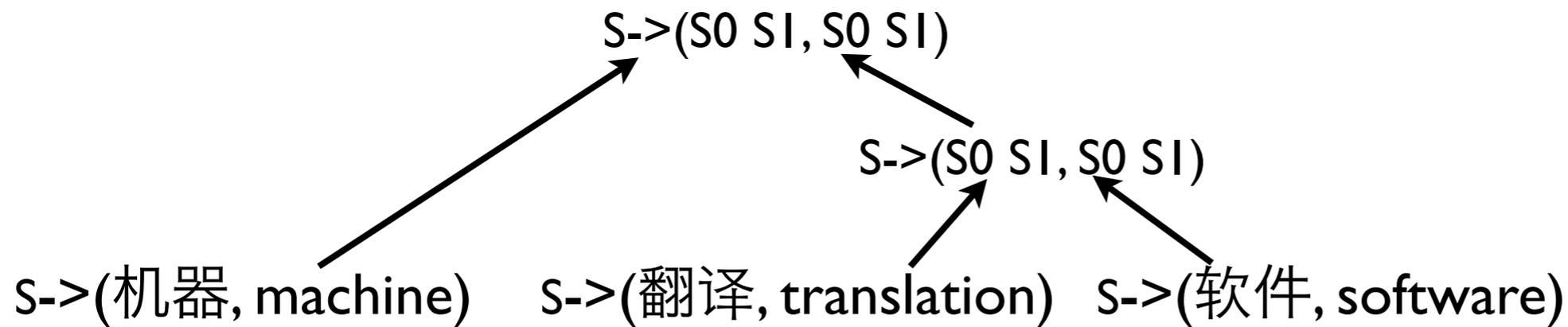
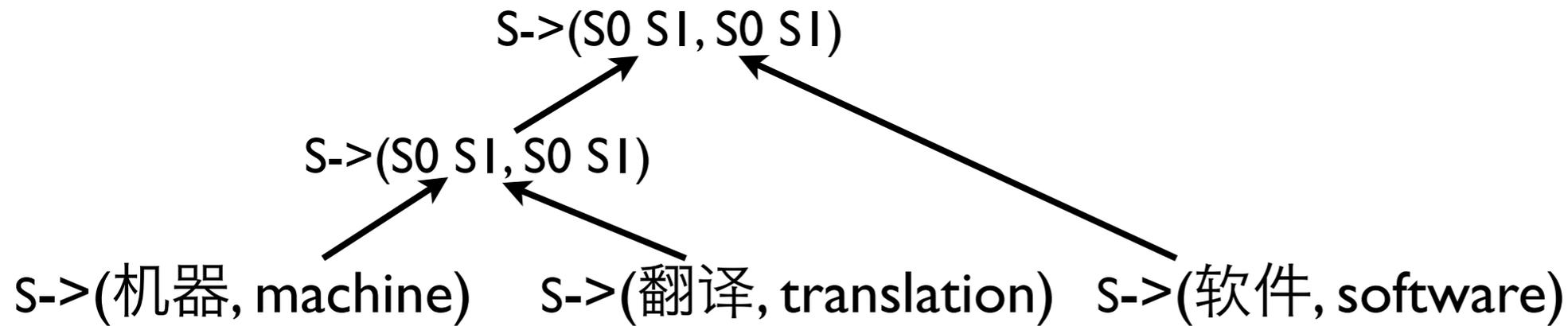
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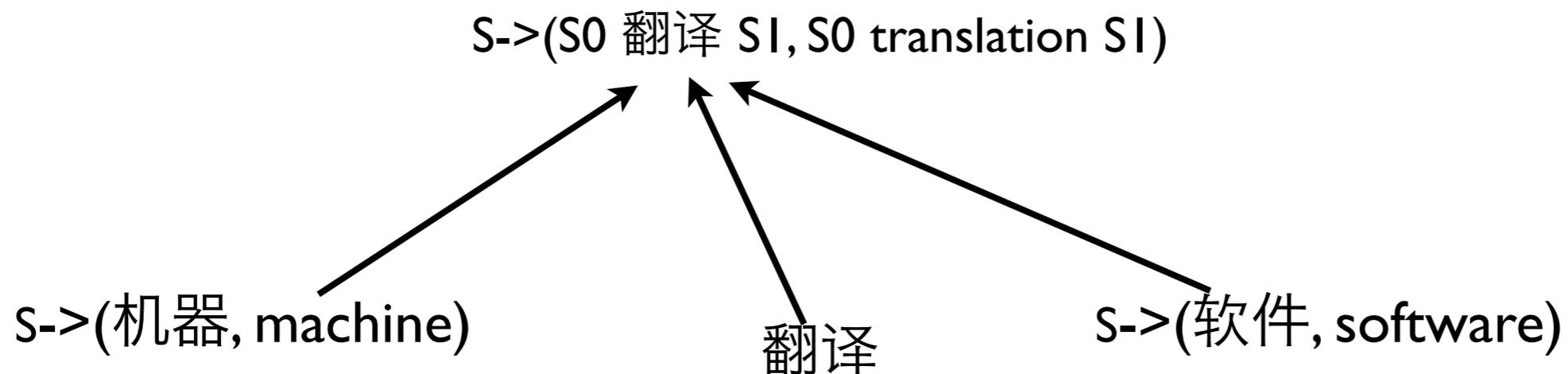
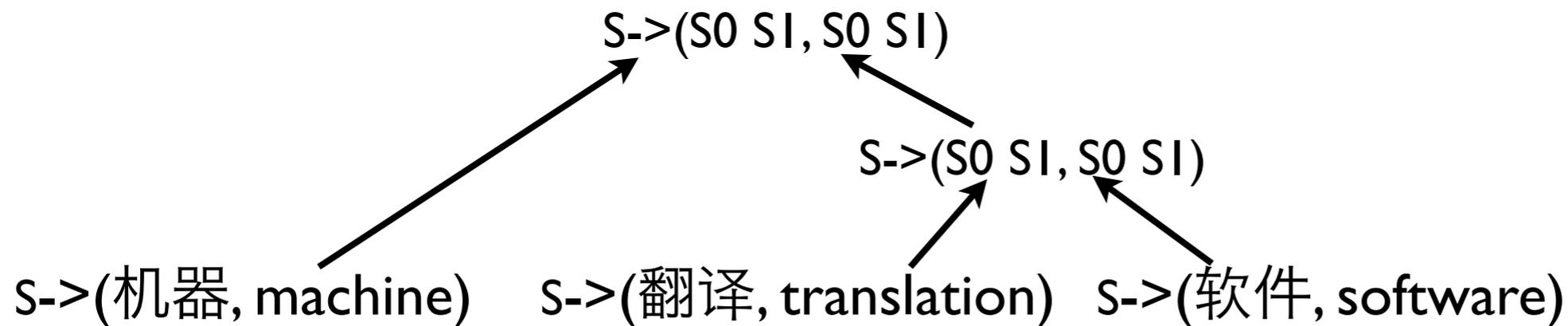
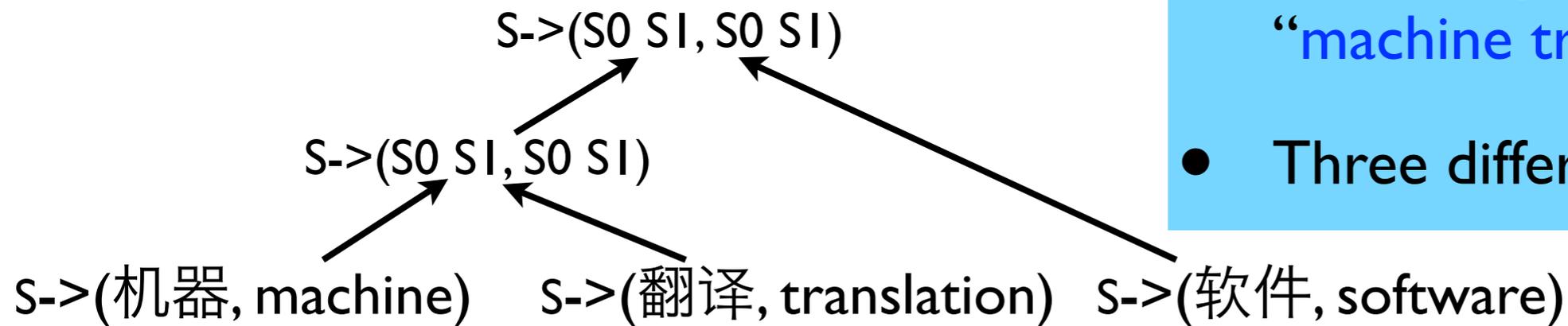


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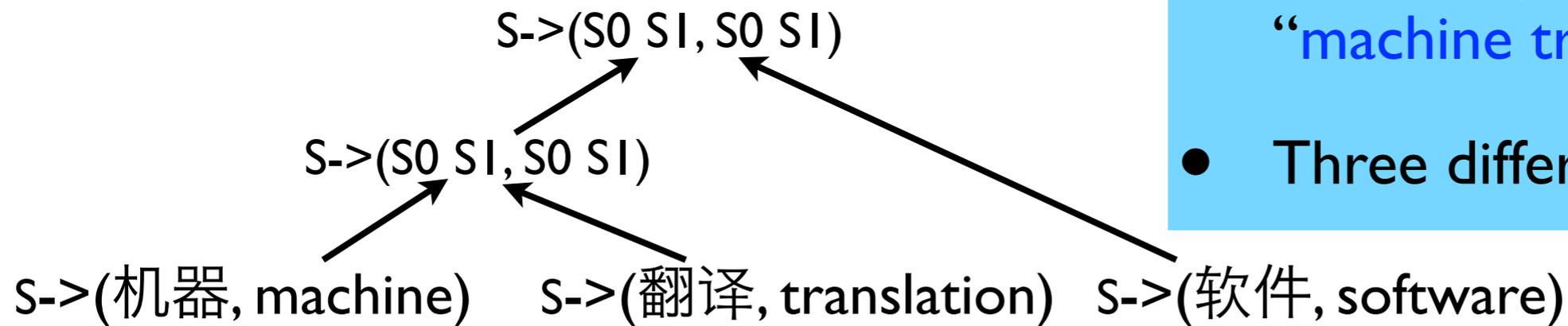
- Same output: “machine translation software”
- Three different derivation trees



# Spurious Ambiguity in Derivation Trees

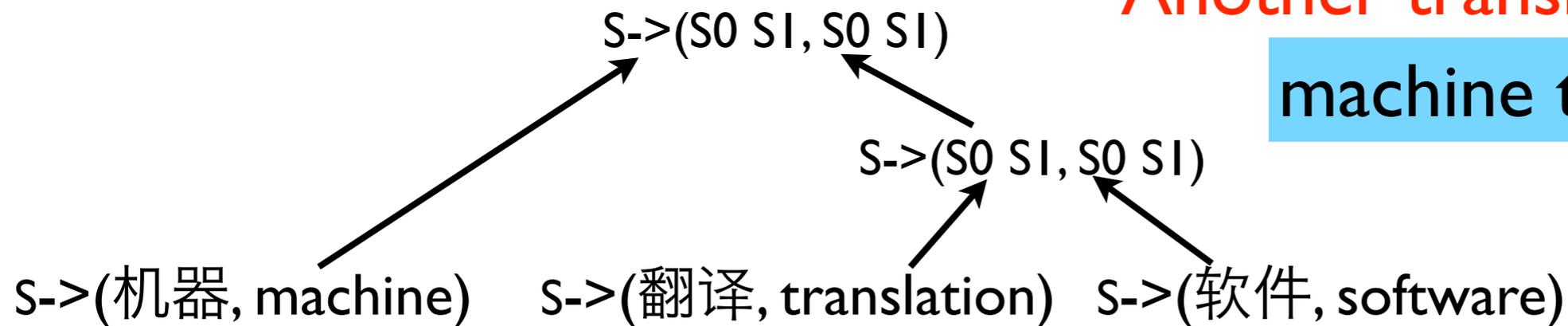
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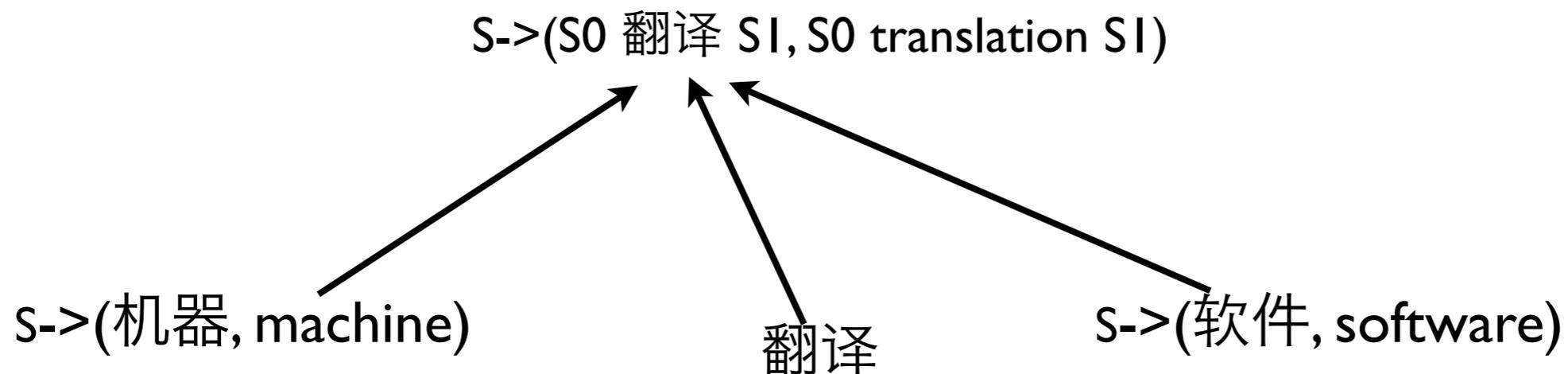


- Same output: “machine translation software”
- Three different derivation trees

Another translation:



machine transfer software



# MAP, Viterbi and N-best Approximations

# MAP, Viterbi and N-best Approximations

- Exact MAP decoding

$$\begin{aligned} y^* &= \arg \max_{y \in \text{Trans}(x)} p(y|x) \\ &= \arg \max_{y \in \text{Trans}(x)} \sum_{d \in D(x,y)} p(y, d|x) \end{aligned}$$

# MAP, Viterbi and N-best Approximations

- Exact MAP decoding

$$y^* = \arg \max_{y \in \text{Trans}(x)} p(y|x) \quad \text{NP-hard (Sima'an 1996)}$$

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- N-best approximation (**crunching**) (May and Knight 2006)

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# MAP vs. Approximations

translation string	MAP	Viterbi	4-best crunching	derivation	probability
red translation	0.28	0.16	0.16		0.16
blue translation	0.28	0.14	0.28		0.14
green translation	0.44	0.13	0.13		0.13
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- Exact MAP decoding under spurious ambiguity is **intractable** on HG
- Viterbi and crunching are efficient, but ignore most derivations
- Our goal: develop an **approximation** that considers **all** the derivations **but** still allows **tractable** decoding

# Variational Decoding

# Variational Decoding

**Decoding** using **Variational** approximation

**Decoding** using a sentence-specific  
**approximate** distribution

# Variational Decoding for MT: an Overview

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Sentence-specific decoding

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Sentence-specific decoding

Three steps:

# Variational Decoding for MT: an Overview

## Sentence-specific decoding

### Three steps:

- 1 Generate a hypergraph for the foreign sentence

# Variational Decoding for MT: an Overview

## Sentence-specific decoding

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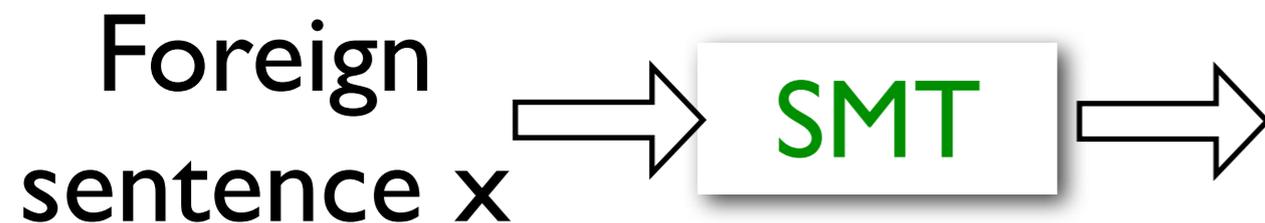
Foreign  
sentence  $x$

# Variational Decoding for MT: an Overview

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# Variational Decoding for MT: an Overview

Sentence-specific decoding

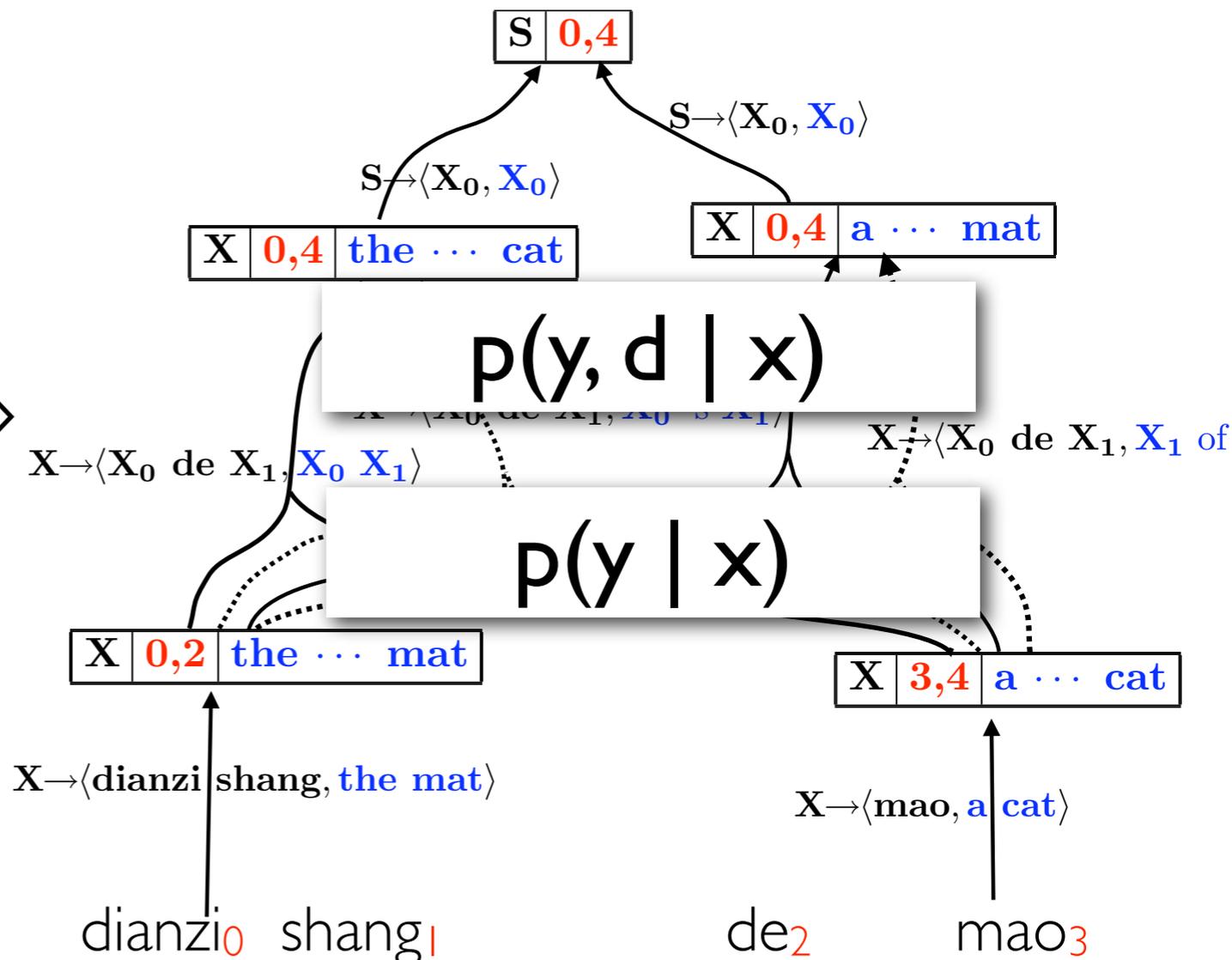
Three steps:

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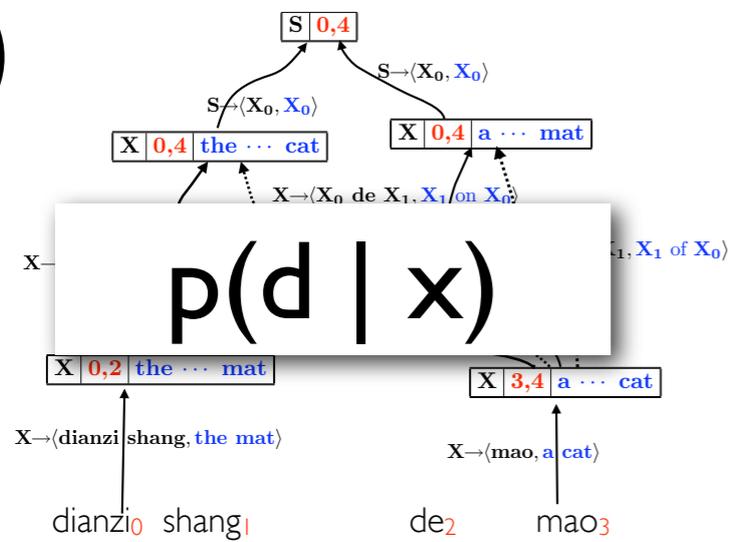
MAP decoding under P is intractable

Foreign sentence  $x$

SMT

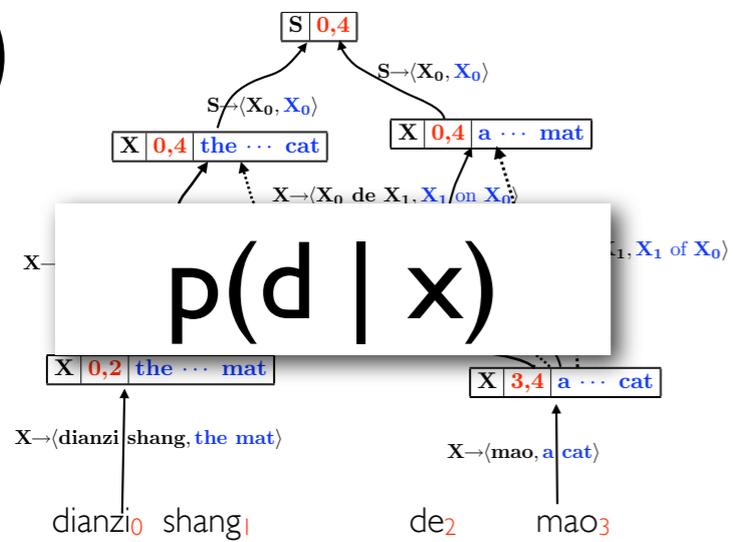


1



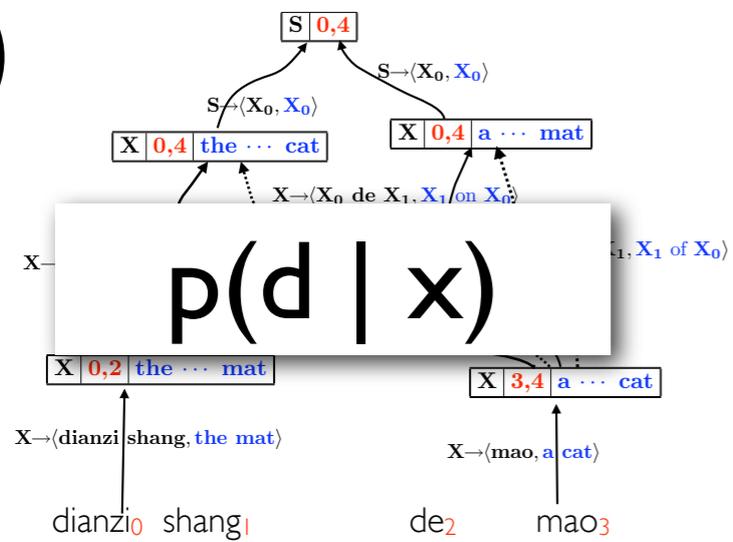
Generate a hypergraph

1



Generate a hypergraph

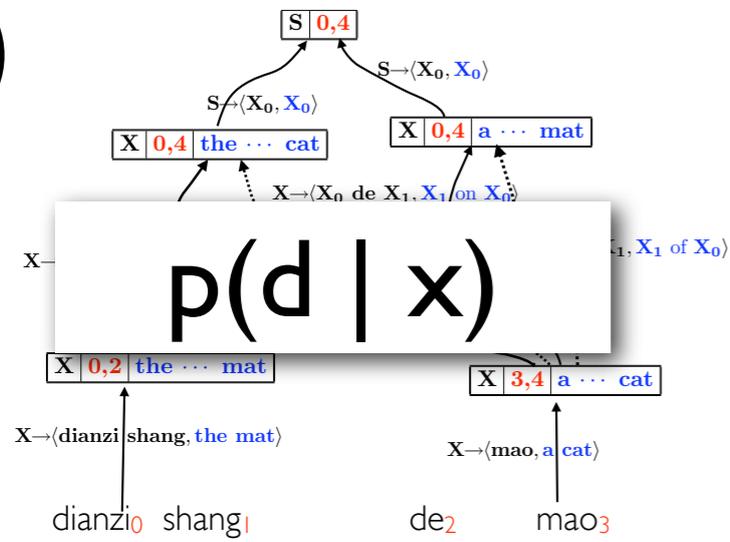
1



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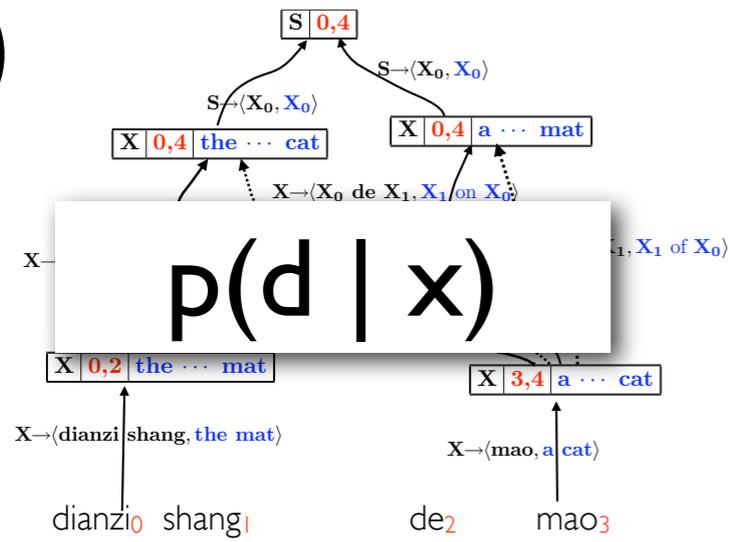
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1

