

600.271 Key to HW9
sp '13

I At least 2 satisfying assignments prob.

a) This prob is in NP.

1) Guess 2 assignments for the variables (e.g. T F T T F
F T F F T)

2) check that they differ for at least one variable.

3) check that both satisfy the CNF exp.

If all the tests succeed, then 'yes' else 'no'.

Speed $O(n)$. Correctness: complete it.

b) will prove SAT prob \leq poly this prob

Typical inst E

Given CNF exp E transform it to CNF exp E' s.t.

E is satisfiable iff E' is satisfiable by at least 2 diff assignments.

Let $E' = (\gamma_1 + \gamma_2) E$ s.t. γ_1 and γ_2 don't occur in E .

Any satisfying assignment of E with $\gamma_1 = T; \gamma_2 = F$
or $\gamma_1 = F; \gamma_2 = T$

will satisfy E' .

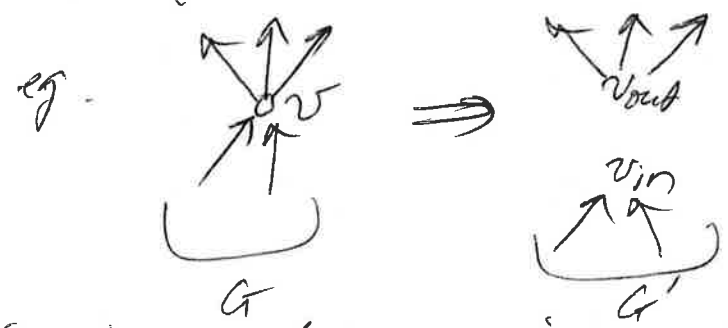
If E' has ≥ 2 satisfying assignment any one of the assignments ignoring γ_1 and γ_2 values satisfies E .

The transformation is computable in poly. time.

let v be any vertex of G .

split v into 2 vertices v_{out}, v_{in} s.t.

if (v, u) is an edge in G then (v_{out}, u) is an edge of G'
& if (u, v) is an edge in G then (u, v_{in}) is an edge of G'



if G has the cycle $v \rightarrow i_1 \rightarrow \dots \rightarrow i_{n-1}$ then G' has

the path $v_{out} \rightarrow i_1 \rightarrow \dots \rightarrow i_{n-1} \rightarrow v_{in}$

if G' has a Ham-path it must start with v_{out} (since v_{out} has no in-edges) & must end with v_{in} .

Then G has the Ham-cycle $v \rightarrow i_1 \rightarrow \dots \rightarrow i_n \rightarrow v$

Note that the trans. is computable in poly-time.

11 1) The problem is in NP. Guess k variables that take the value \pm . Evaluate the exp. If it evaluates to T , say yes else no.

Will show Hitting Set prob \leq poly time prob.

Typical inst: S_1, \dots, S_m, k E, k' E is universe.

Transform S_1, \dots, S_m, k to E, k' s.t. S_1, \dots, S_m have a hitting set of size k iff E is satis. with at most k' vars set to T .