600.271 Key to HW7
Sp'13

I 10 (a) $PCP \le_m$ Halting prob.

Typical instance, $E = (x_1, y_1), \cdots (x_n, y_n)$ $[M], x$

Given E, transform it to $[M], x$ s.t. E has a solution iff TM M halts on x.

Fix x = blank tape.

Description of M:

a) On any input other than blank tape, we don't care --- say it halts.

b) On blank tape: M checks ~~[illegible]~~ whether

~~sequence 1, 2, 3~~

$x_1 = y_1$; then $x_2 = y_2, \cdots$; then $x_n = y_n$; then

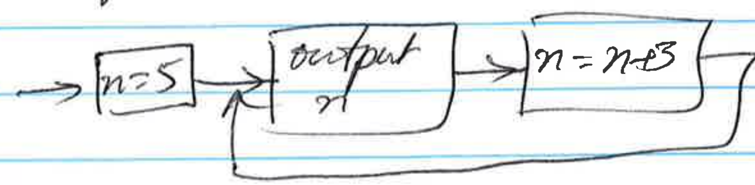
$x_1 x_1 = y_1 y_1$; then $x_1 x_2 = y_1 y_2, \cdots$; then $x_{i_1} x_{i_2} \cdots x_{i_m} = y_{i_1} y_{i_2} \cdots y_{i_m}$; $\cdots$

If one of them results in equality, M halts.

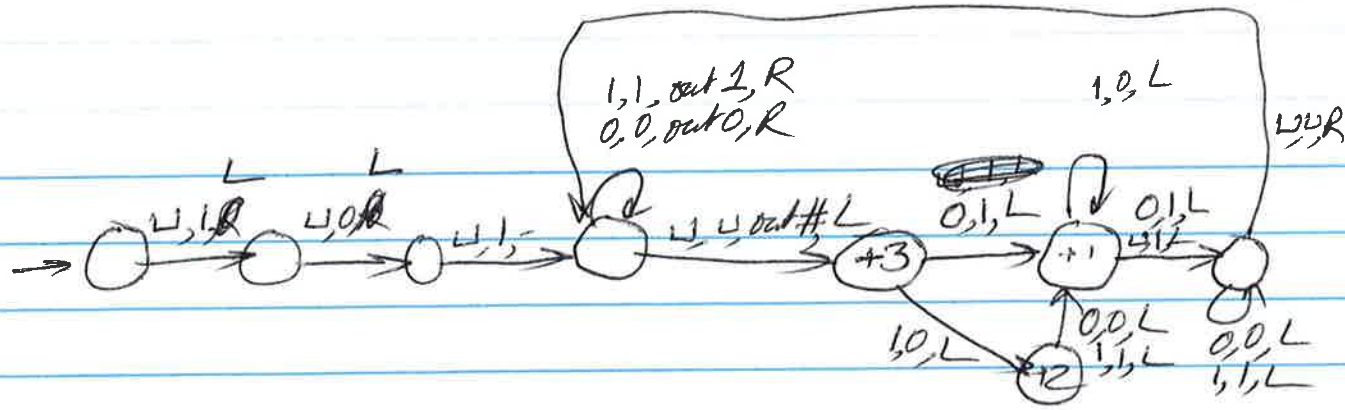
If E has a solution, M will detect it & halt.
If E has no solution, M will run forever.

Hence the transformation is correct. Note also that the transformation is a computable function.

II (d)



We will give full credit for the above.
A TM enumerator is given below.



(b) {[M] | M halts on input 101}

Generate [M_1], [M_2] . . . & dovetail their computations on input 101. output all [M_i] s.t. M_i halts

more detail:

1) At step 1, generate [M_1], perform one step of its simulation on input 101. Place the status in a queue.

At step i, i ≥ 2, generate [M_i], perform one more step of simulation for every process in the queue. Also perform ~~one~~ 1st step of M_i on input 101 & place it in queue.
If any M_k halts, output [M_k], and remove ~~@~~ its process from the queue.

III Prob 12: (a) If A and B are r.e. then A ∩ B is r.e.

X & Y

start the enumerators of A & B and enumerate them in parallel.

$\boxed{X} \to x_1, x_2, \ldots$ for A

$\boxed{Y} \to y_1, y_2, \ldots$ for B

a) when any x_i comes out of X, check whether it is one of the y_j's already enumerated. If so enumerate x_i.

b) When any y_i comes out of Y, check whether it is one of the x_j's already enumerated. If so, enumerate y_i.

Prob 14 $A \cup B$ is r.e.

Proceed as above. Enumerate all the x_i's that come out of X & all the y_j's that come out of Y.