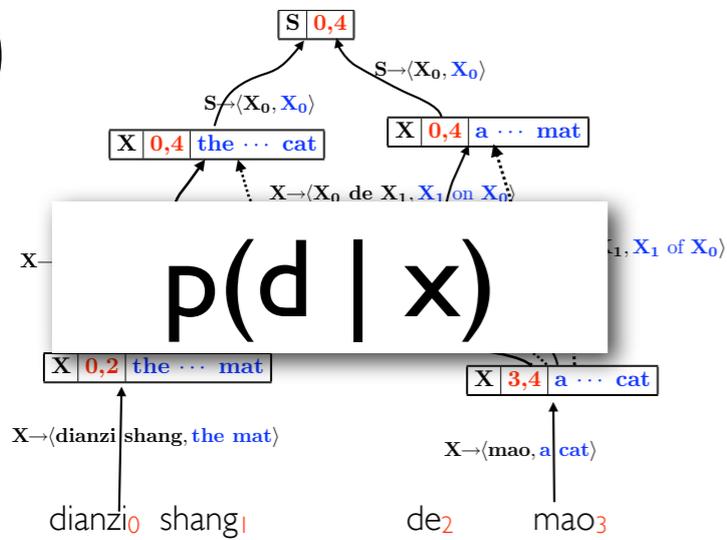


Generate a hypergraph

2

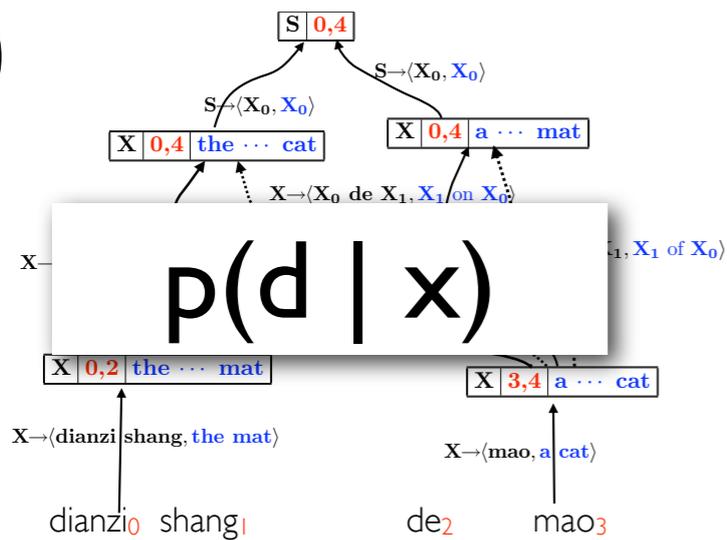


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Generate a hypergraph

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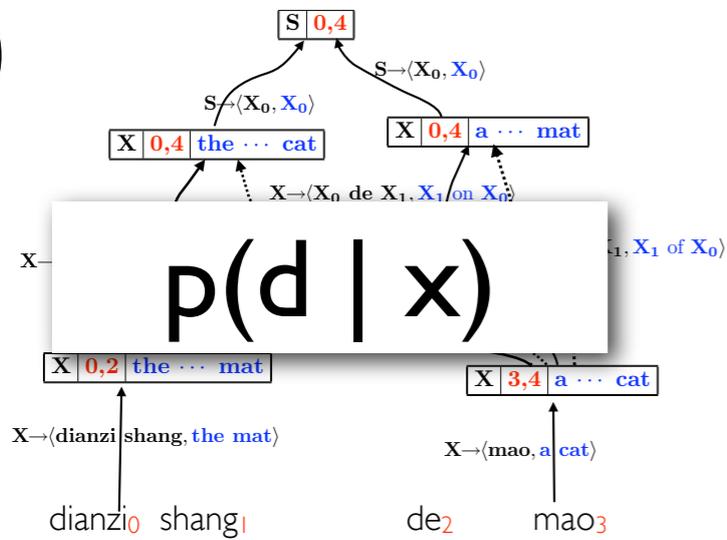


Estimate a model from the hypergraph



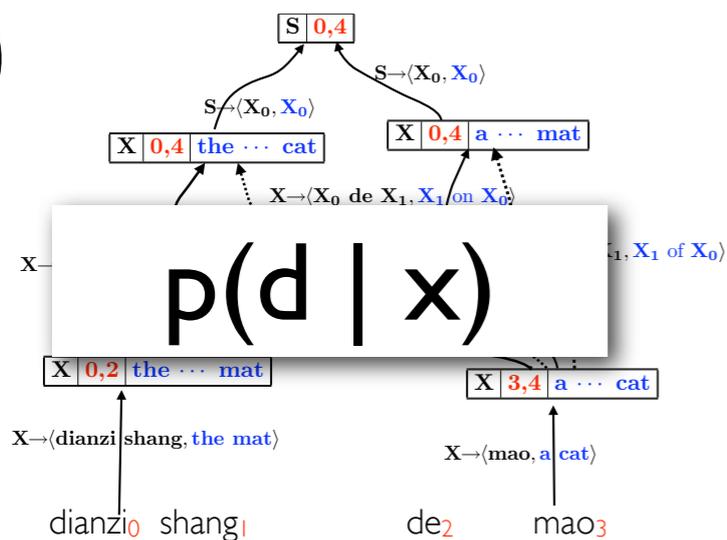
$$q^*(y | x)$$

1



Generate a hypergraph

2



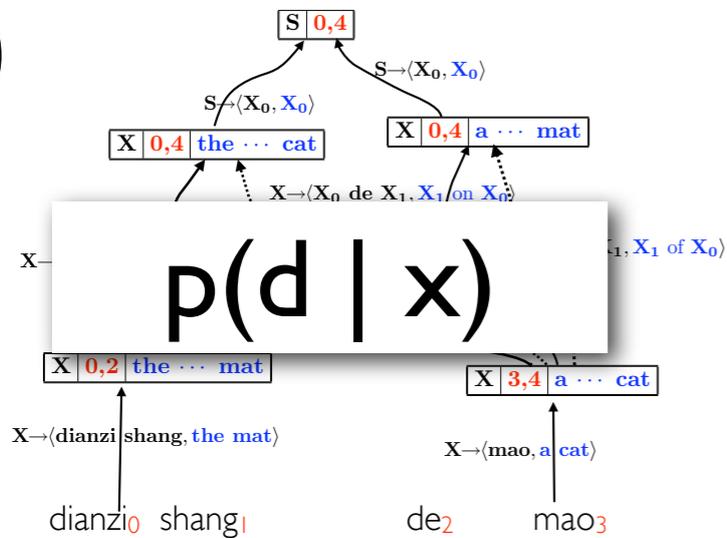
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$q^*$  is an n-gram model over output strings.



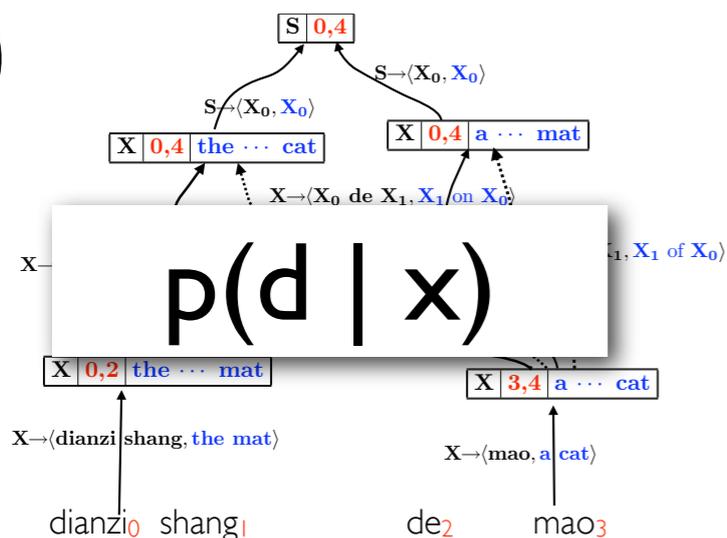
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1



Generate a hypergraph

2



Estimate a model from the hypergraph

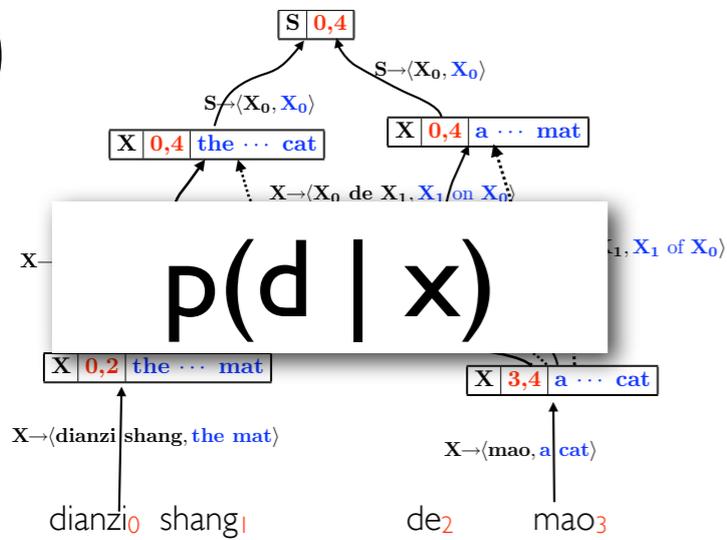


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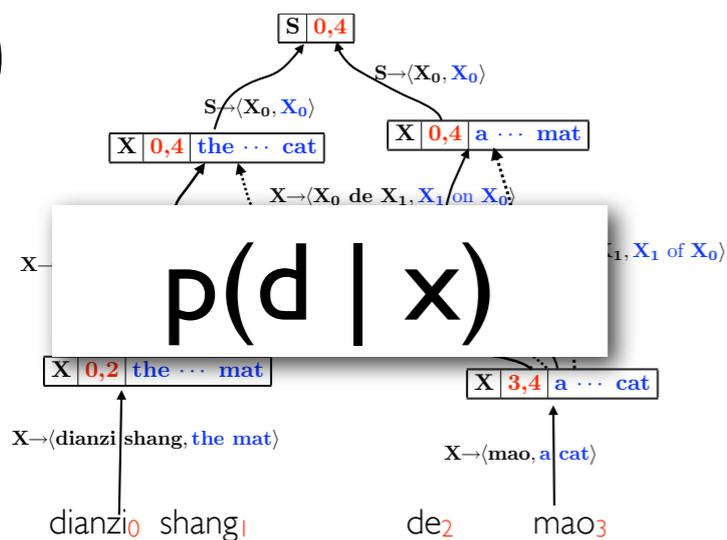
$$\approx \sum_{d \in D(x,y)} p(d|x)$$

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Generate a hypergraph

2



Estimate a model from the hypergraph

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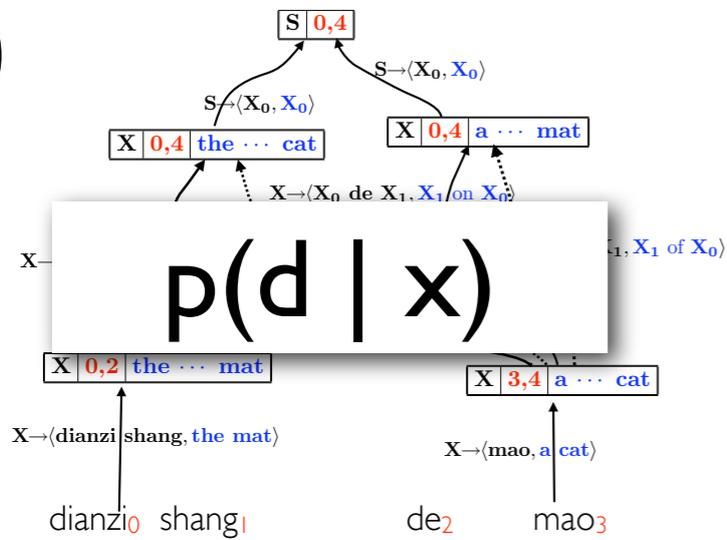


$$q^*(y | x)$$

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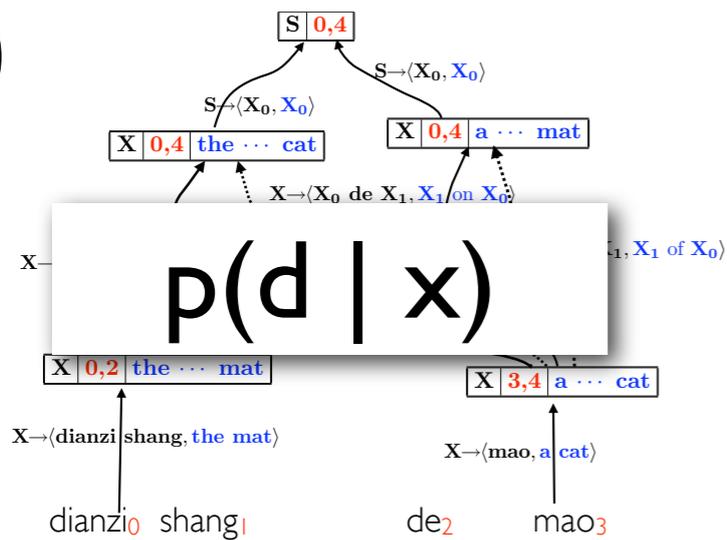
3

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Generate a hypergraph

2



Estimate a model from the hypergraph

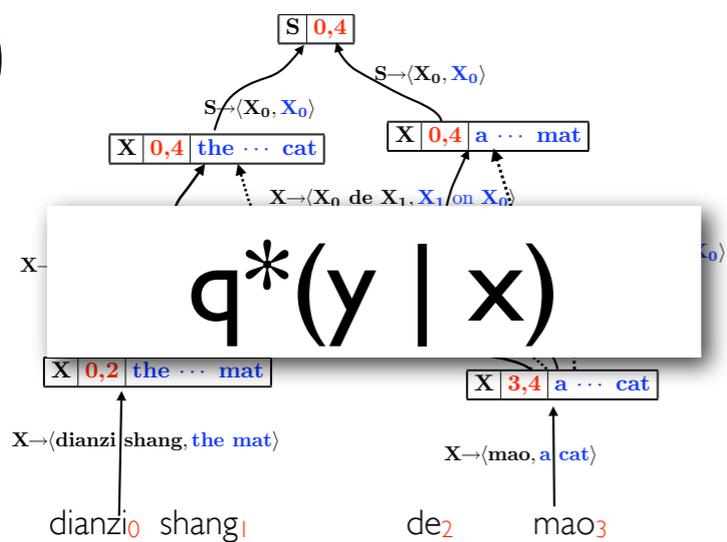
$q^*$  is an n-gram model over output strings.



$$q^*(y | x)$$

$$\approx \sum_{d \in D(x,y)} p(d|x)$$

3



Decode using  $q^*$  on the hypergraph

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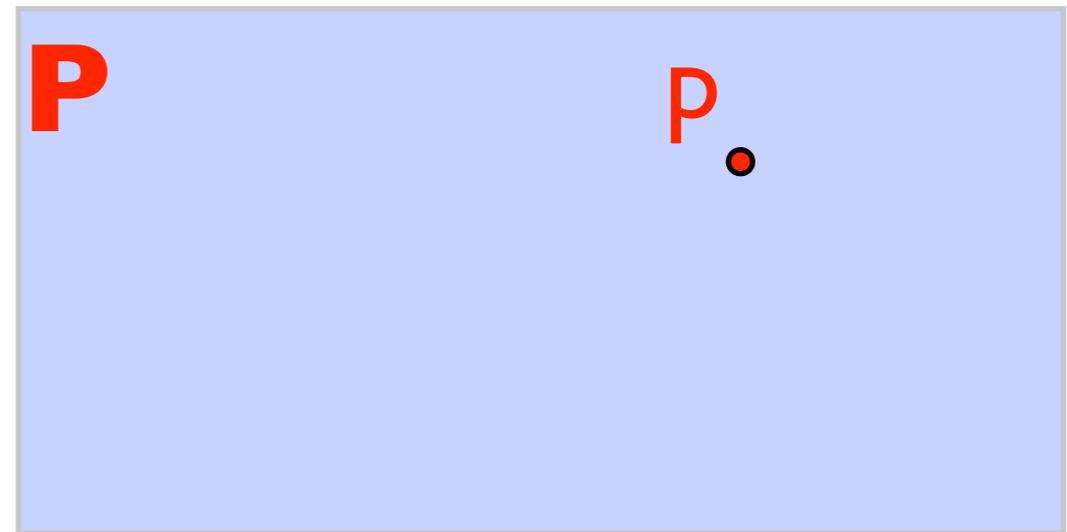
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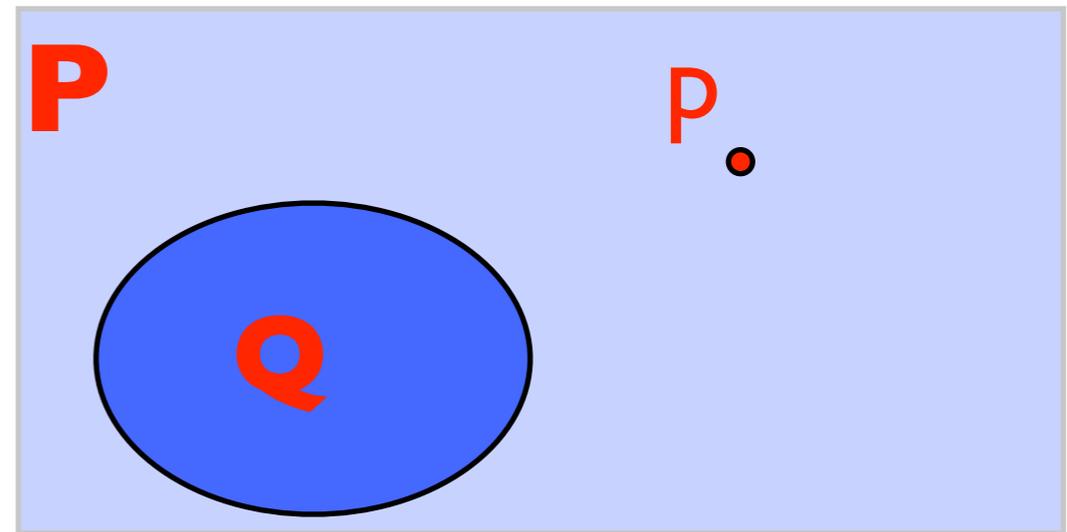
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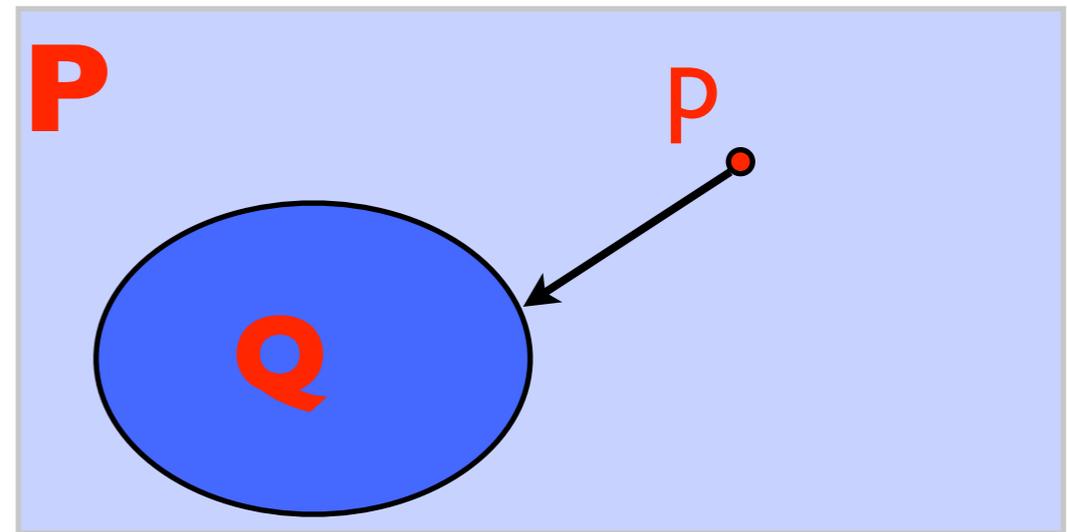
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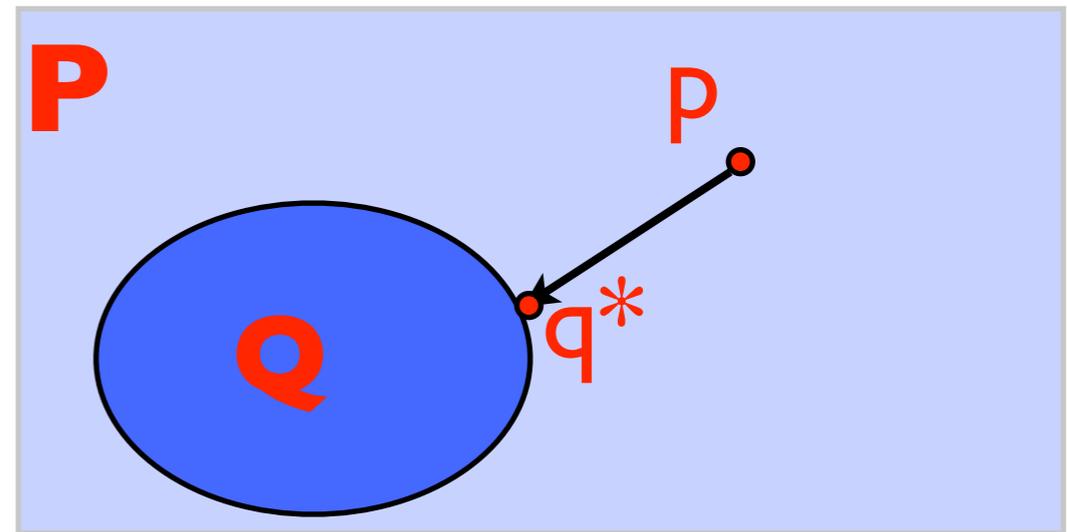
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compute expected n-gram counts and normalize

score the hypergraph with the n-gram model

# KL divergences under different variational models

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$$q^* = \arg \min_{q \in \mathcal{Q}} \text{KL}(p||q) = \bar{H}(p, q) - \bar{H}(p)$$

Measure bits/word	$\bar{H}(p)$	$\bar{\text{KL}}(p  \cdot)$			
		$q_1^*$	$q_2^*$	$q_3^*$	$q_4^*$
MT'04	1.36	0.97	0.32	0.21	0.17
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- Larger **n** ==> better approximation  $q_n$  ==> smaller KL divergence from  $p$
- The reduction of KL divergence happens mostly when switching from unigram to bigram

# BLEU Results on Chinese-English NIST MT 2004 Tasks

	Decoding scheme	BLEU
	Viterbi	35.4
(Kumar and Byrne, 2004)	MBR ( $K=1000$ )	35.8
(May and Knight, 2006)	Crunching ( $N=10000$ )	35.7
	Crunching+MBR ( $N=10000$ )	35.8
<b>New!</b>	Variational (1to4gram+wp+vt)	<b>36.6</b>

- variational decoding improves over Viterbi, MBR, and crunching

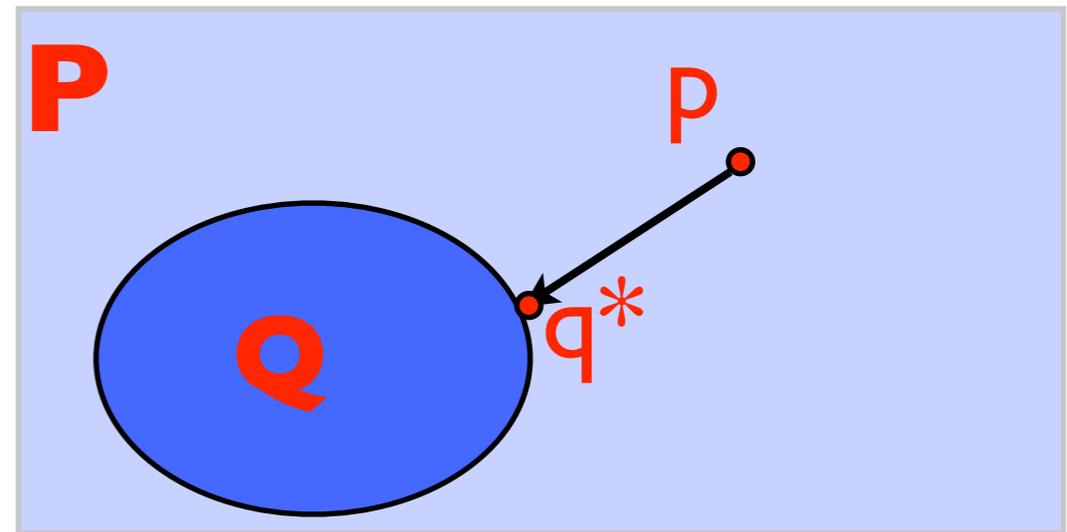
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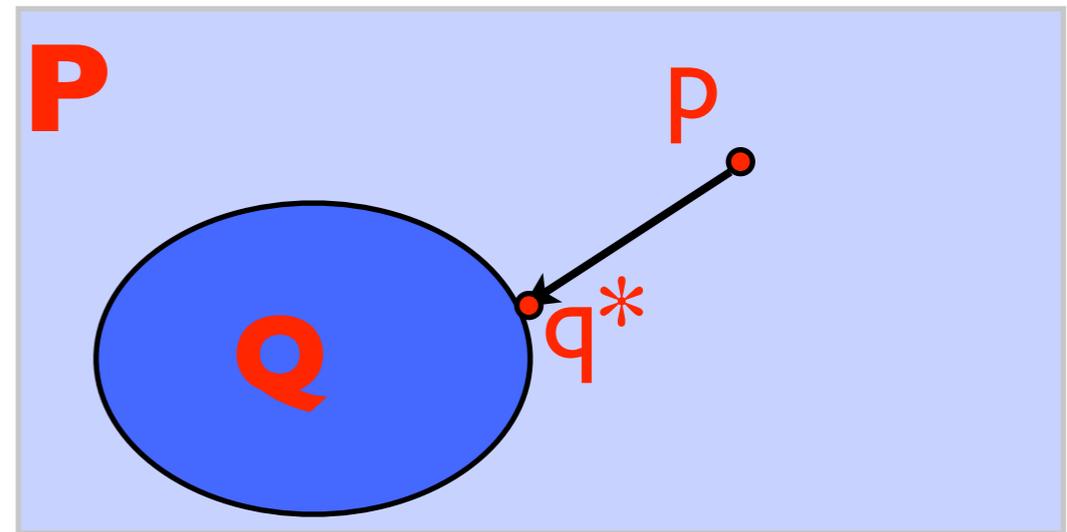
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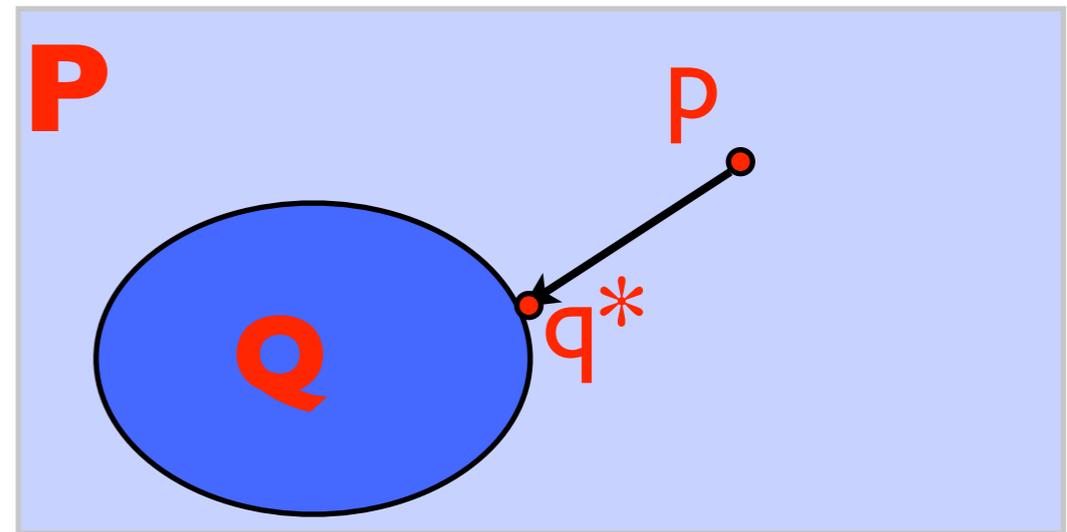
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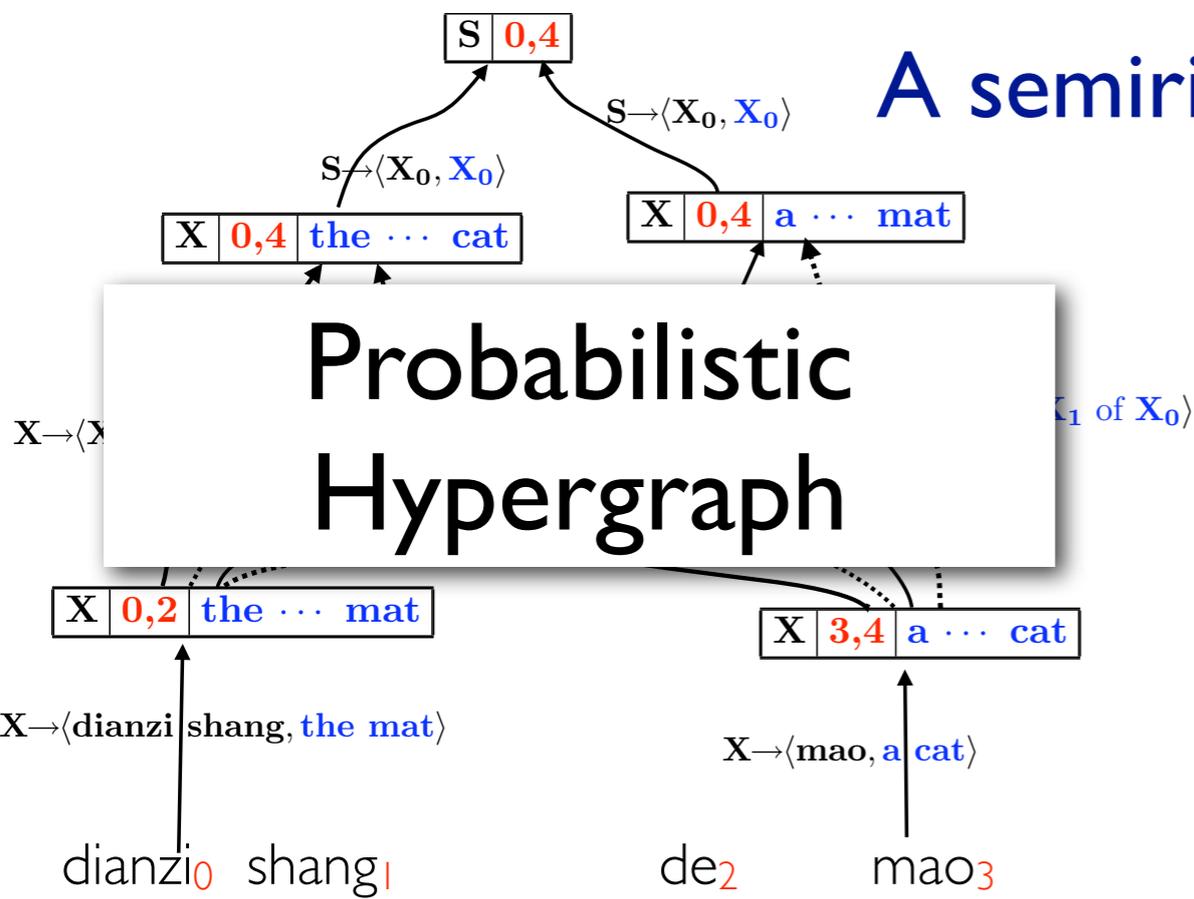
# Outline

