Example Questions for Chapter 3

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February 24, 2009

1. Here are point transformation questions to think about.

    1. Suppose you are a robot in the plane. You start at the origin facing in the x direction. You travel 10 units along x, then rotate 45 degrees counterclockwise. What is the homogeneous transformation that describes the relationship of points in your coordinate system to the origin at which you started?
    2. Suppose you now travel another 10 units in the direction you are facing, then rotate another 45 degrees counterclockwise. What is the new homogeneous transformation? Can you express it in terms of the original transformation?
    3. What is the inverse transformation?

2. Here are some kinematics questions to think about:

    1. Consider a robot in the plane that has a prismatic joint arranged at an angle of 45 degrees to the x axis with (variable) length L_1, and a revolute joint of length L_2 making an angle θ with the x axis attached at the end. Write down the forward kinematics for this system for a point at the end of the revolute joint.
    2. Write the inverse kinematics for this system.
    3. Write the Jacobian.
    4. Does this manipulator have any singular configurations (hint: check the determinant of the Jacobian to see if it is ever zero)?

3. Prove that rotations in the plane commute. That is, $R(\theta_1)R(\theta_2) = R(\theta_2)R(\theta_1)$.

4. Prove the rotations in 3D *do not* commute (e.g. provide an example).

5. Notice that topological equivalence is an equivalence relation, and thus it forms a partition on a given set. What is the partition induced on the sans-serif lowercase letters (that is, which letters are topologically equivalent). What if I require the transformation to be a diffeomorphism?

6. What are the kinematic equations for a car with wheels of radius 1 ft, an axle that is 6 ft wide, and a wheelbase of 10 ft?

7. Try questions 3.2, 3.8, 3.17, and 3.21.