Homework 5 Computational Geometry Due: 4/30/97

1. Prove that a polygon P is completely visible from an edge e on P if all the vertices of P are visible from e.

2. Given k monotone polygonal chains (say, with respect to the x-axis) with n total vertices, compute their vertical visibility map (i.e., the pieces that are visibile from a point at $-\infty$) in $O(n \log k)$ time.

3. Suppose you are given a set $\{f_1, f_2, \ldots, f_n\}$ of n functions from $\mathbf{R}$ to $\mathbf{R}$, each of the form
$$f_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i.$$
Assuming you can compute the intersection points between two functions f_i and f_j in $O(1)$ time, describe an efficient algorithm to construct a representation of the function
$$F(x) = \min_{1 \le i \le n} \{f_i(x)\}.$$
What is the running time of your method?

4. Prove the zone theorem for a line L in an arrangement $\mathcal{A}$ of n lines using Davenport-Schinsel sequences. (Hint: think of the "functions" defined by the line portions that L "sees" above itself; also, consider each line M in $\mathcal{A}$ as defining two functions—one for each side of M.)

5. Given red, blue and green sets, R, B and G, of n points each in general position in the plane (i.e., no three points being colinear), how would you test whether there is line that bisects each set simultaneously? (How much time would your procedure take in terms of $n = |R| + |B| + |G|$?)