- Hypergraph as Hypothesis Space
- Unsupervised Discriminative Training
  - ▶ minimum imputed risk
  - ▶ contrastive language model estimation
- Variational Decoding
- **First- and Second-order Expectation Semirings**

<b>decoding</b> (e.g., mbr)	<b>training</b> (e.g., mert)
<b>atomic inference operations</b> (e.g., finding one-best, k-best or expectation, inference can be <i>exact</i> or <i>approximate</i> )	

# A semiring framework to compute all of these

## Probabilistic Hypergraph



- “decoding” quantities:

- Viterbi
- K-best
- Counting
- .....

- First-order quantities:

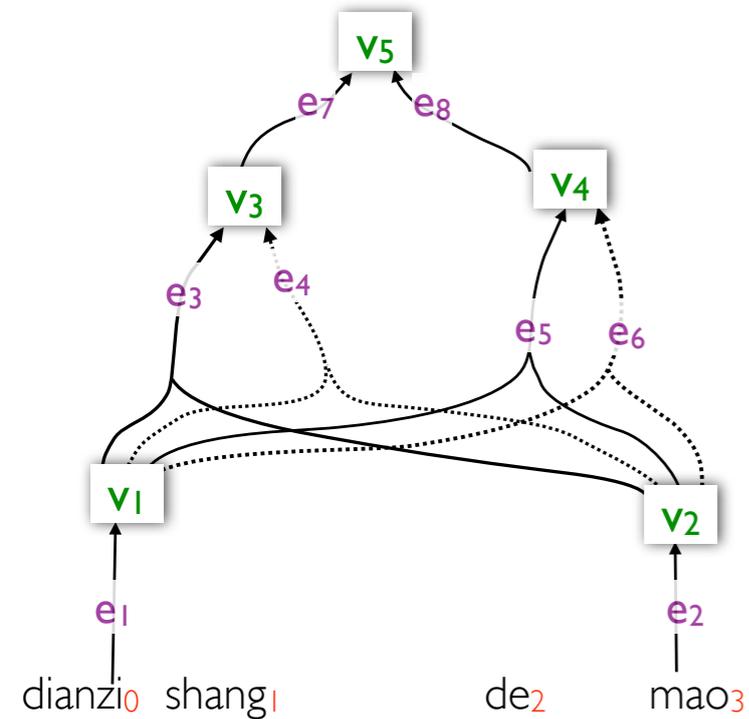
- expectation
- entropy
- Bayes risk
- cross-entropy
- KL divergence
- feature expectations
- first-order gradient of  $Z$

- Second-order quantities:

- expectation over product
- interaction between features
- Hessian matrix of  $Z$
- second-order gradient descent
- gradient of expectation
  - gradient of entropy or Bayes risk

# Compute Quantities on a Hypergraph: a Recipe

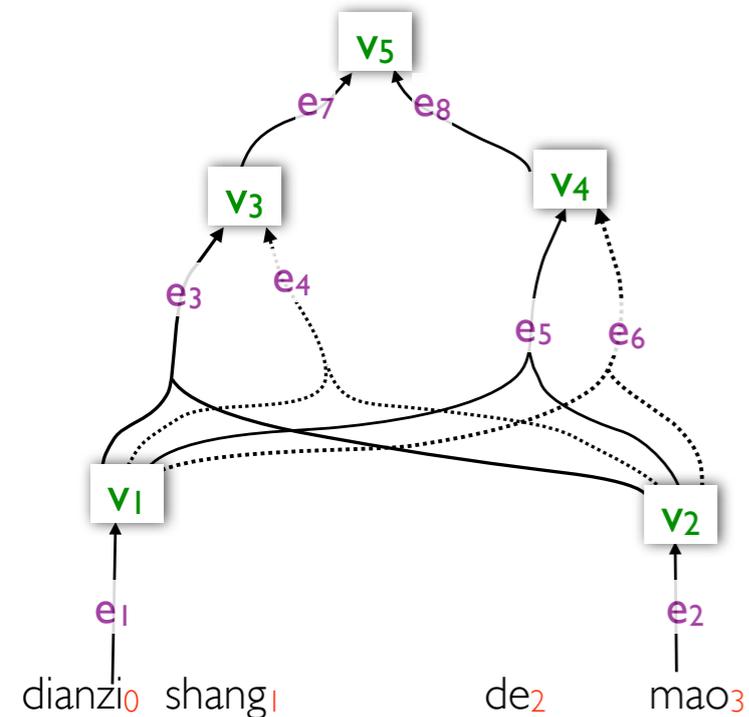
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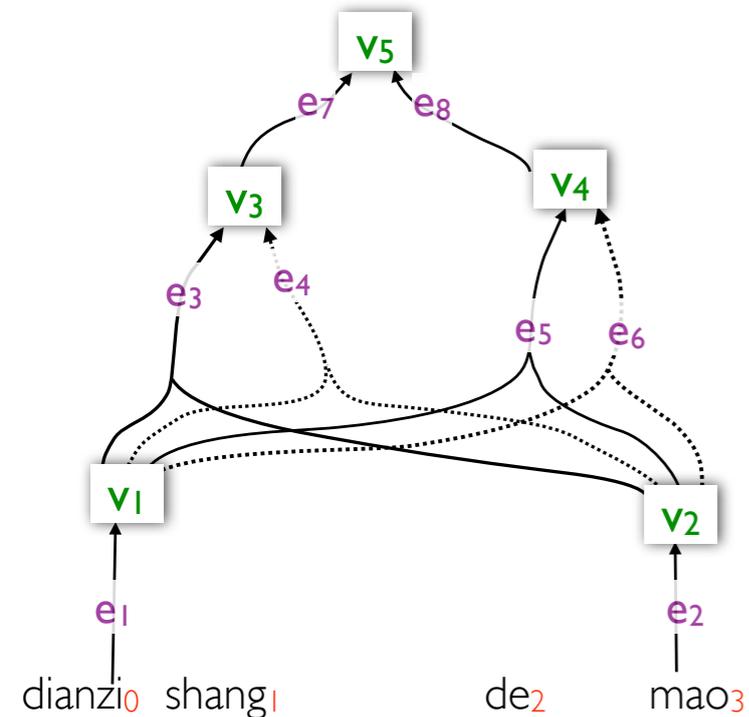
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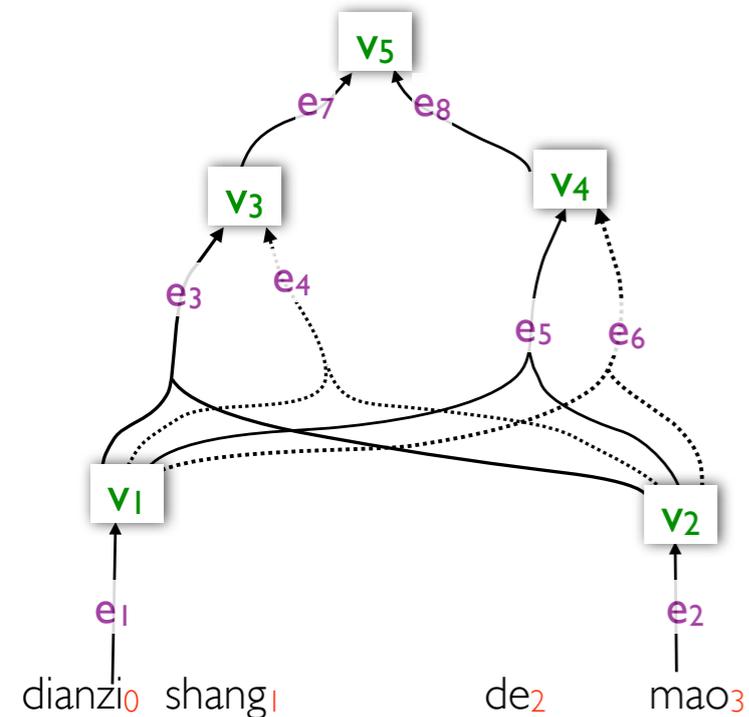


▶ specify a weight for each hyperedge

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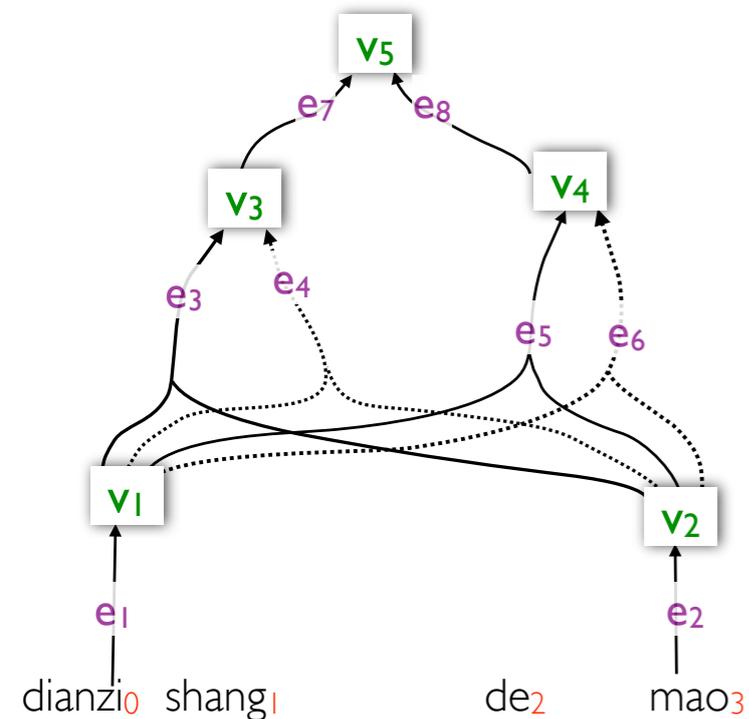
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  - three steps:

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$\langle K, \oplus, \otimes \rangle$   
a **set** with **plus** and **times** operations

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▶ run the inside algorithm



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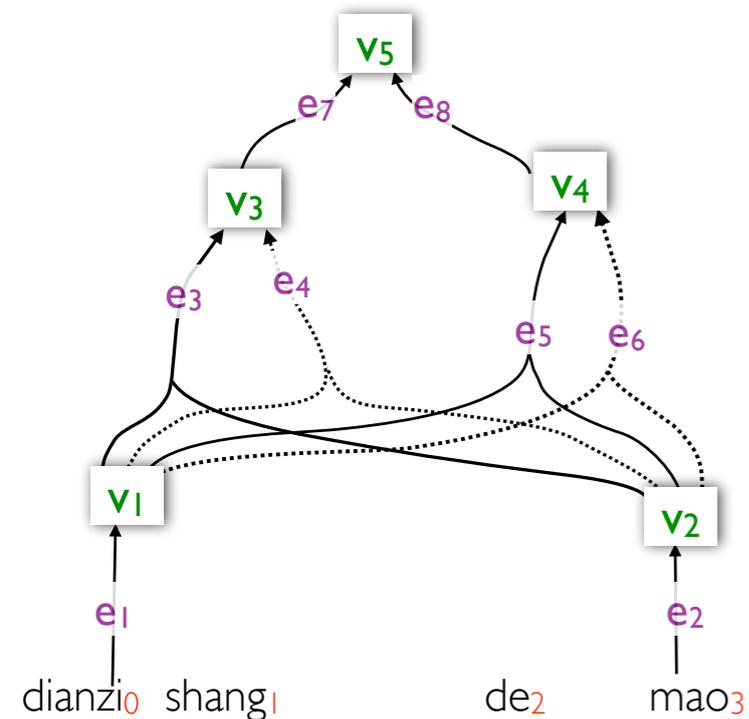
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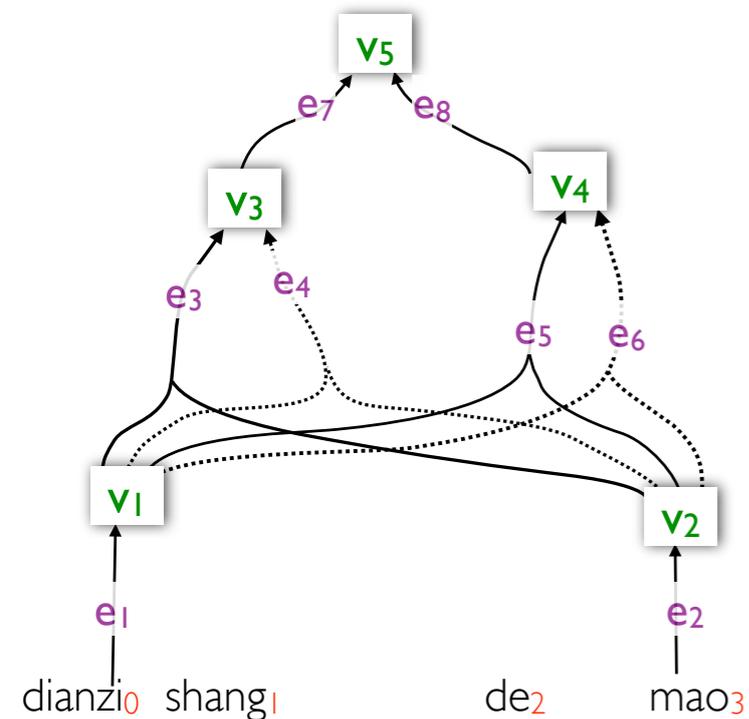
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each weight is a semiring member

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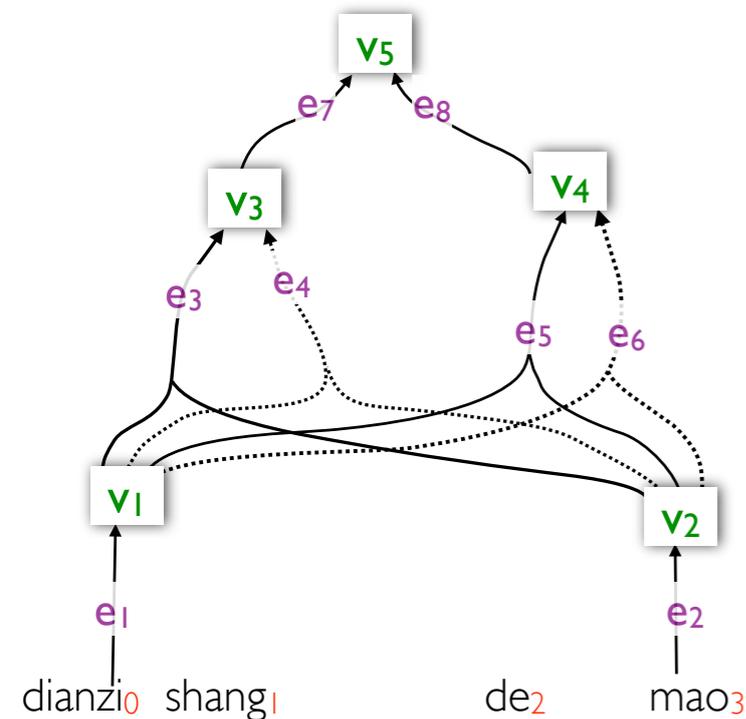
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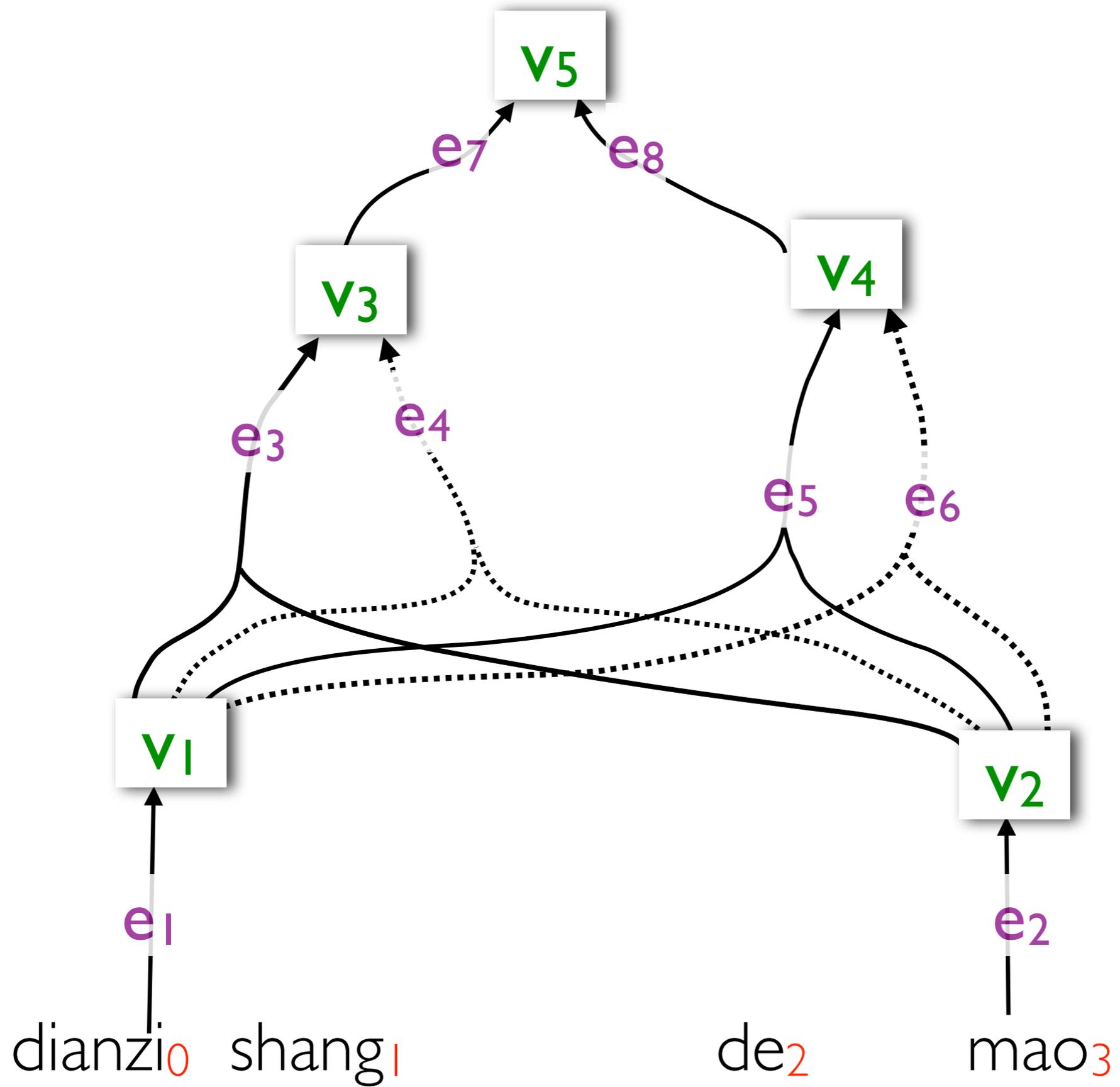
▶ run the inside algorithm

complexity is  $O(\text{hypergraph size})$

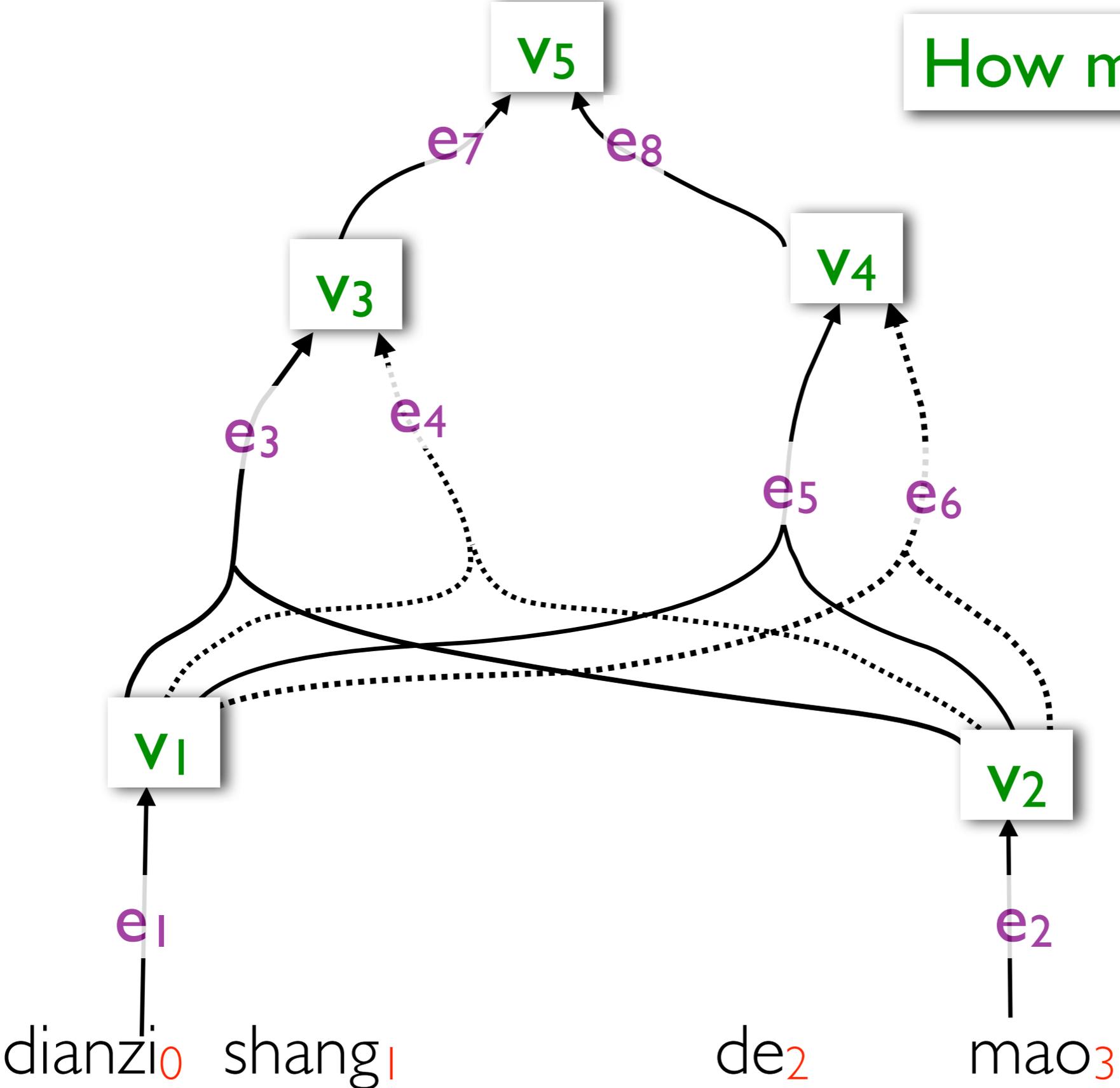


# Semirings

- “Decoding” time semirings (Goodman, 1999)
  - counting, Viterbi, K-best, etc.
- “Training” time semirings
  - first-order expectation semirings (Eisner, 2002)
  - second-order expectation semirings (new)
- Applications of the Semirings (new)
  - entropy, risk, gradient of them, and many more

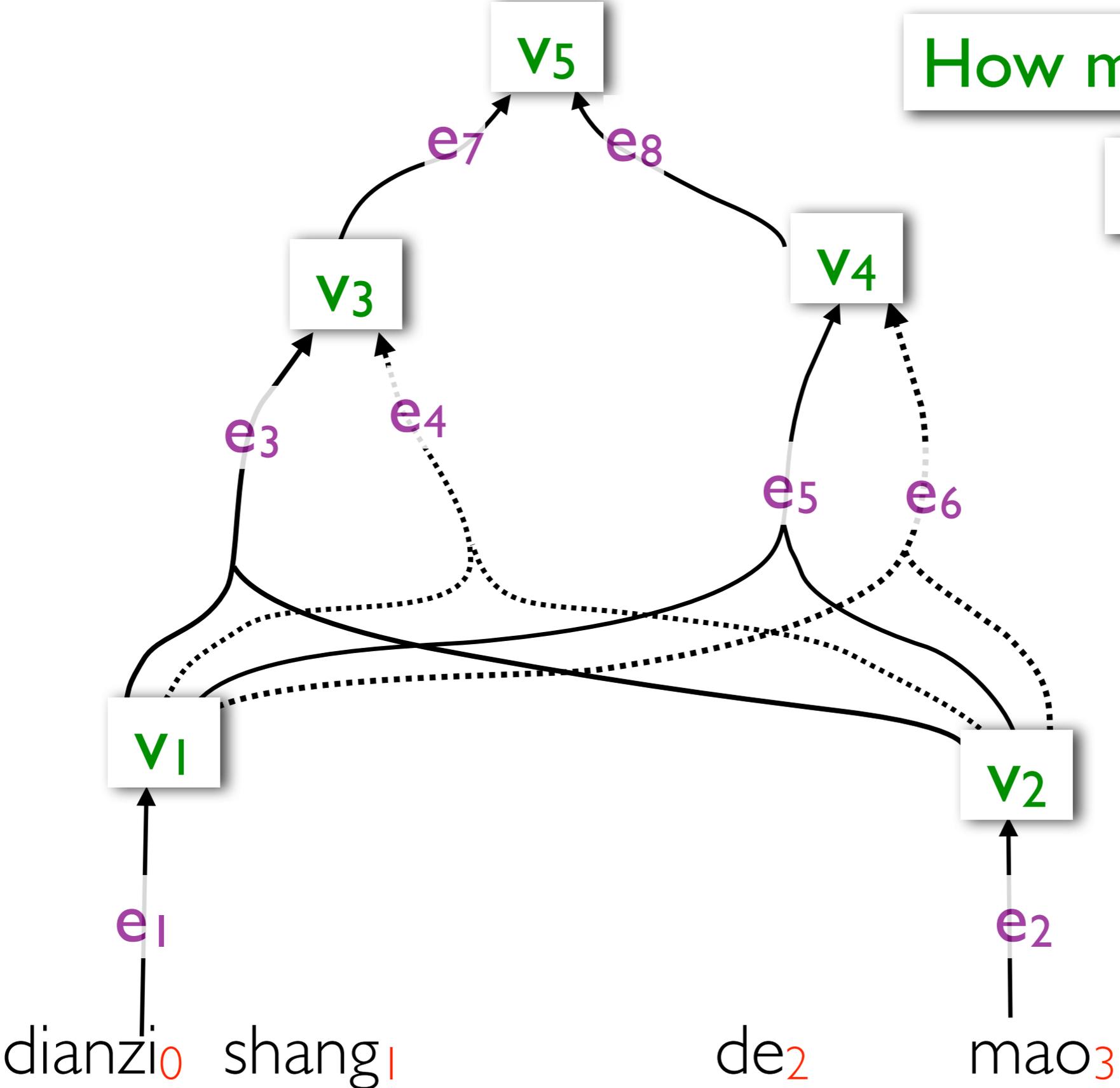


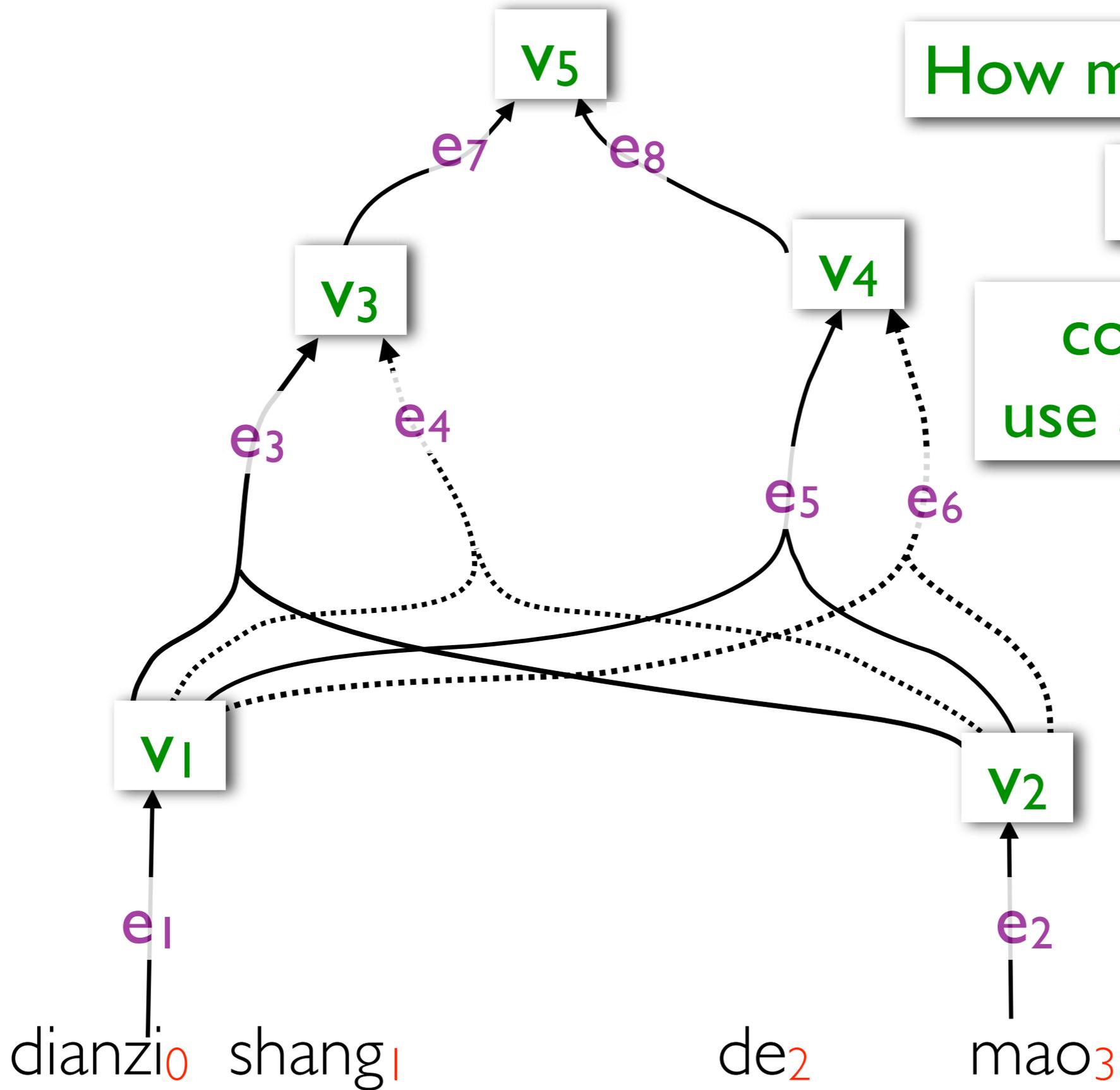
How many trees?



How many trees?

four 😊





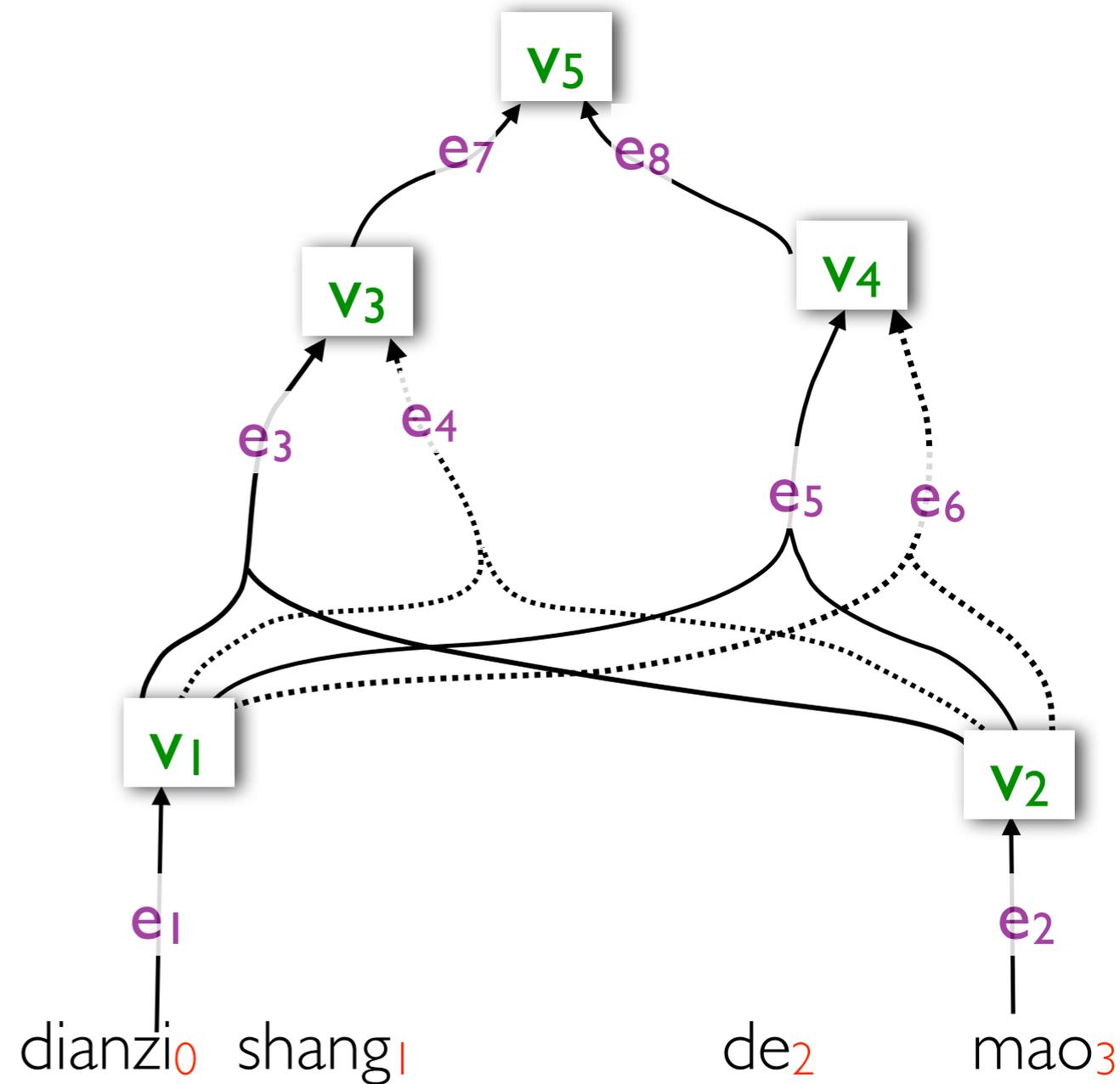
How many trees?

four 😊

compute it  
use a semiring?

# Compute the Number of Derivation Trees

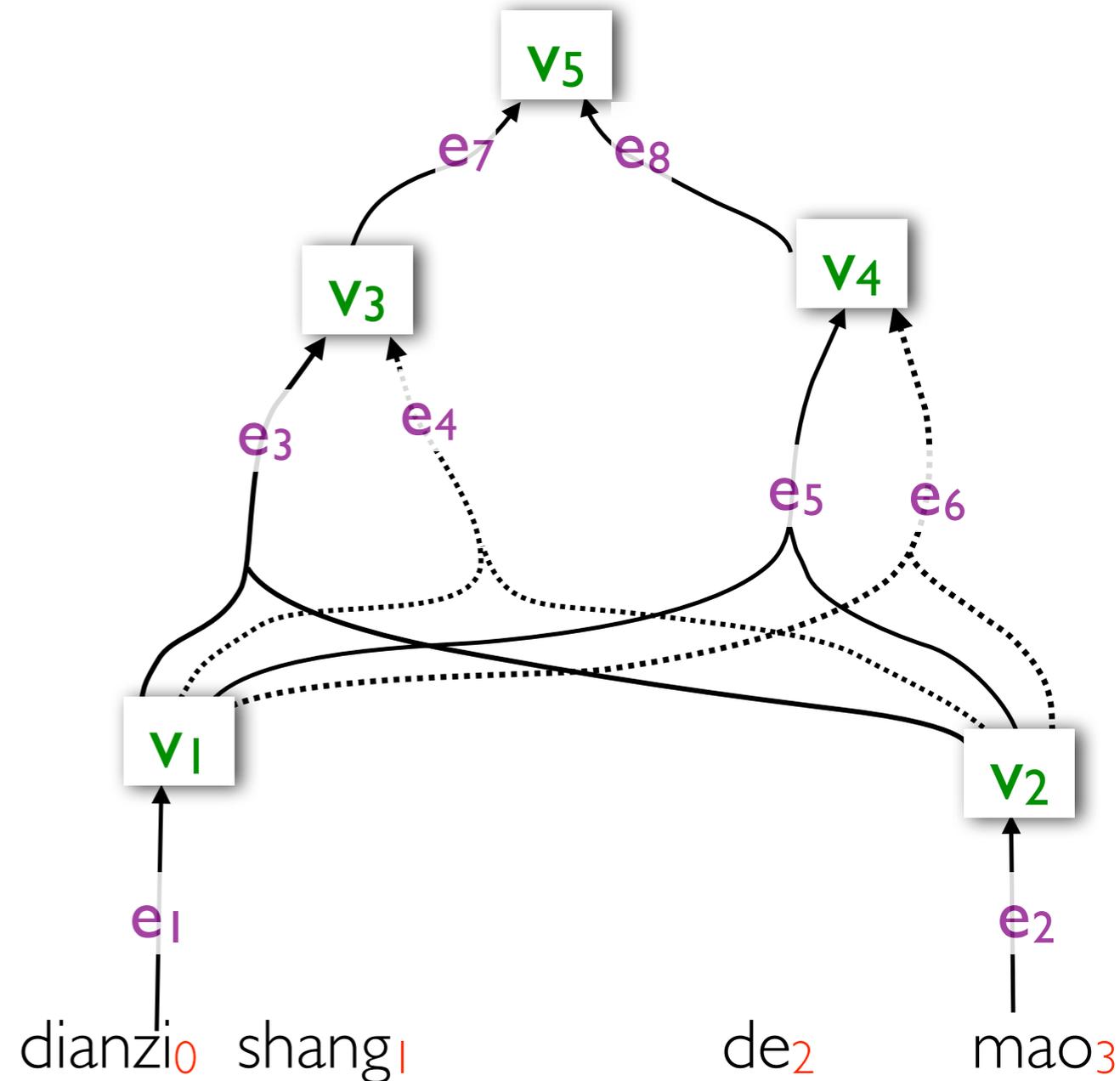
**Three steps:**



# Compute the Number of Derivation Trees

## Three steps:

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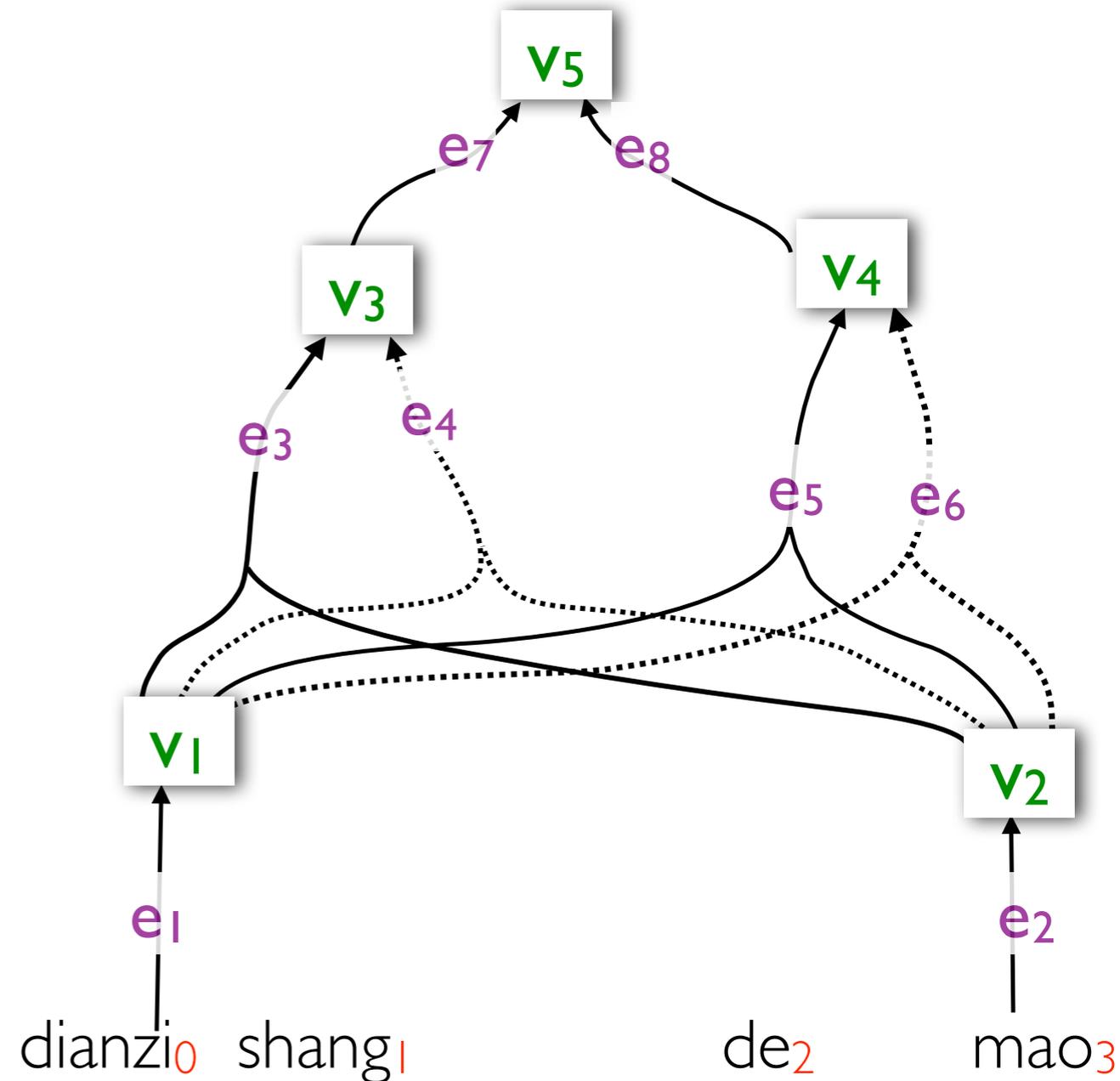
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counting semiring:

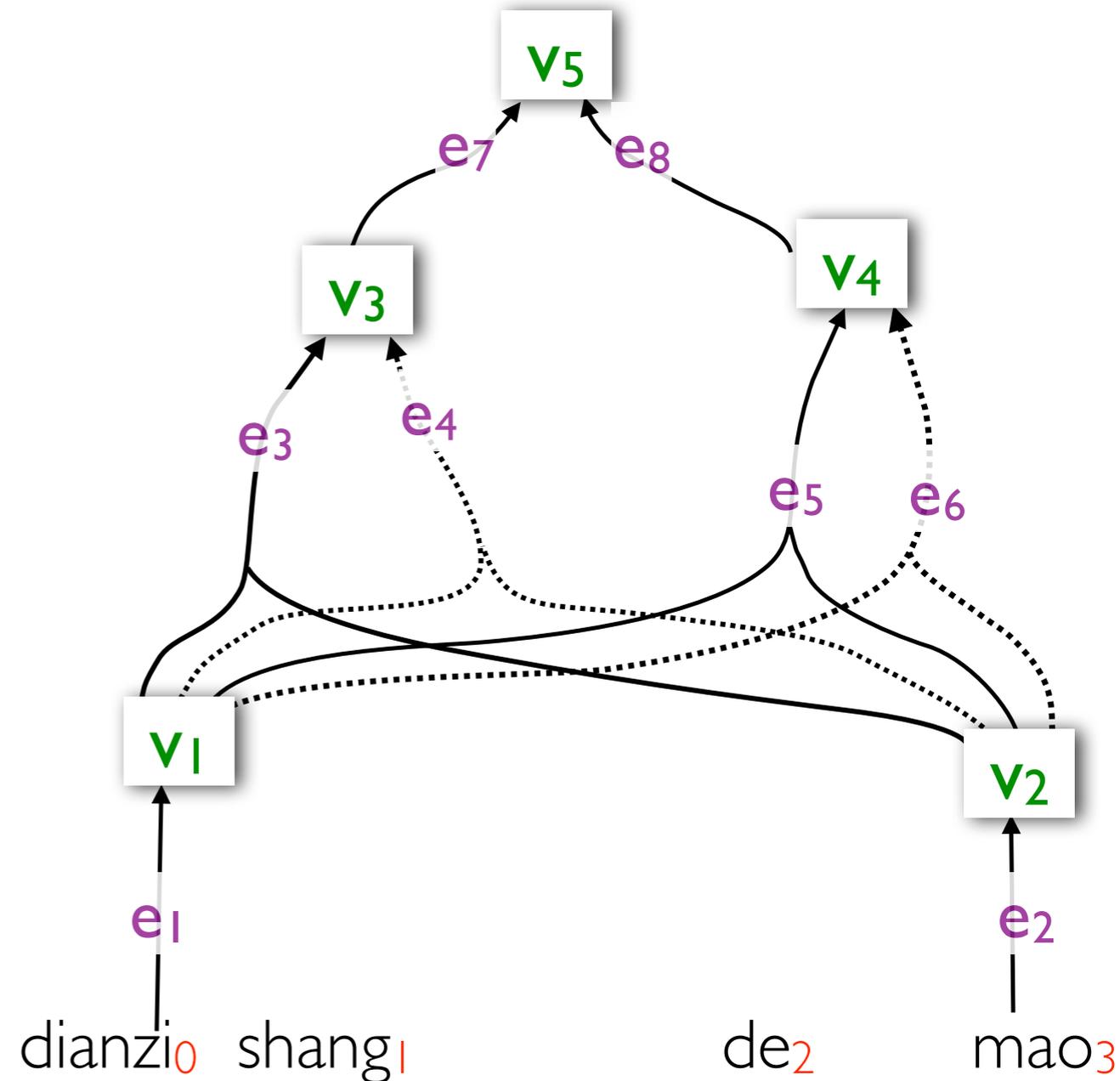
ordinary integers with  
regular  $+$  and  $\times$



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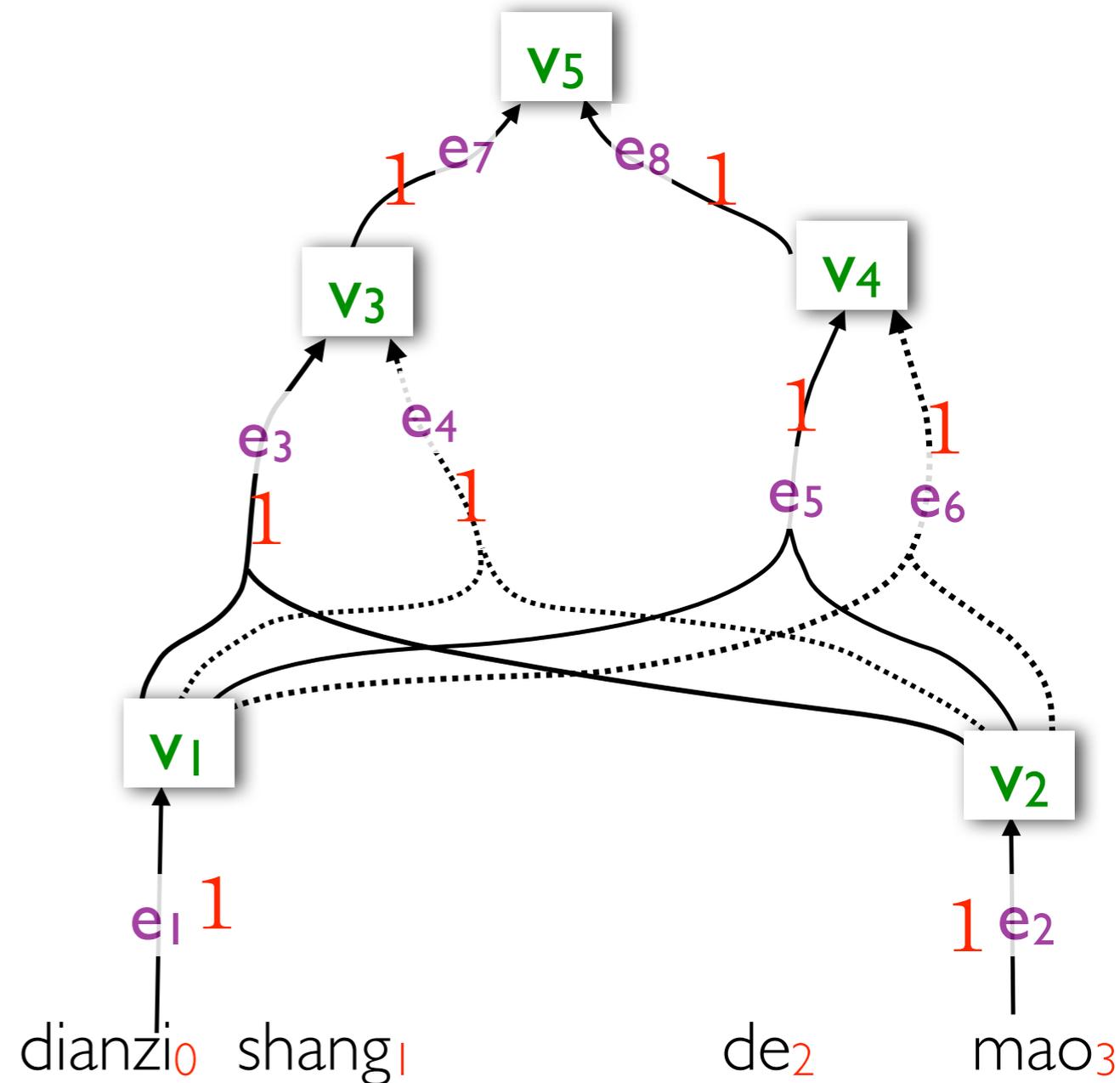
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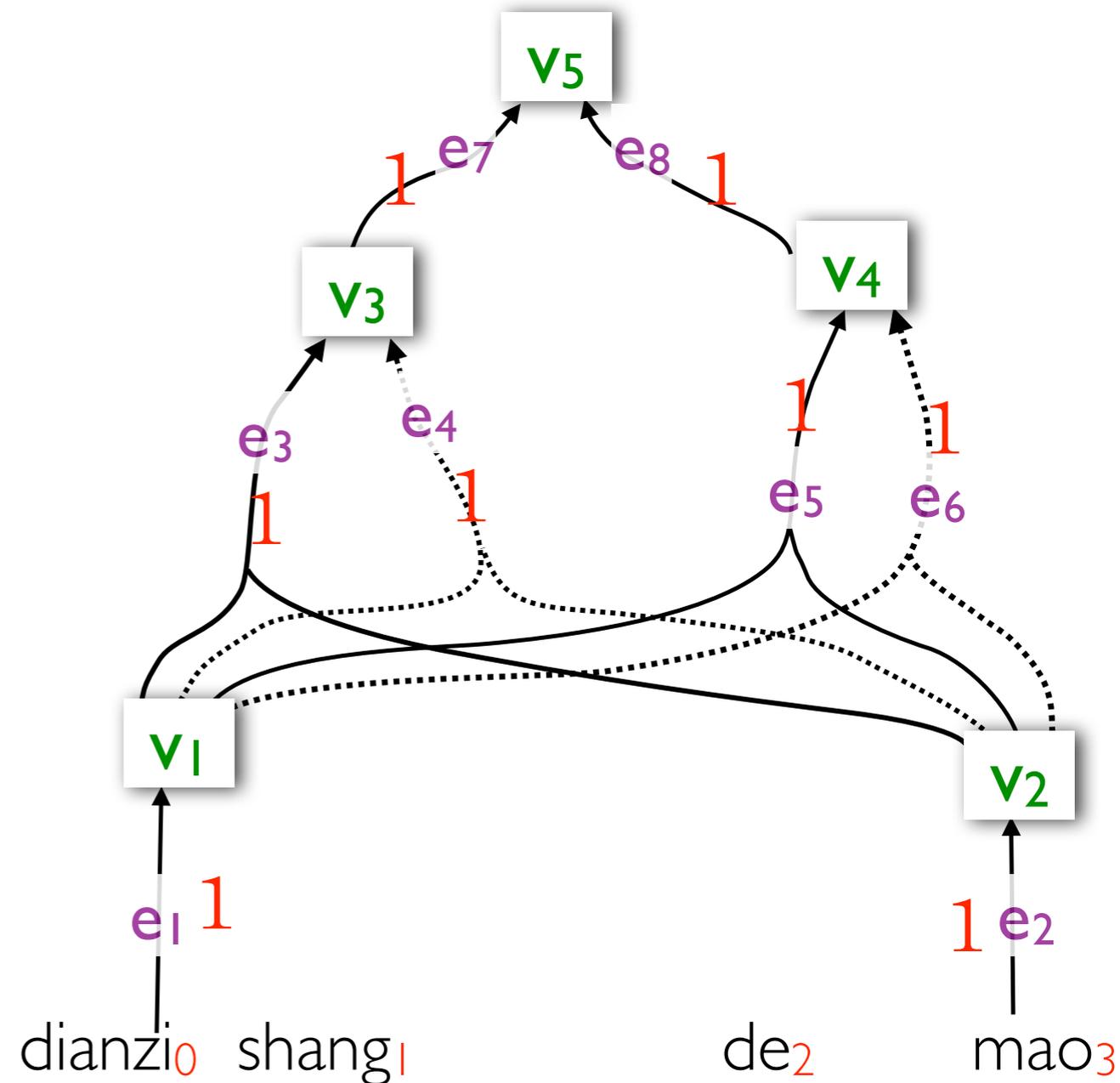
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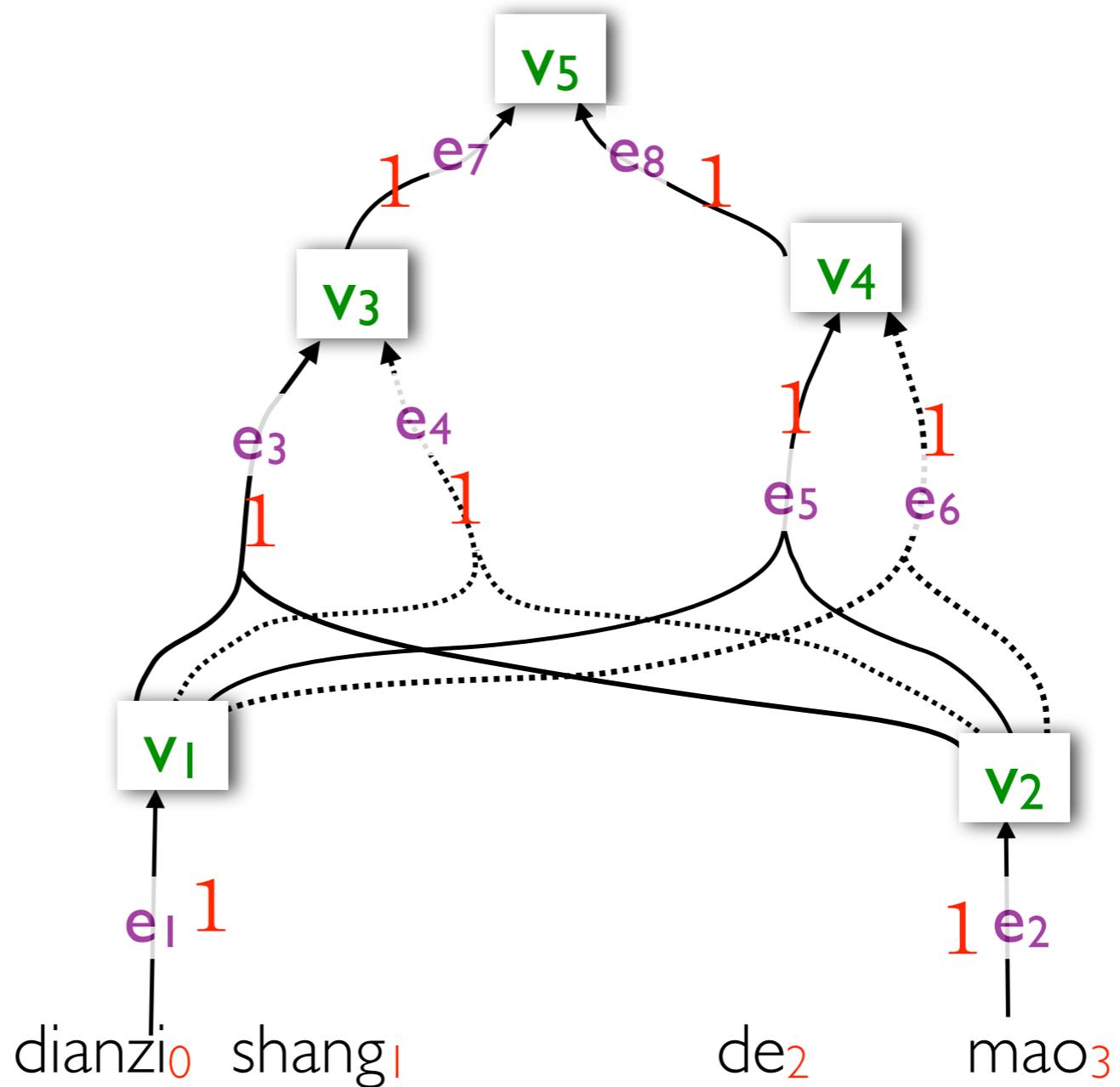
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- ▶ run the inside algorithm

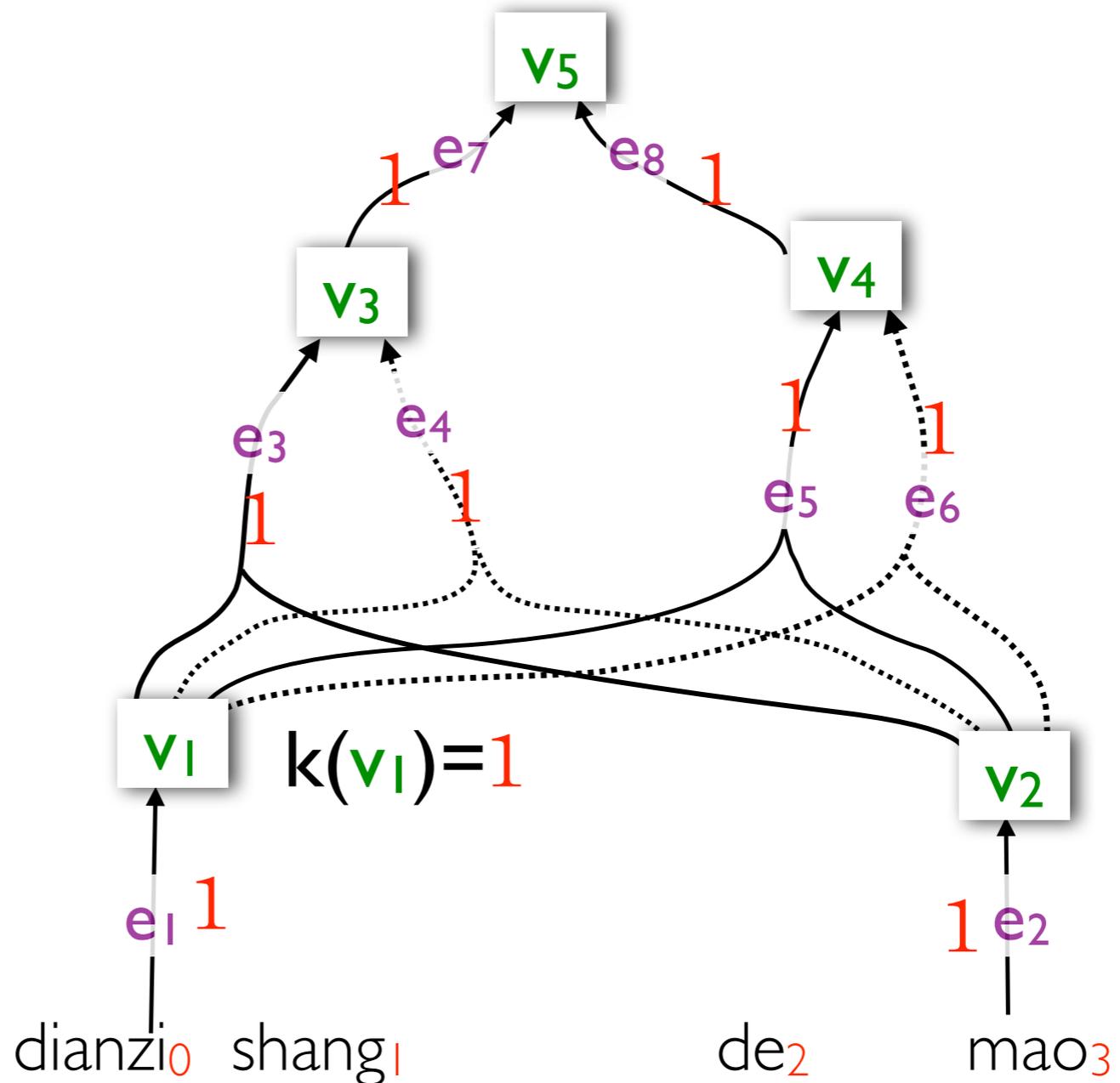


**Bottom-up**  
process in  
computing the  
number of trees



$$k(v_i) = k(e_i)$$

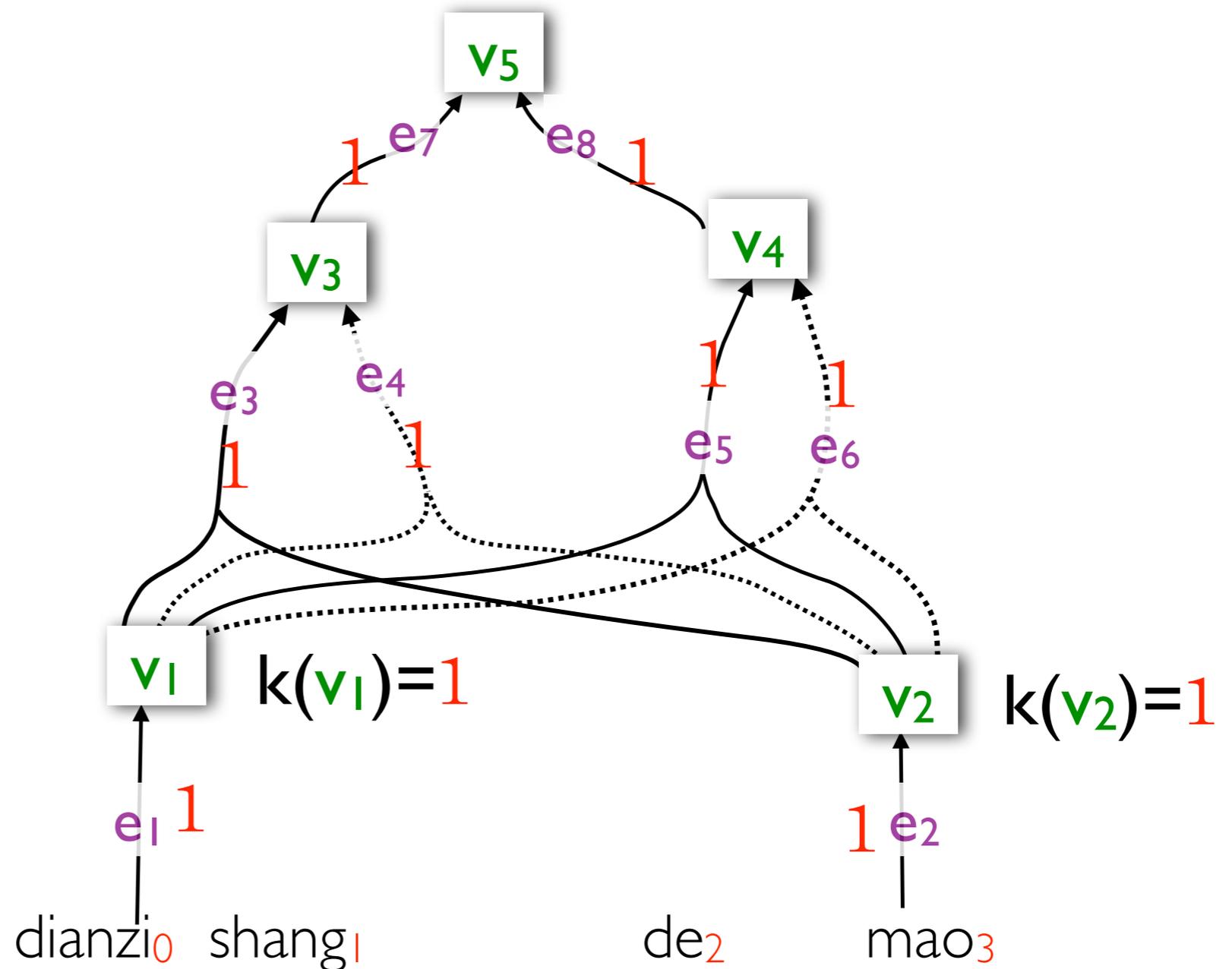
**Bottom-up**  
process in  
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number of trees



$$k(v_1) = k(e_1)$$

$$k(v_2) = k(e_2)$$

**Bottom-up**  
process in  
computing the  
number of trees

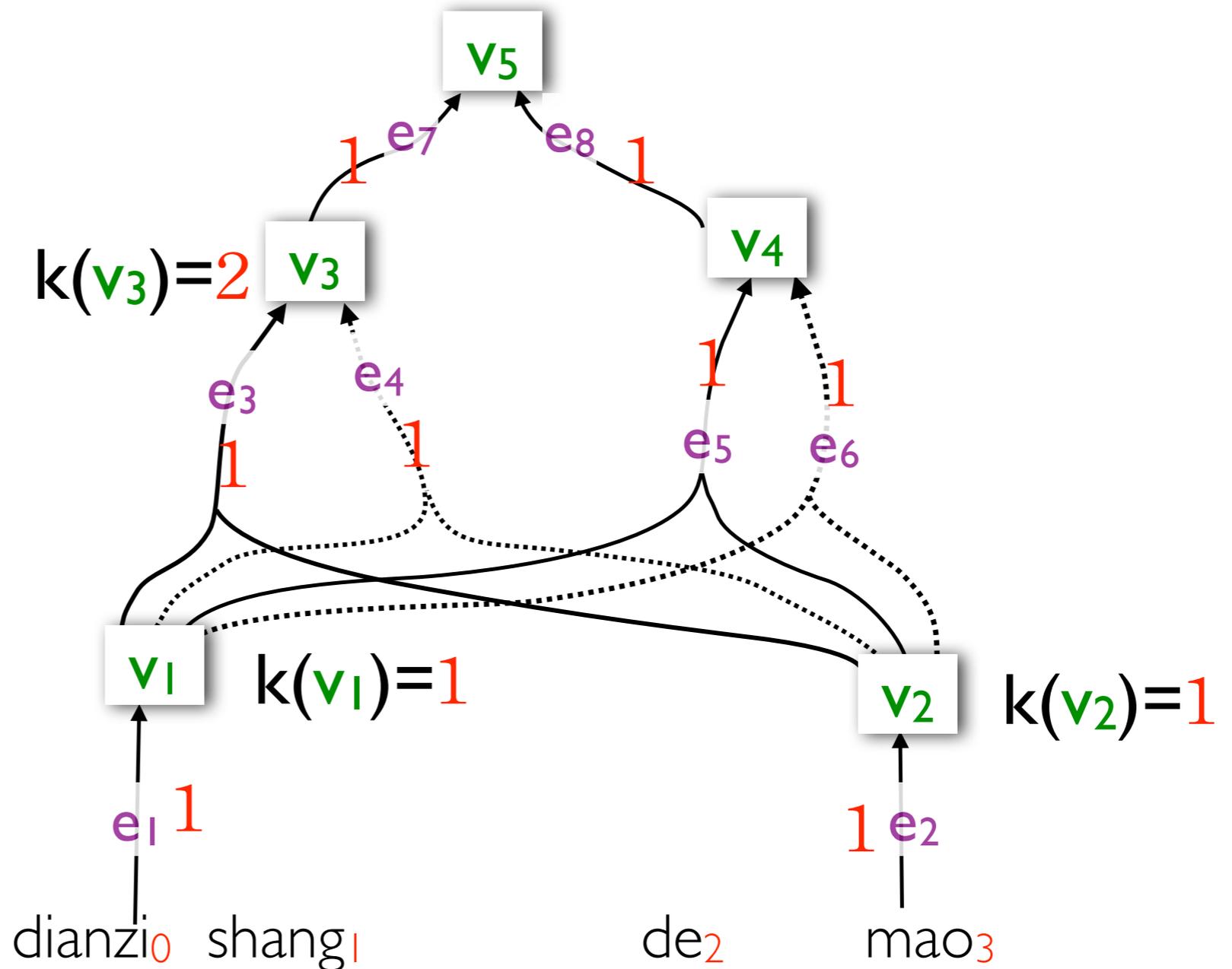


$$k(v_1) = k(e_1)$$

$$k(v_2) = k(e_2)$$

$$k(v_3) = k(e_3) \otimes k(v_1) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

**Bottom-up**  
process in  
computing the  
number of trees



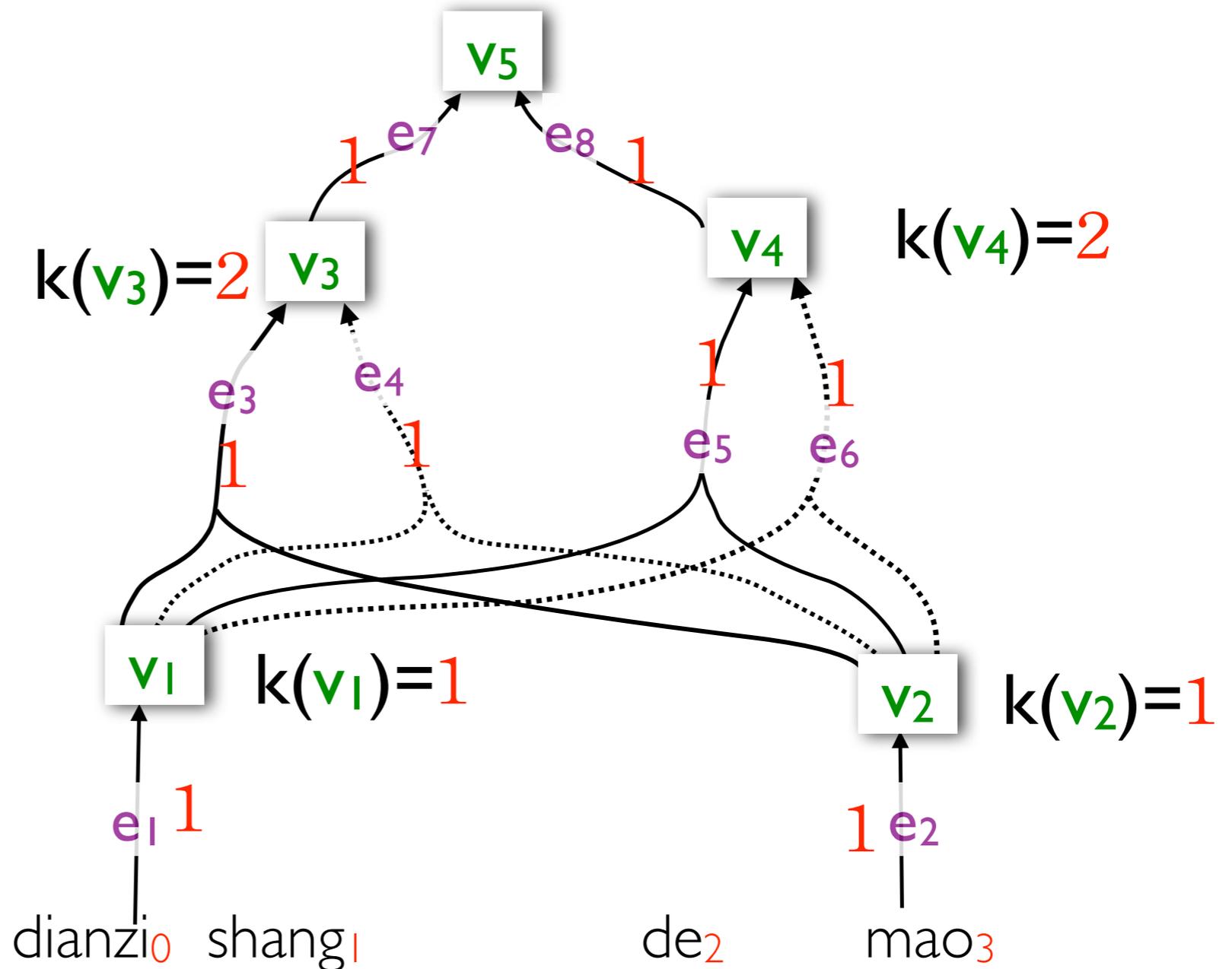
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**Bottom-up**  
process in  
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number of trees



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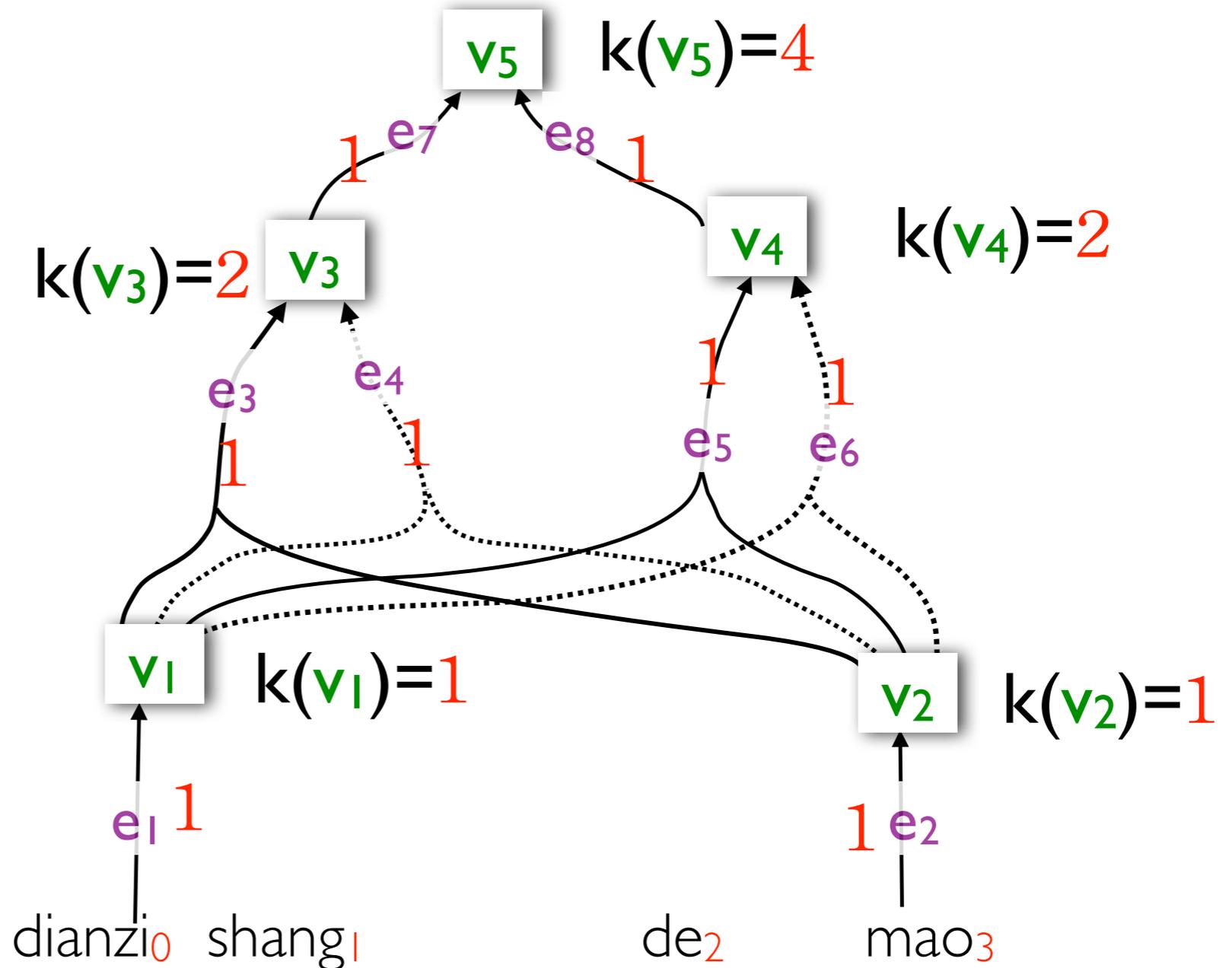
$$k(v_2) = k(e_2)$$

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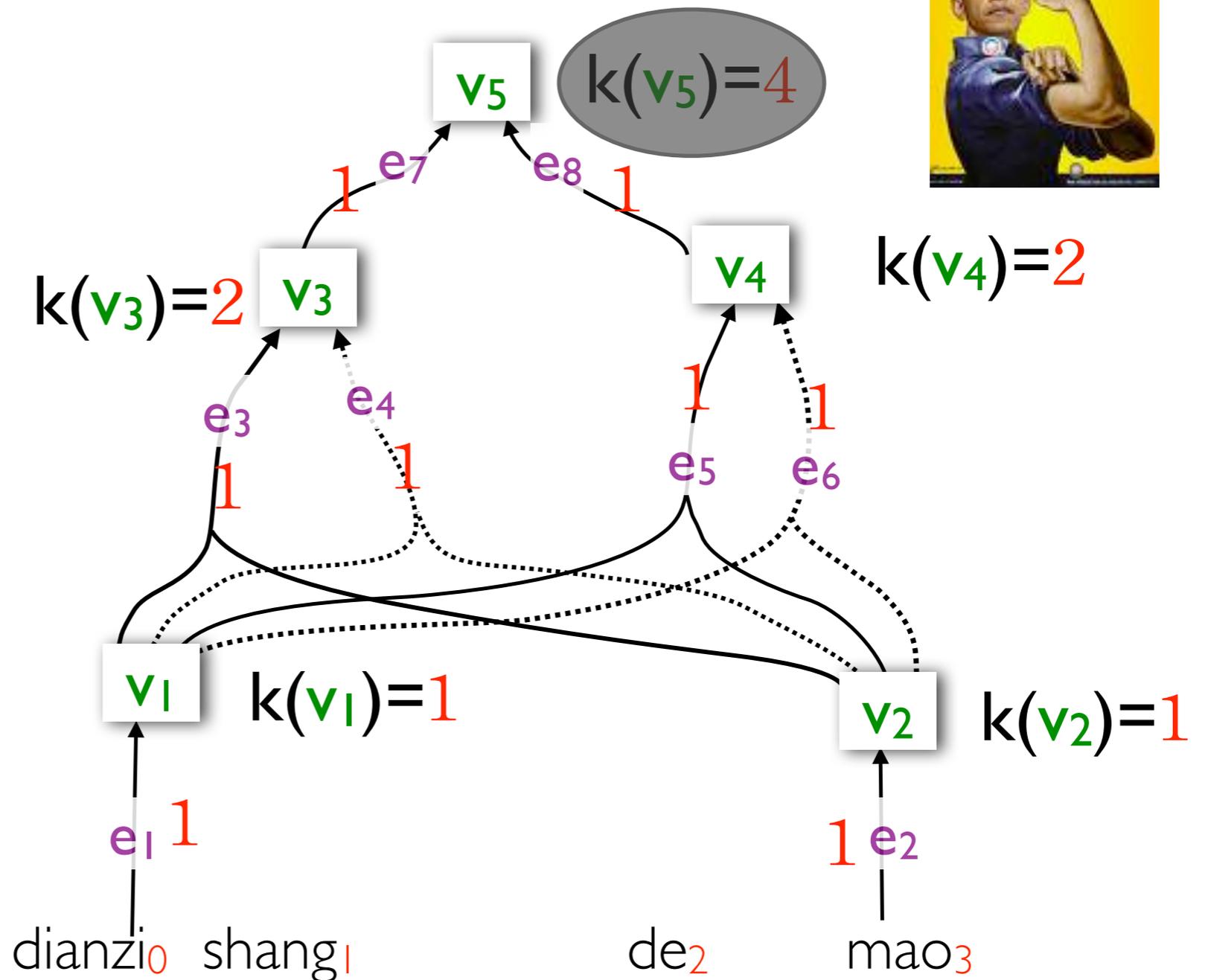
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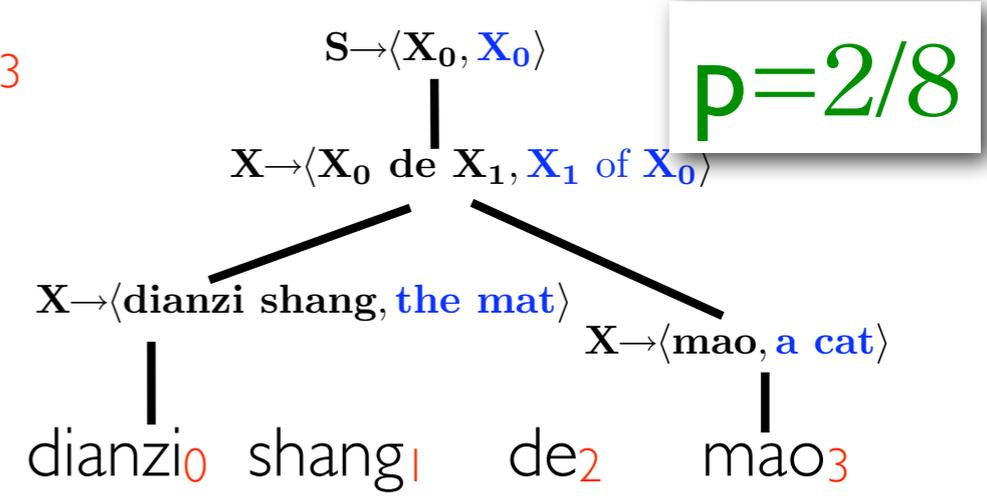
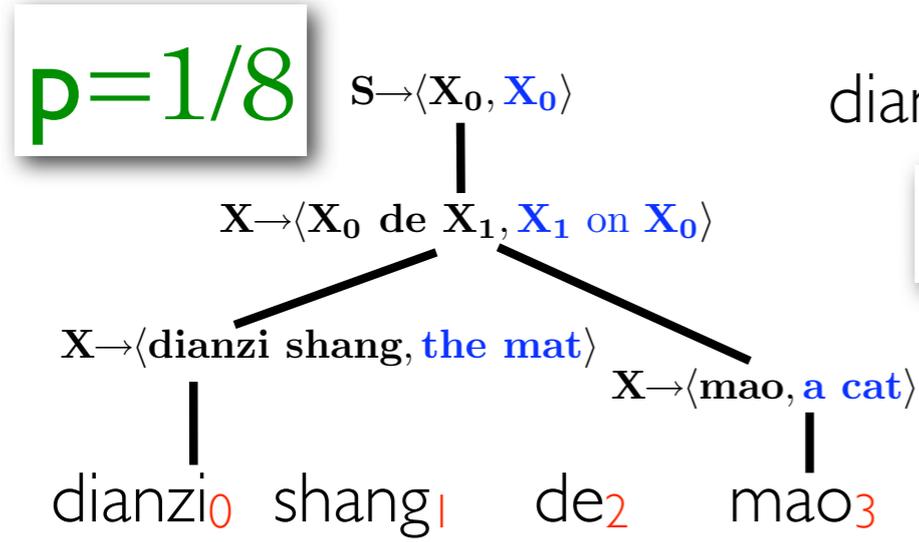
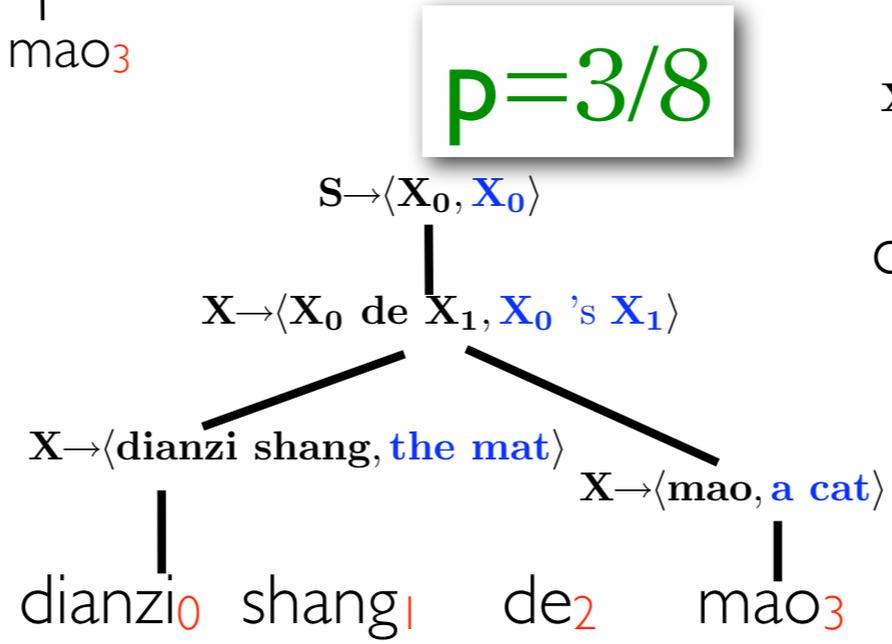
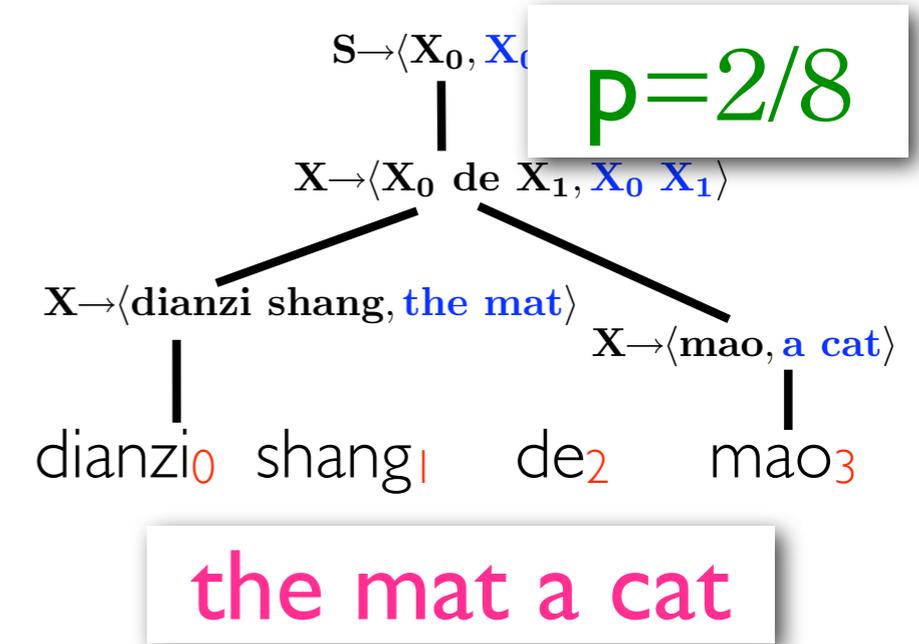
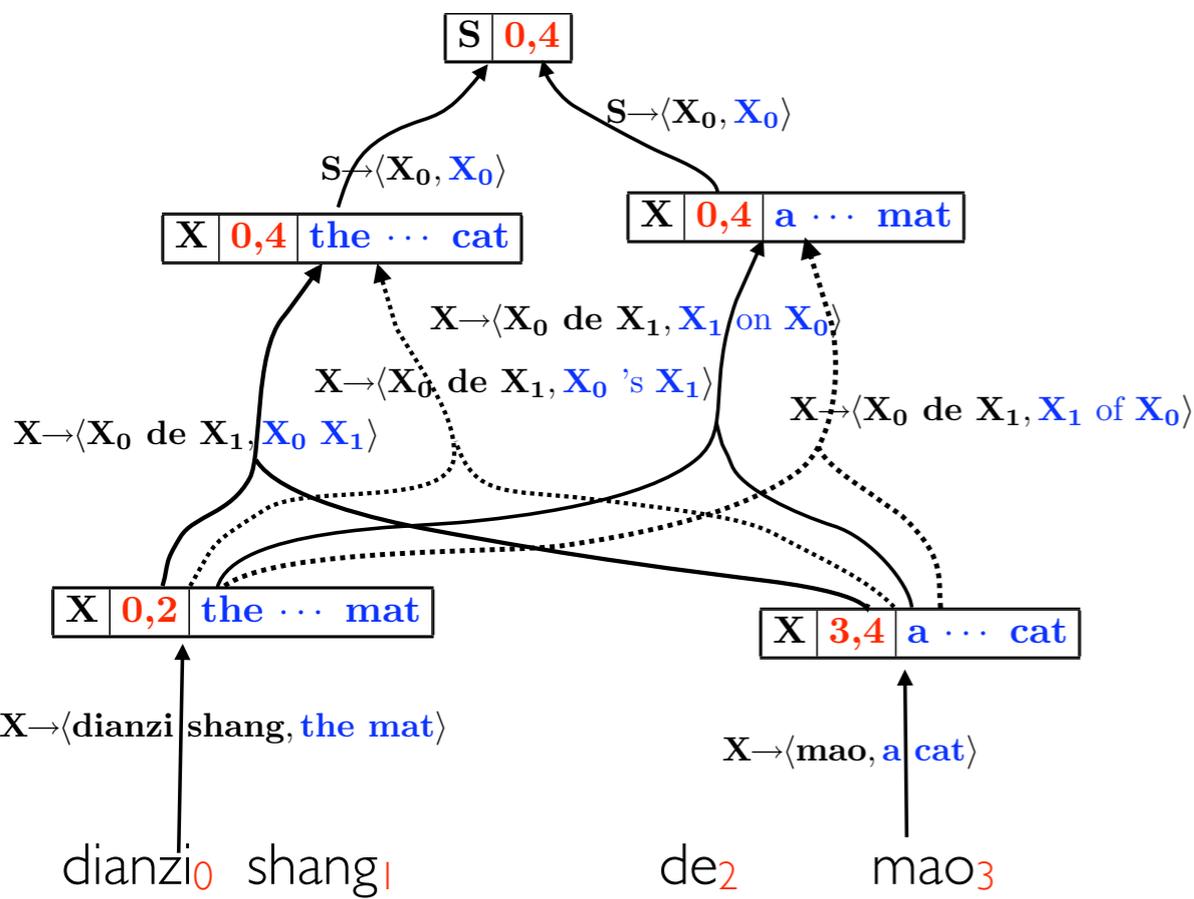
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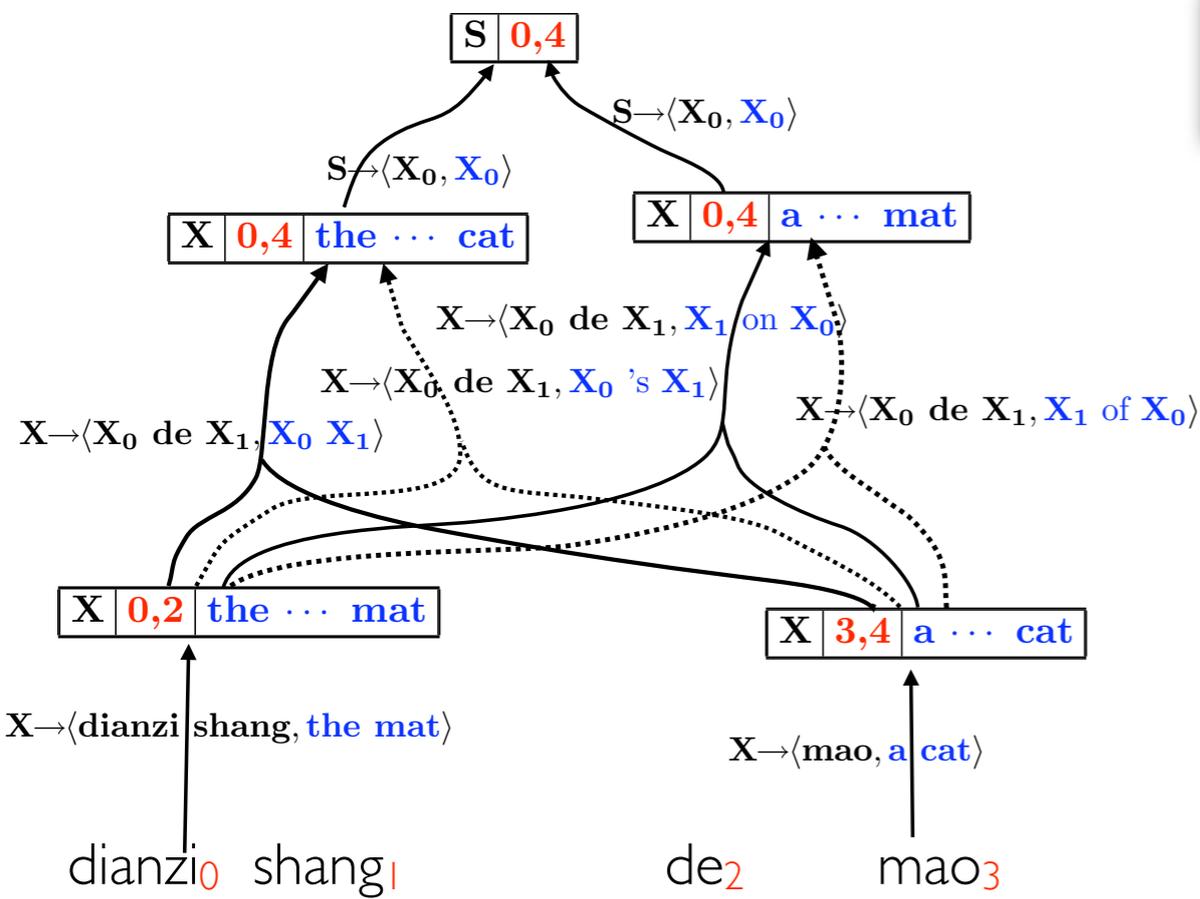


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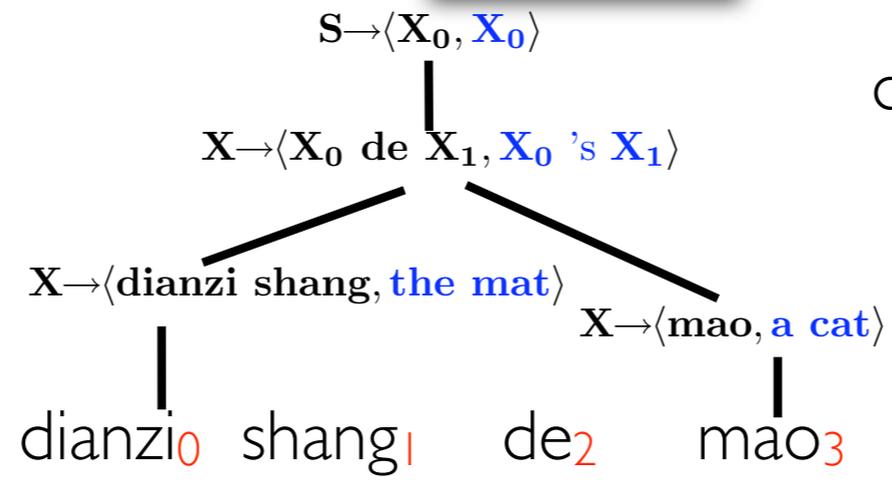




# expected translation length?

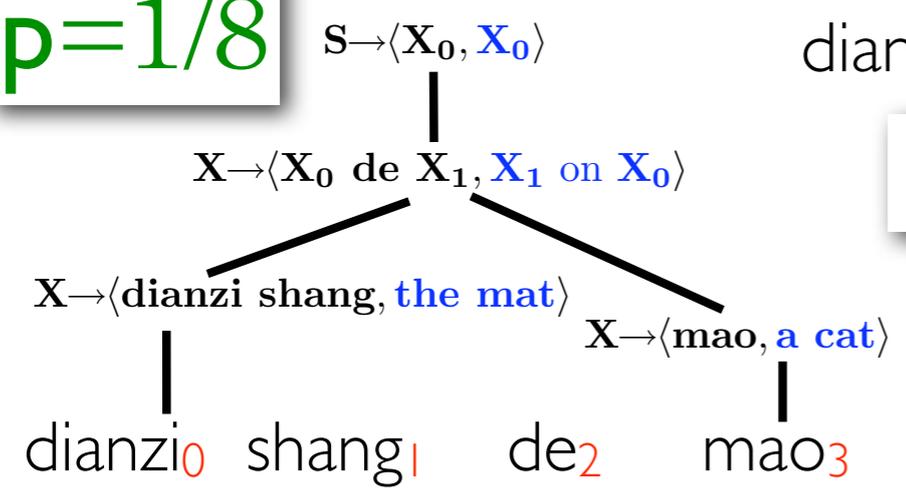


$p=3/8$

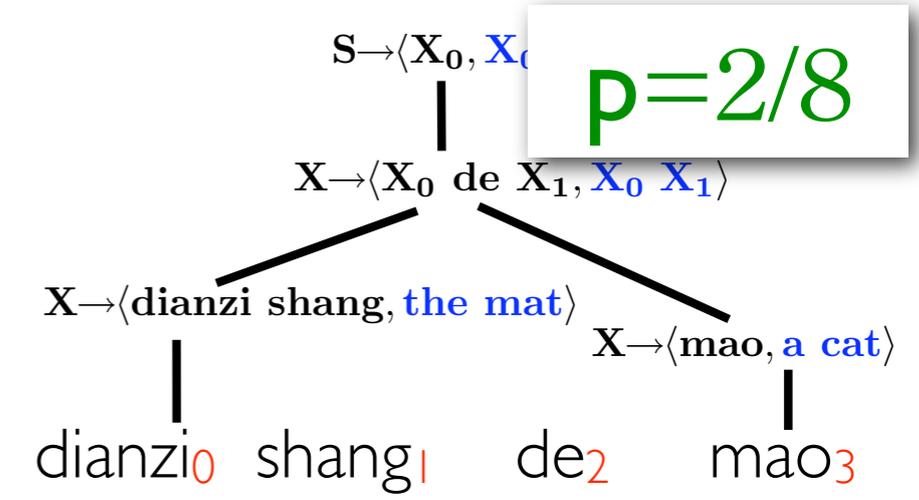


the mat's a cat

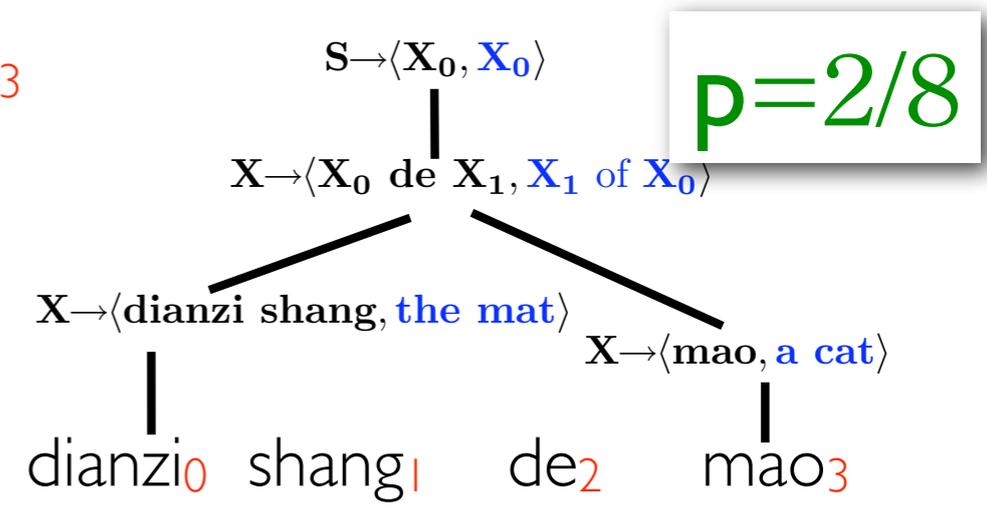
$p=1/8$



a cat on the mat

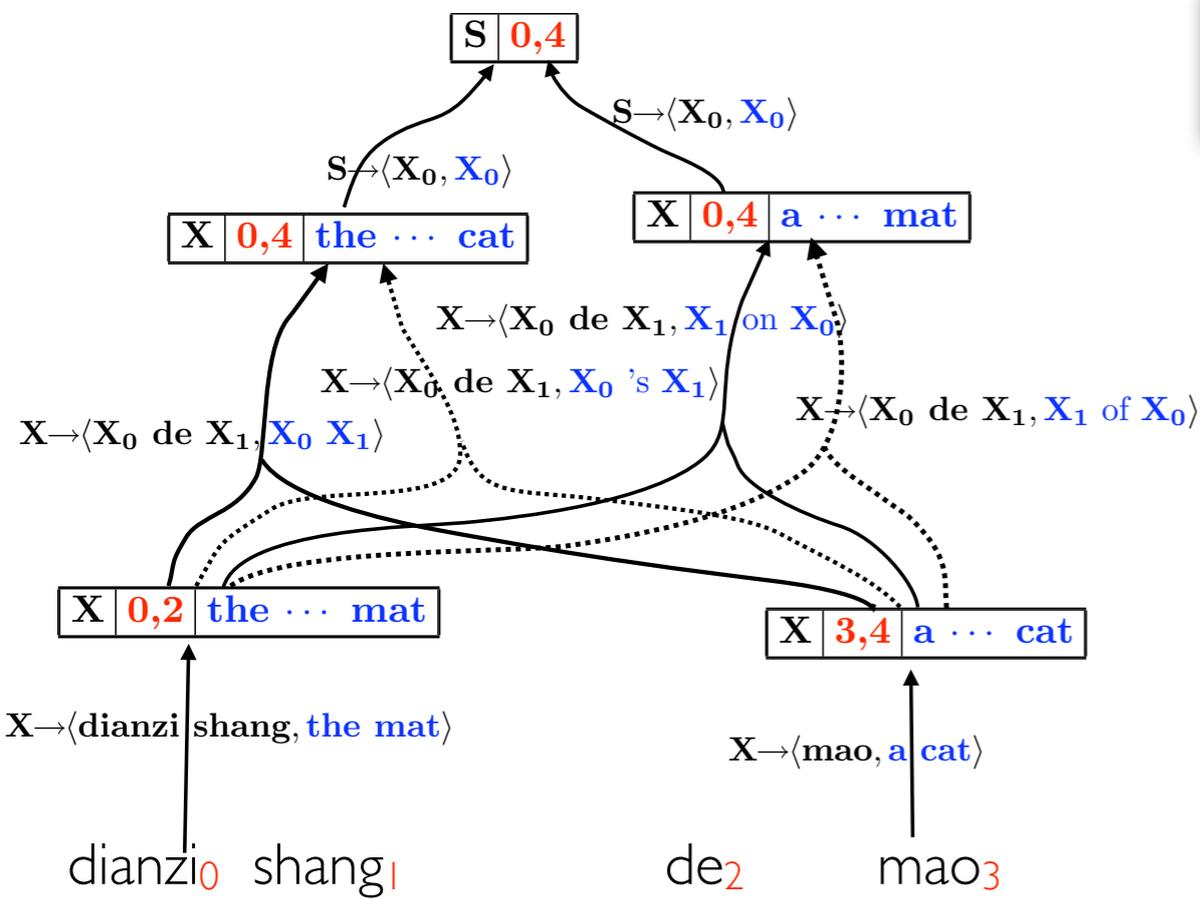


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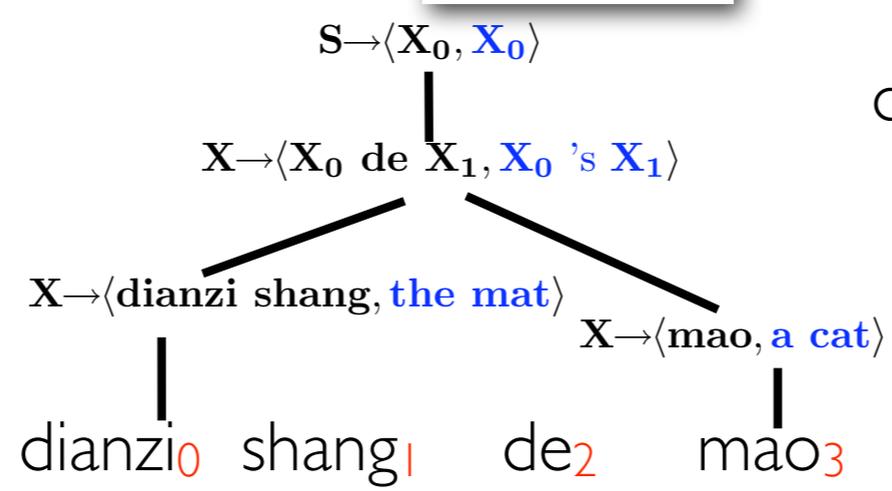


a cat of the mat

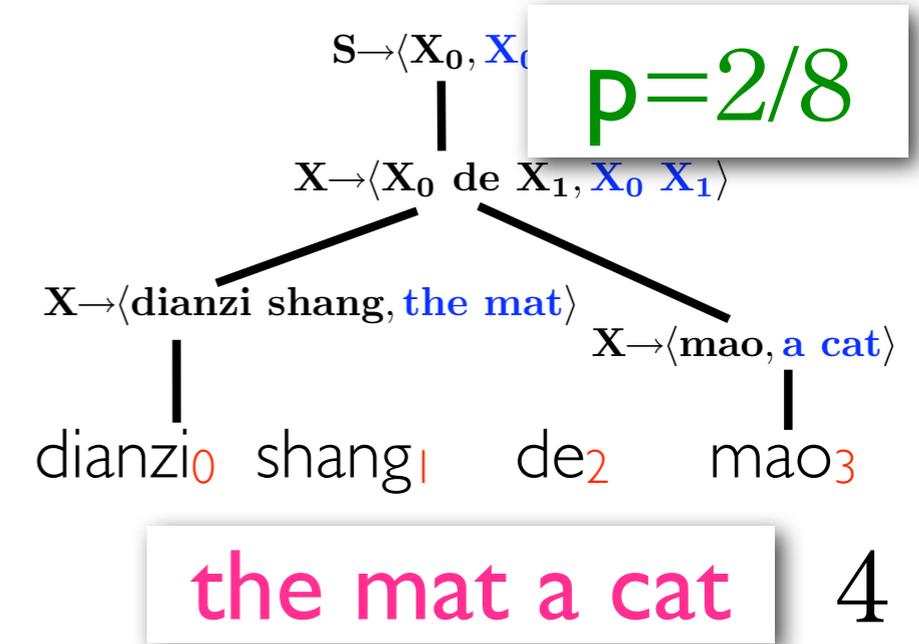
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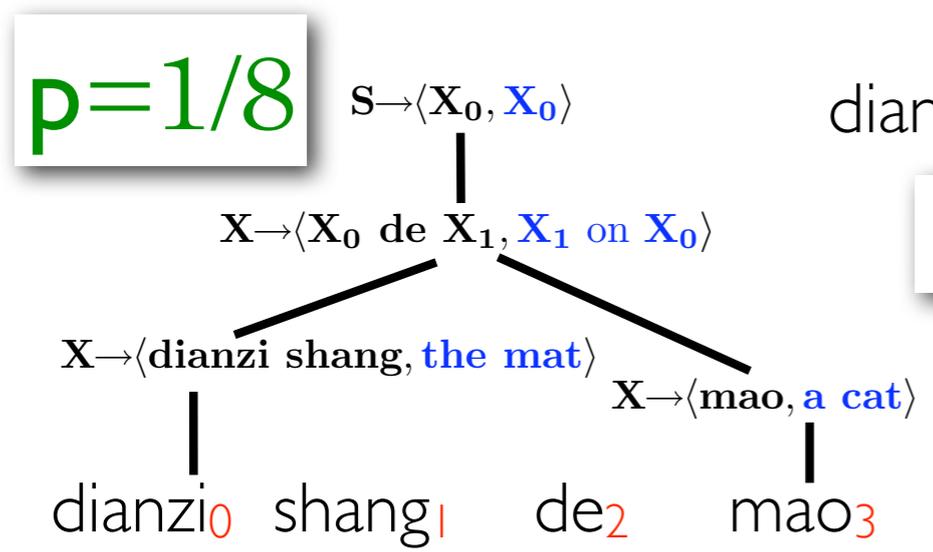
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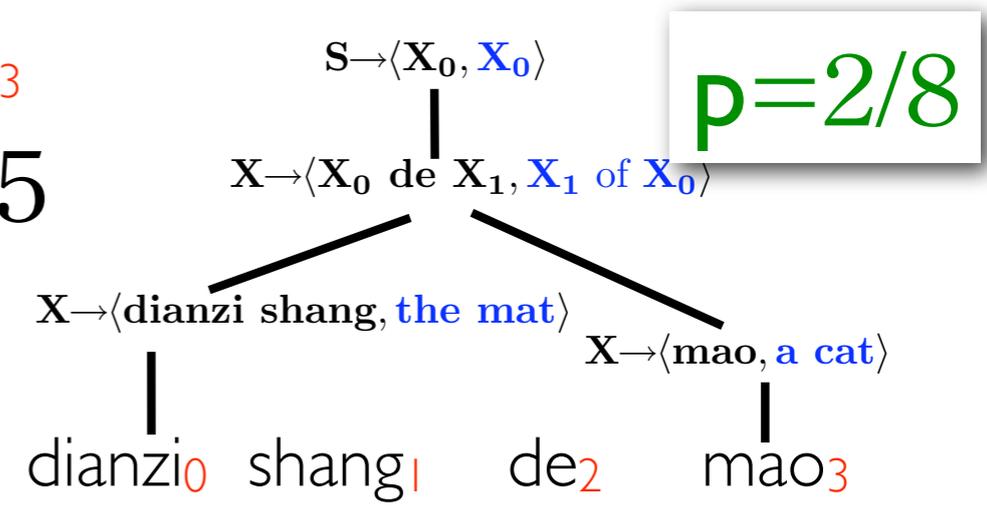
the mat 's a cat 5



the mat a cat 4



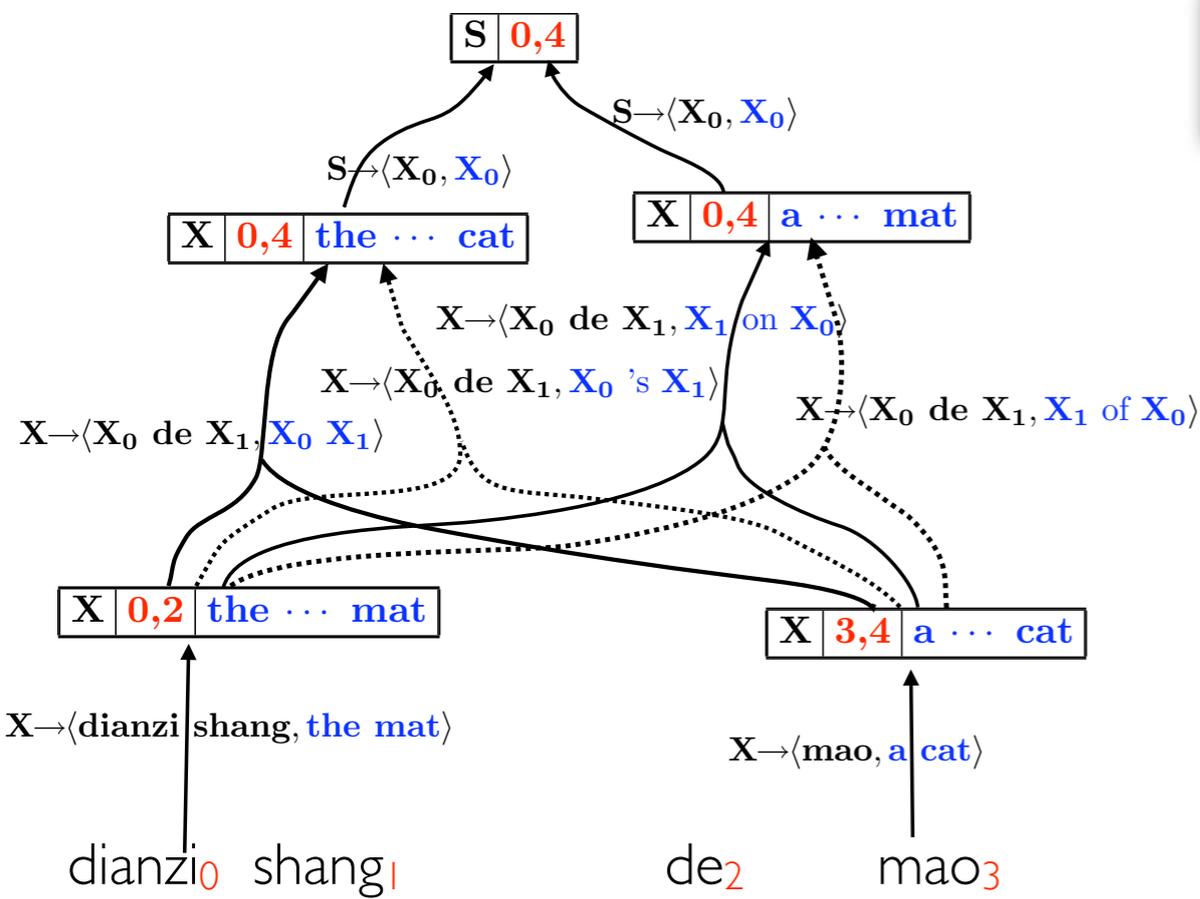
a cat on the mat 5



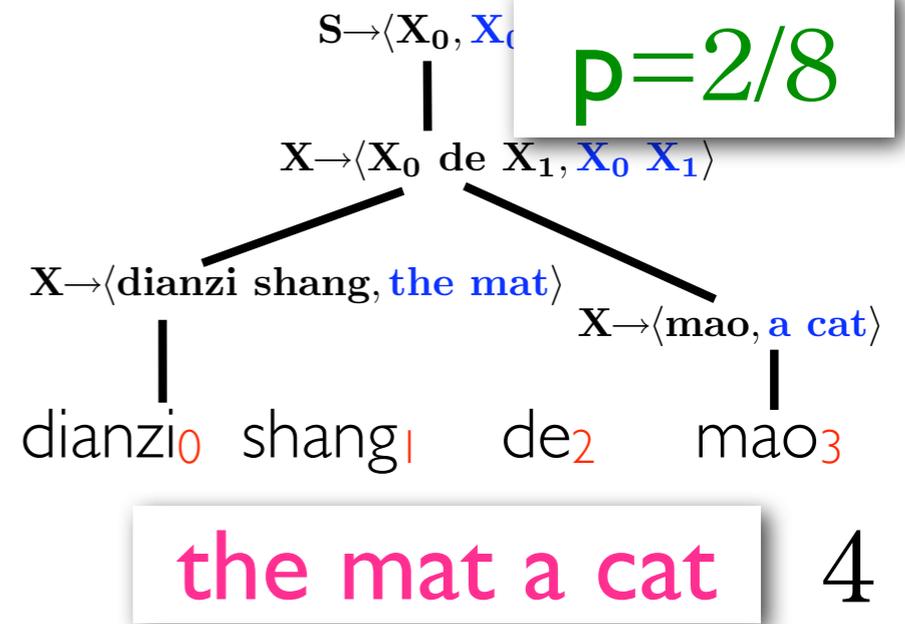
a cat of the mat 5

# expected translation length?

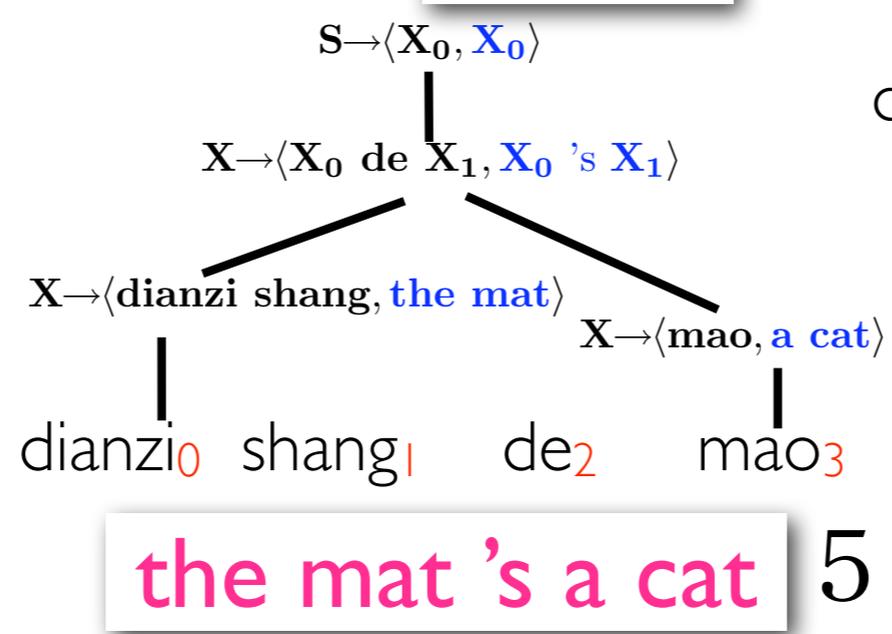
$$2/8 \times 4 + 6/8 \times 5 = 4.75$$



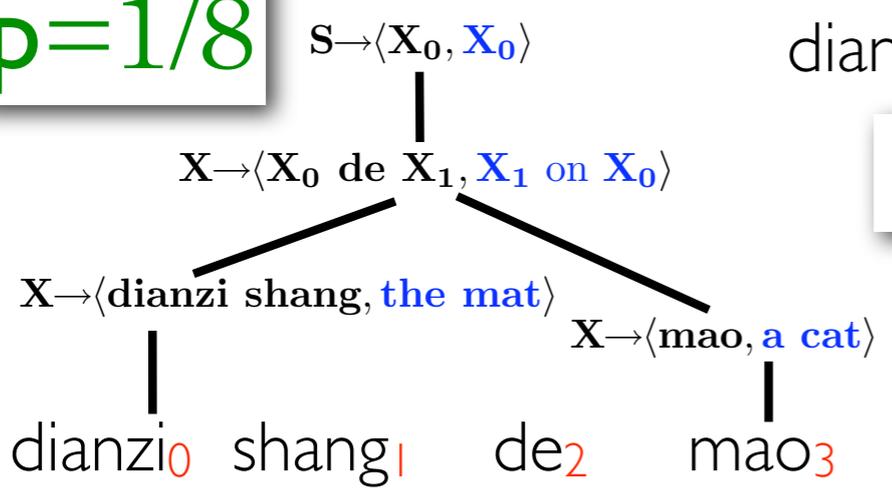
$p=2/8$



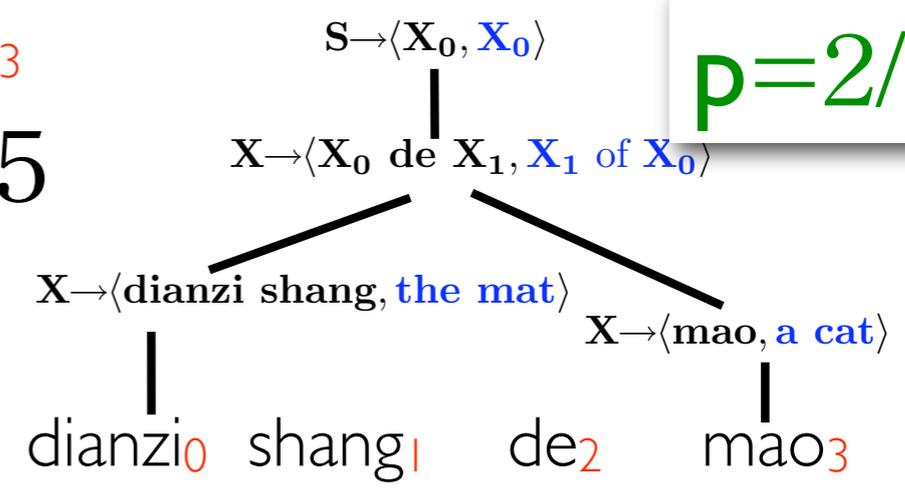
$p=3/8$



$p=1/8$



$p=2/8$



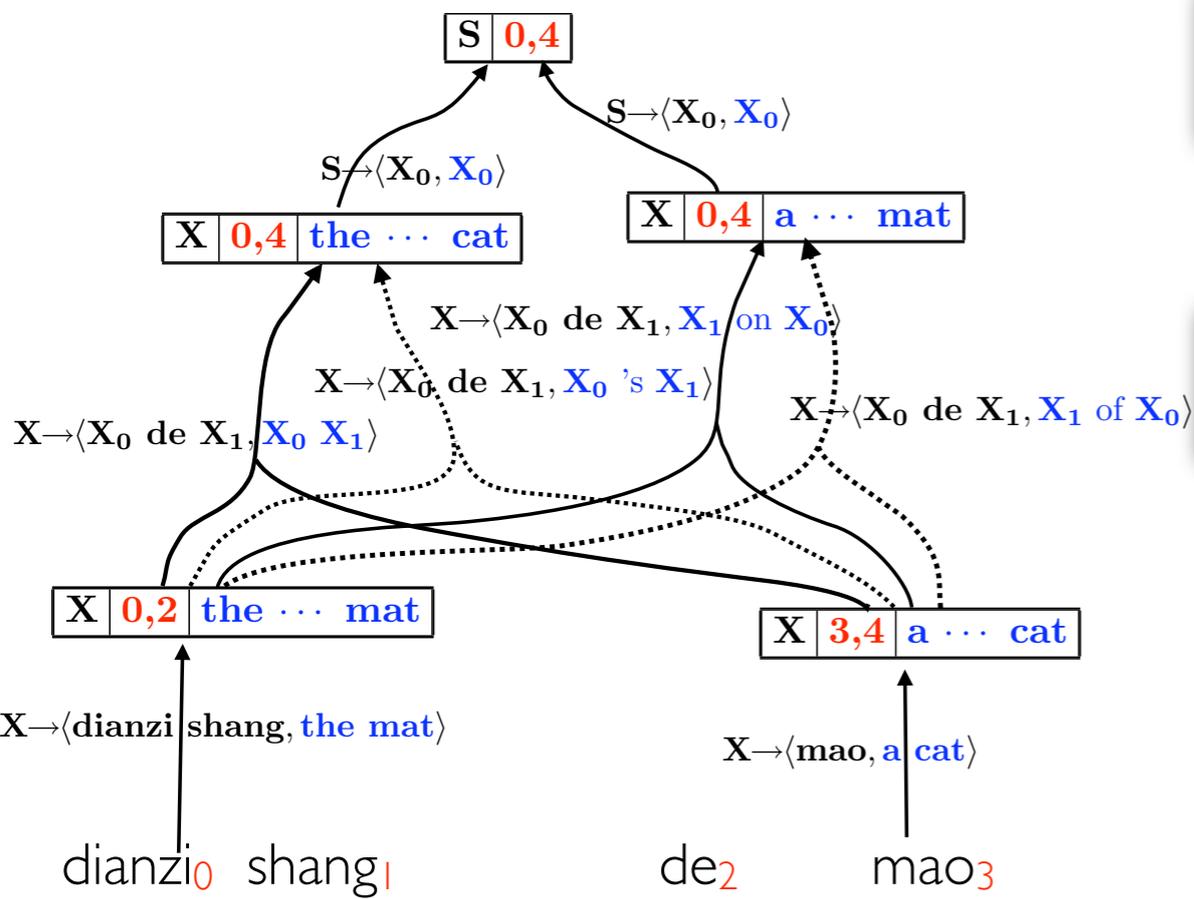
a cat on the mat 5

a cat of the mat 5

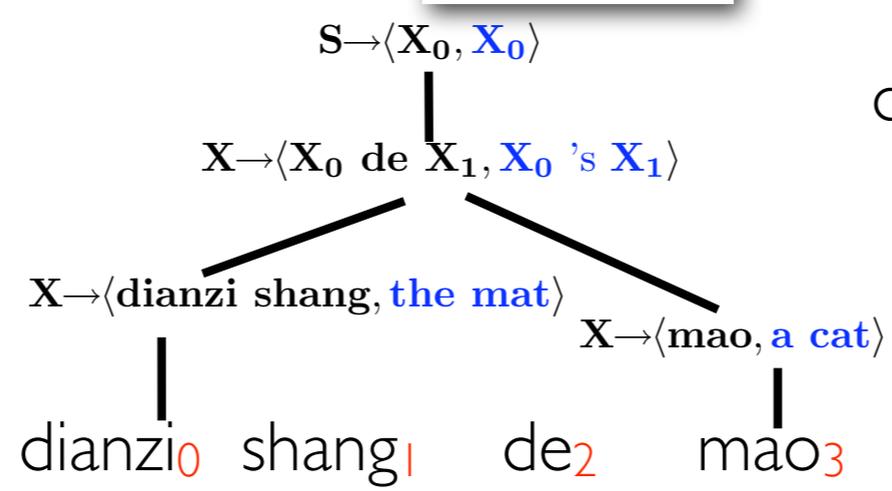
expected translation length?

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variance?

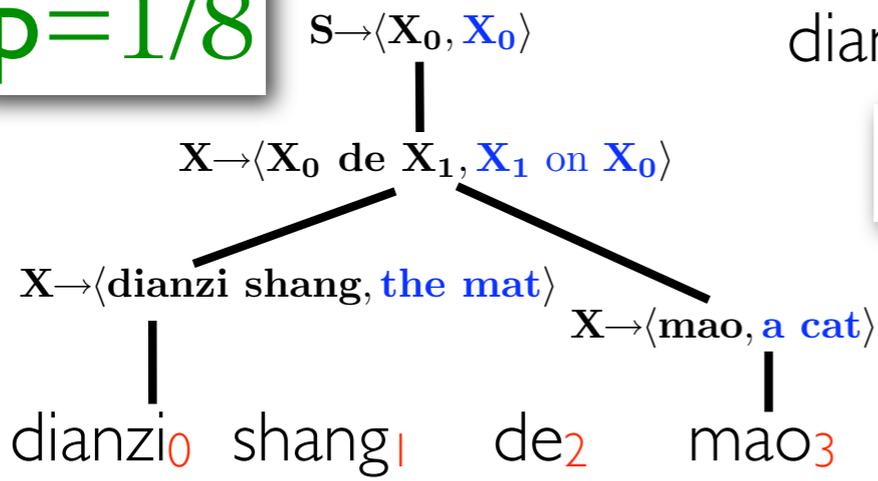


$p=3/8$



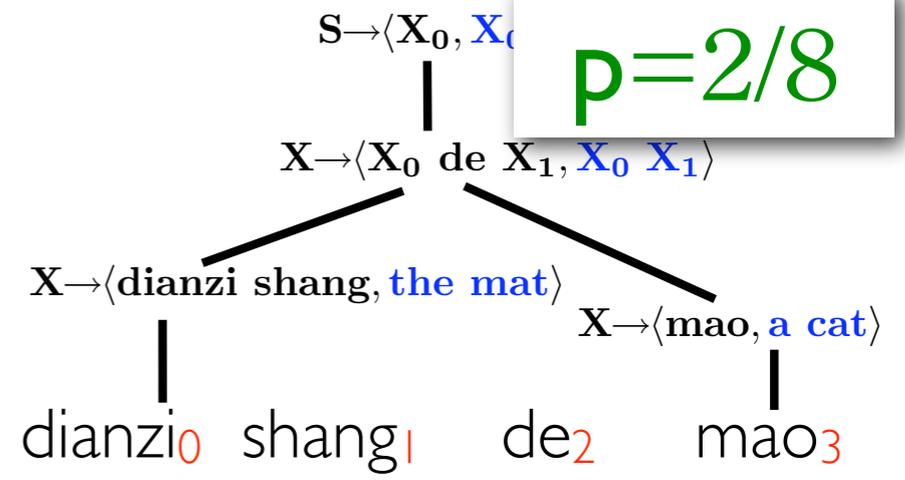
the mat 's a cat 5

$p=1/8$



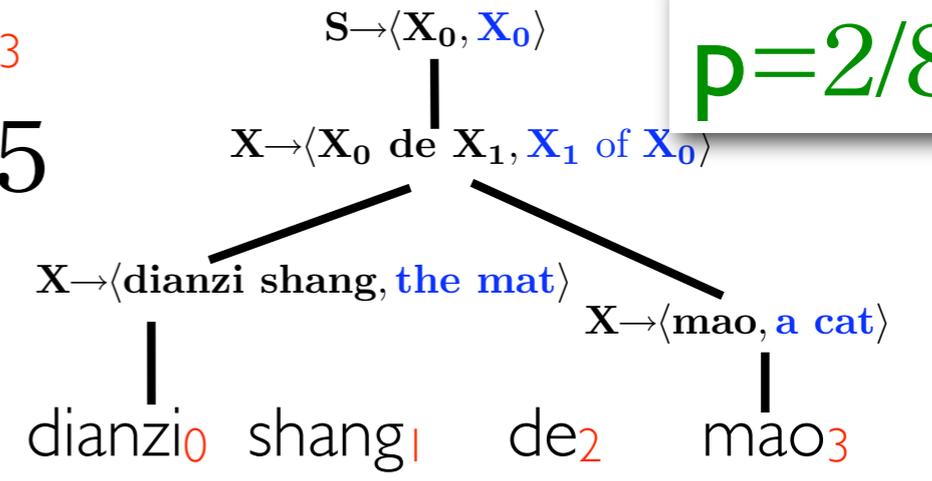
a cat on the mat 5

$p=2/8$



the mat a cat 4

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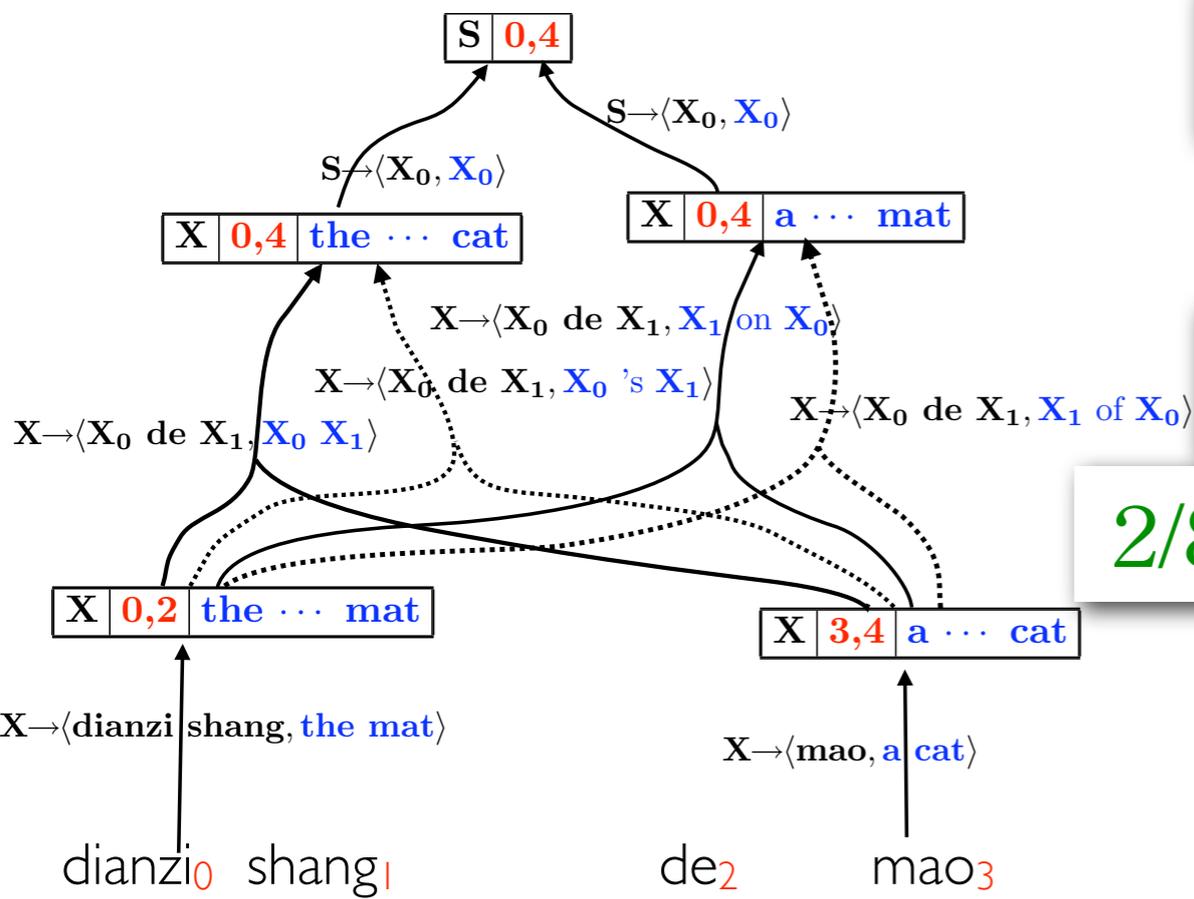
a cat of the mat 5

expected translation length?

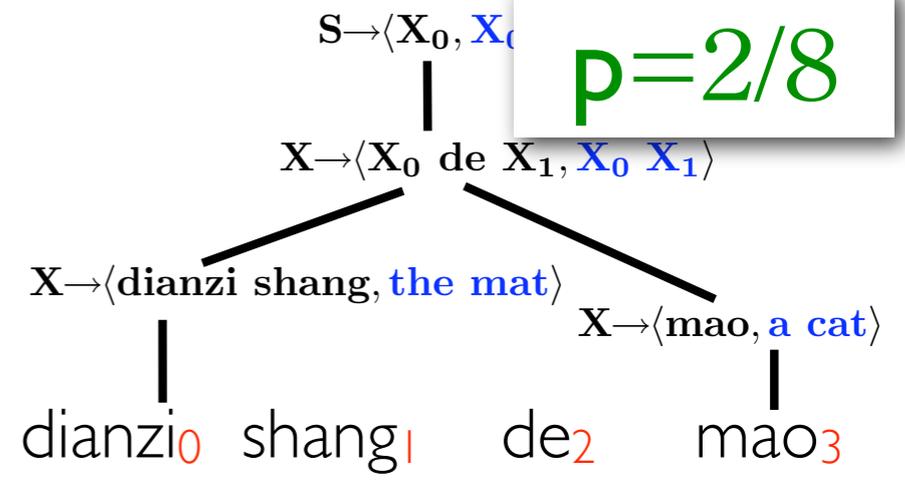
$$2/8 \times 4 + 6/8 \times 5 = 4.75$$

variance?

$$2/8 \times (4-4.75)^2 + 6/8 \times (5-4.75)^2 \approx 0.19$$

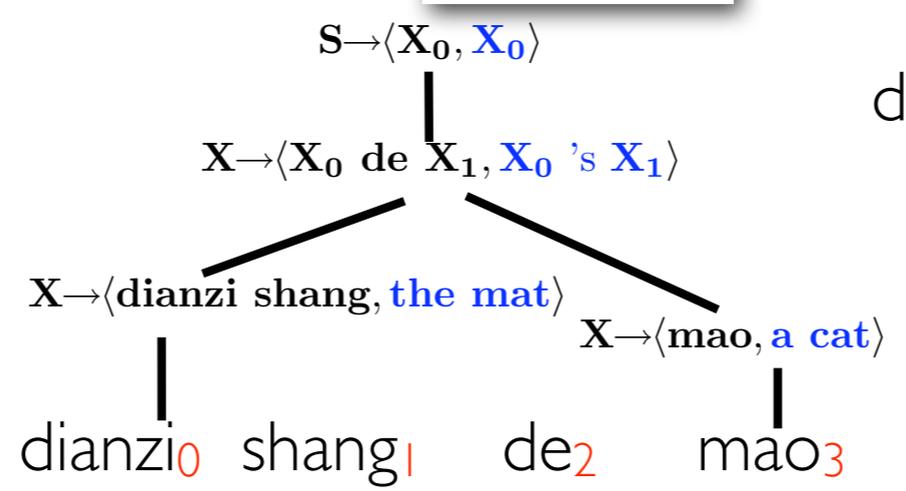


$p=2/8$



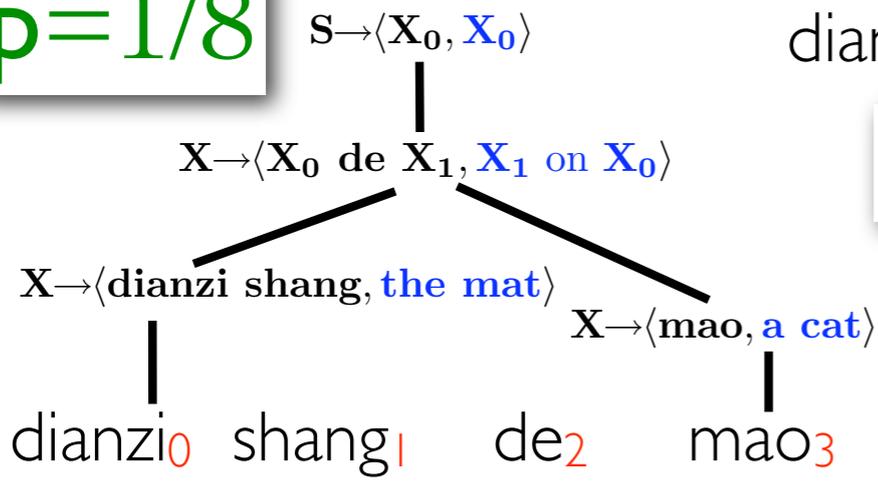
the mat a cat 4

$p=3/8$



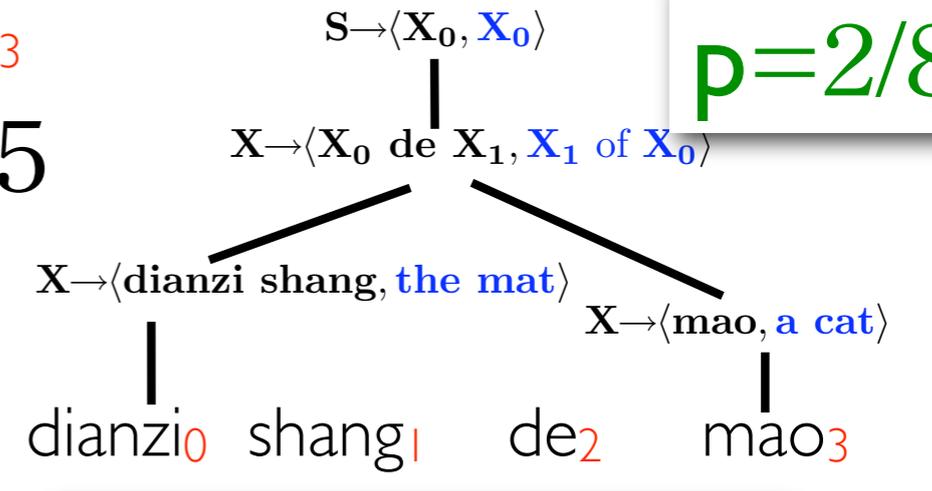
the mat 's a cat 5

$p=1/8$



a cat on the mat 5

$p=2/8$



a cat of the mat 5

# First- and Second-order Expectation Semirings

First-order:

(Eisner, 2002)

- each member is a 2-tuple:  $\langle p, r \rangle$

$\langle p_1, r_1 \rangle \otimes \langle p_2, r_2 \rangle$	$\langle p_1 p_2, p_1 r_2 + p_2 r_1 \rangle$
$\langle p_1, r_1 \rangle \oplus \langle p_2, r_2 \rangle$	$\langle p_1 + p_2, r_1 + r_2 \rangle$

Second-order:

- each member is a 4-tuple:  $\langle p, r, s, t \rangle$

$\langle p_1, r_1, s_1, t_1 \rangle \otimes \langle p_2, r_2, s_2, t_2 \rangle$	$\langle p_1 p_2, p_1 r_2 + p_2 r_1, p_1 s_2 + p_2 s_1, p_1 t_2 + p_2 t_1 + r_1 s_2 + r_2 s_1 \rangle$
$\langle p_1, r_1, s_1, t_1 \rangle \oplus \langle p_2, r_2, s_2, t_2 \rangle$	$\langle p_1 + p_2, r_1 + r_2, s_1 + s_2, t_1 + t_2 \rangle$

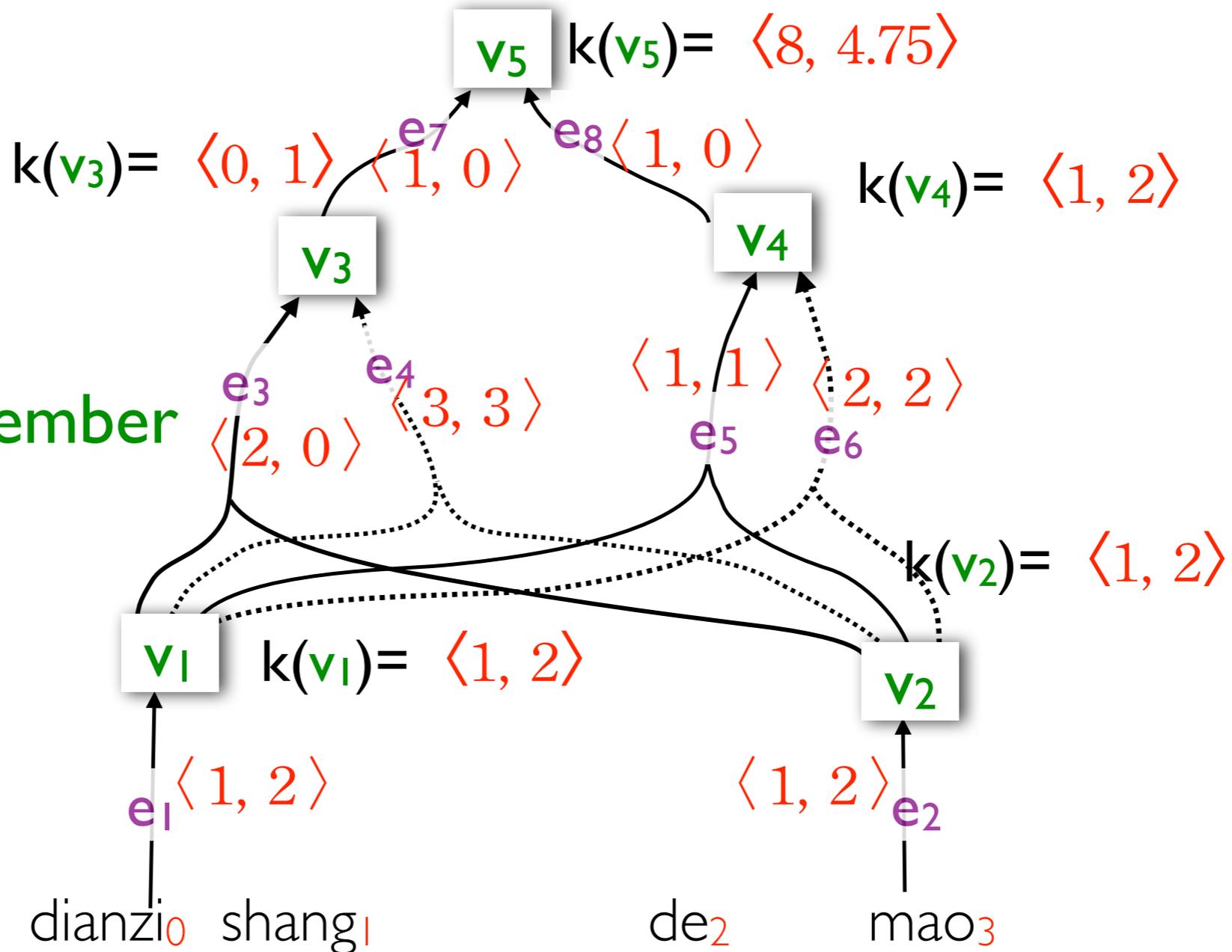
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**First-order:**  
each semiring member  
is a **2-tuple**



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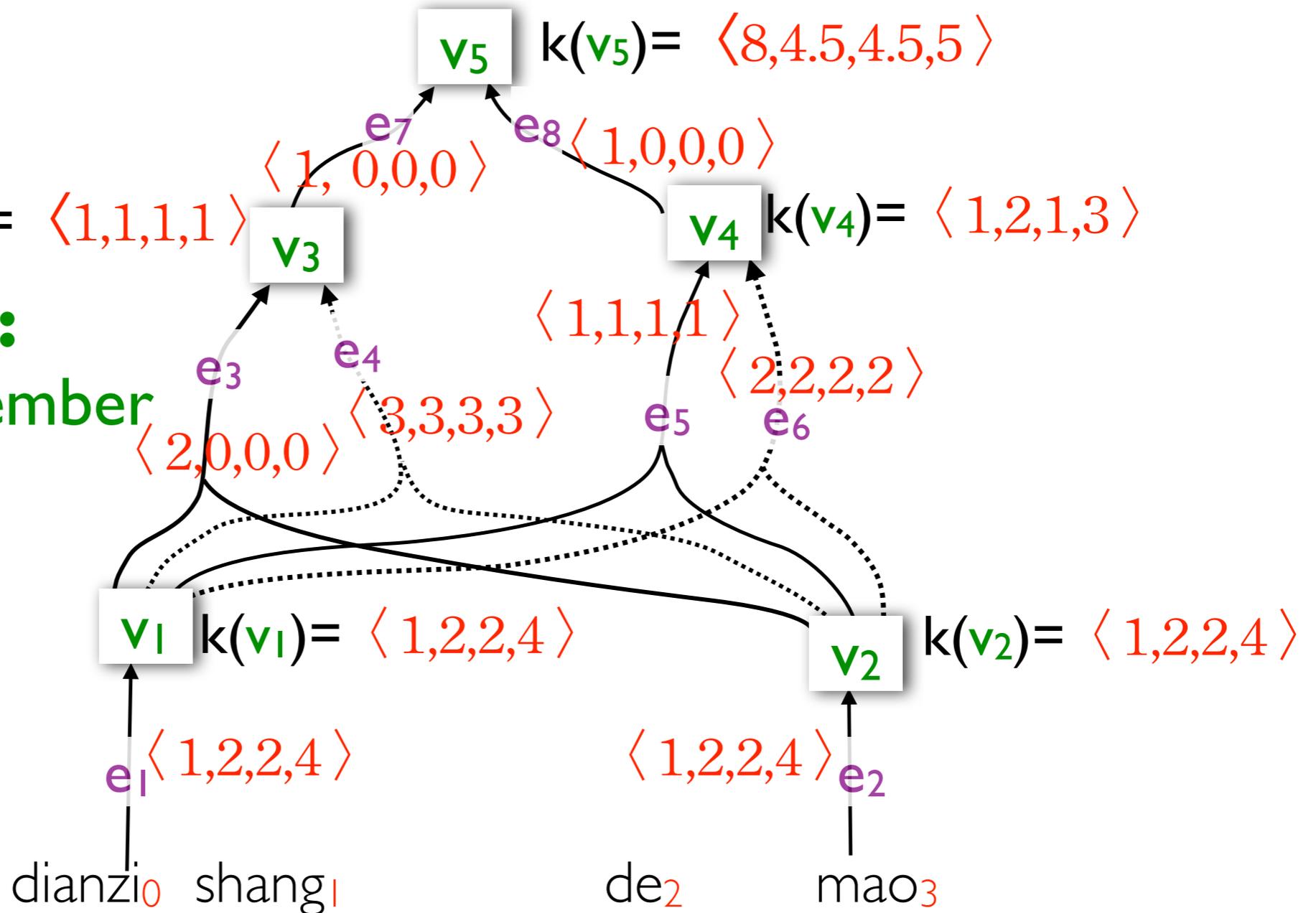
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**Second-order:**  
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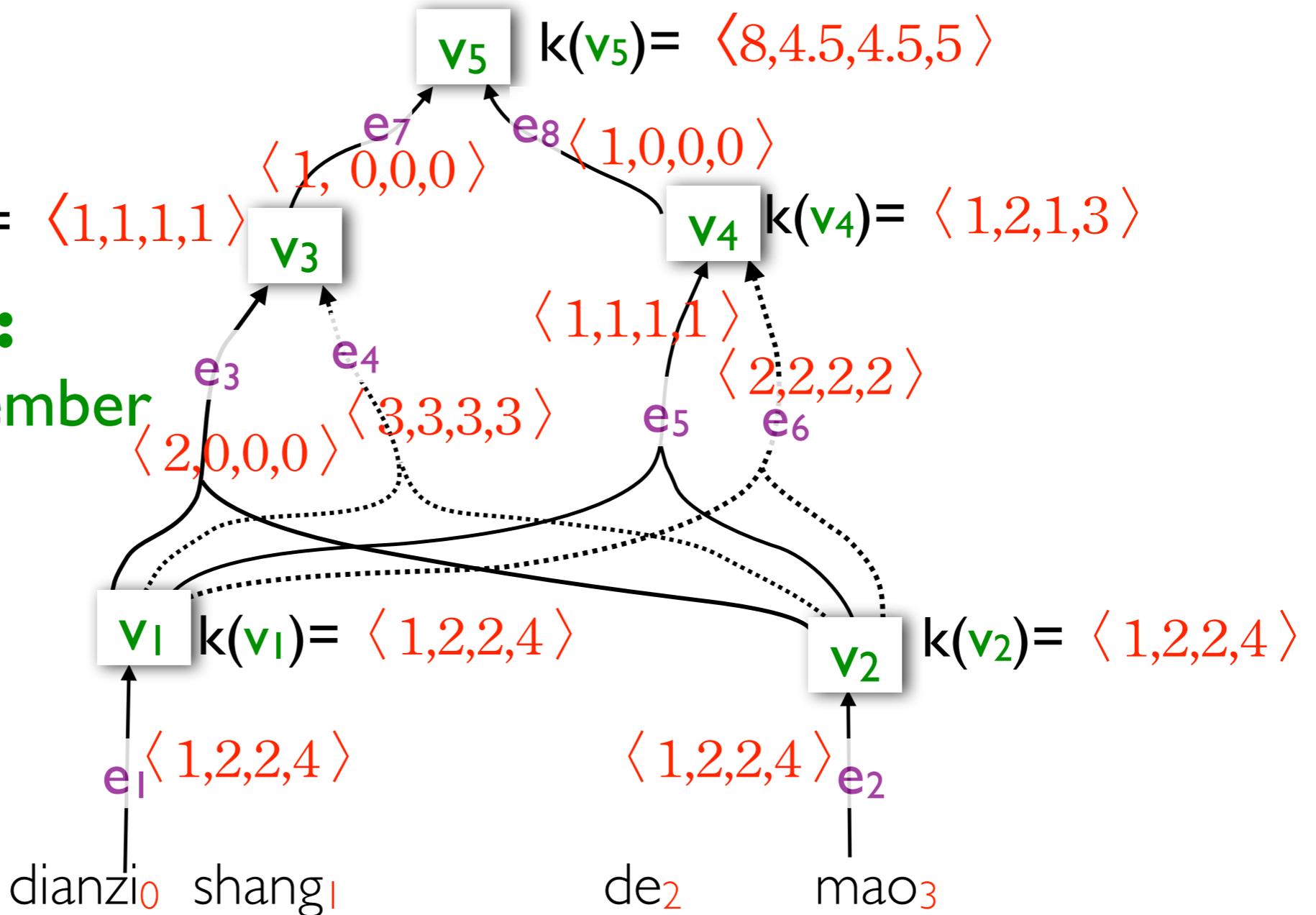
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e.g., the length of the translation yielded by  $d$

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← exponential size

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← exponential size

- $r(d)$  is a function over a derivation  $d$   
e.g., the length of the translation yielded by  $d$
- $r(d)$  is **additively** decomposed

$$r(d) \stackrel{\text{def}}{=} \sum_{e \in d} r_e$$

e.g., translation length is **additively** decomposed!

# Second-order Expectations on Hypergraphs

- Expectation of **products** over a hypergraph

$$\bar{t} \stackrel{\text{def}}{=} \mathbb{E}_p[r \cdot s] = \sum_{d \in \text{HG}} p(d)r(d)s(d)$$

← exponential size

- **r** and **s** are additively decomposed

$$r(d) \stackrel{\text{def}}{=} \sum_{e \in d} r_e$$

$$s(d) \stackrel{\text{def}}{=} \sum_{e \in d} s_e$$

**r** and **s** can be identical or different functions.

Compute expectation using expectation semiring:

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$p_e$ : transition probability or log-linear score at edge  $e$   
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$$H(p) = \mathbb{E}_p[-\log p] = - \sum_{d \in \text{HG}} p(d) \log p(d)$$

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$$\text{Risk} = \mathbb{E}_p(L) = - \sum_{d \in \text{HG}} p(d) \cdot L(Y(d))$$

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Bayes risk is an **expectation**

$$\text{Risk} = \mathbb{E}_p(L) = - \sum_{d \in \text{HG}} p(d) \cdot L(Y(d))$$

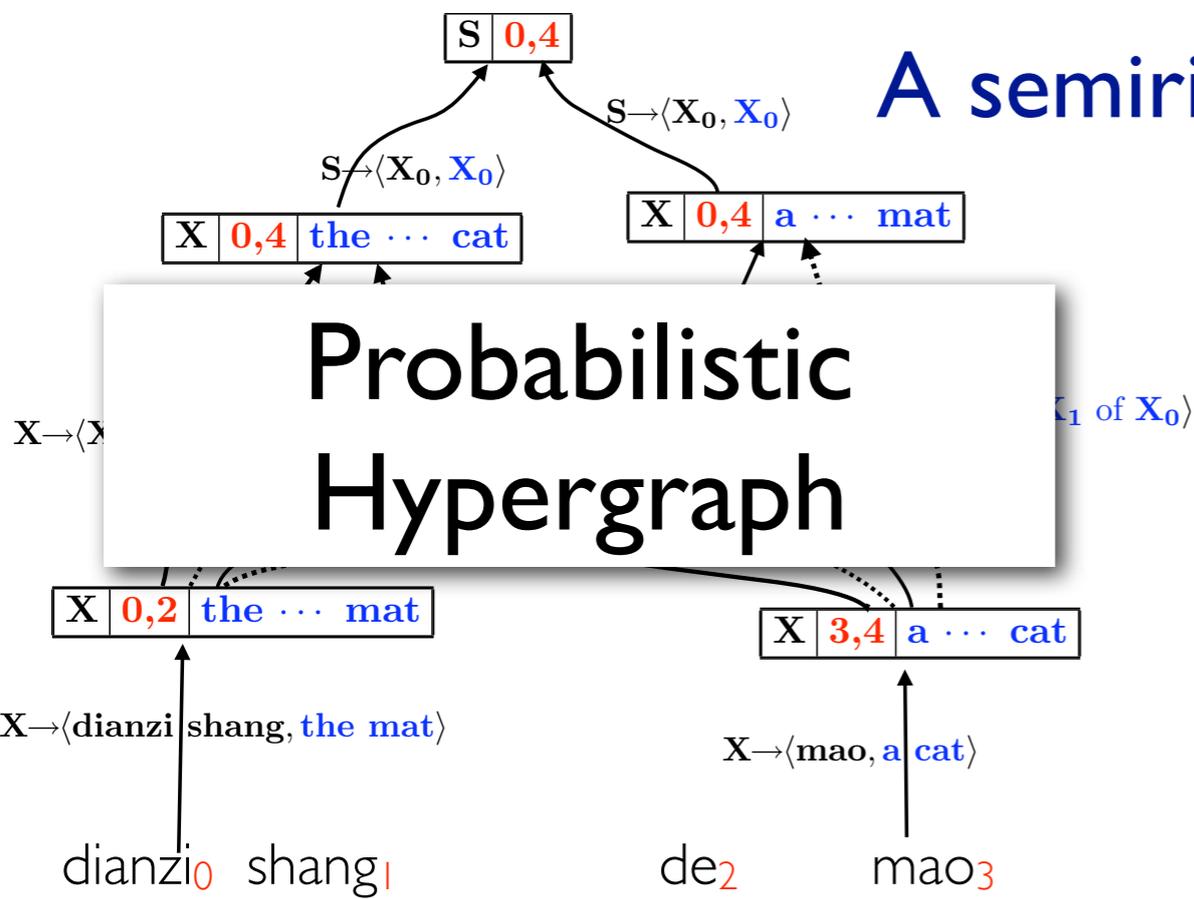
$L(Y(d))$  is additively decomposed!

(Tromble et al. 2008)

# Applications of Expectation Semirings: a Summary

Quantity	$k_e$	$k_{\text{root}}$	Final
<b>Expectation</b>	$\langle p_e, p_e r_e \rangle$	$\langle Z, \bar{r} \rangle$	$\bar{r}/Z$
Entropy	$r_e \stackrel{\text{def}}{=} \log p_e$ , so $k_e = \langle p_e, p_e \log p_e \rangle$	$\langle Z, \bar{r} \rangle$	$\log Z - \bar{r}/Z$
Cross-entropy	$\langle q_e \rangle$ $r_e \stackrel{\text{def}}{=} \log q_e$ , so $k_e = \langle p_e, p_e \log q_e \rangle$	$\langle Z_q \rangle$ $\langle Z_p, \bar{r} \rangle$	$\log Z_q - \bar{r}/Z_p$
Bayes risk	$r_e \stackrel{\text{def}}{=} L_e$ , so $k_e = \langle p_e, p_e L_e \rangle$	$\langle Z, \bar{r} \rangle$	$\bar{r}/Z$
<b>First-order gradient</b>	$\langle p_e, \nabla p_e \rangle$	$\langle Z, \nabla Z \rangle$	$\nabla Z$
<b>Covariance matrix</b>	$\langle p_e, p_e r_e, p_e s_e, p_e r_e s_e \rangle$	$\langle Z, \bar{r}, \bar{s}, \bar{t} \rangle$	$\frac{\bar{t}}{Z} - \frac{\bar{r} \bar{s}^T}{Z^2}$
<b>Hessian matrix</b>	$\langle p_e, \nabla p_e, \nabla p_e, \nabla^2 p_e \rangle$	$\langle Z, \nabla Z, \nabla Z, \nabla^2 Z \rangle$	$\nabla^2 Z$
<b>Gradient of expectation</b>	$\langle p_e, p_e r_e, \nabla p_e, (\nabla p_e) r_e + p_e (\nabla r_e) \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$
Gradient of entropy	$\langle p_e, p_e \log p_e, \nabla p_e, (1 + \log p_e) \nabla p_e \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{\nabla Z}{Z} - \frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$
Gradient of risk	$\langle p_e, p_e L_e, \nabla p_e, L_e \nabla p_e \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$

# A semiring framework to compute all of these



- “decoding” quantities:
  - Viterbi
  - K-best
  - Counting
  - .....

- First-order quantities:
  - expectation
  - entropy
  - Bayes risk
  - cross-entropy
  - KL divergence
  - feature expectations
  - first-order gradient of  $Z$

- Second-order quantities:
  - Expectation over product
  - interaction between features
  - Hessian matrix of  $Z$
  - second-order gradient descent
  - gradient of expectation
    - gradient of entropy or Bayes risk