600.271 Key to HW6

Sp '13

I (d) Given $x_1, x_2, \ldots, x_n$, s

$\exists$ distinct $i_1, \ldots, i_m$ s.t. $x_{i_1} + x_{i_2} + \ldots + x_{i_m} = s$.

For every subset of indices $\{i_1, \ldots, i_m\}$ of $\{1, 2, \ldots, n\}$ check whether $x_{i_1} + \ldots + x_{i_m} = s$. If at least one of the tests succeed, respond 'yes' else respond 'no' & halt. There are $2^n - 1$ subsets. Hence the algo. terminates.

One way to generate the subsets of $\{1, 2, \ldots, n\}$ is to represent each subset by a binary no. with n bits in which 1's indicates that the corresponding elements are present. For example,

1011 indicates the set $\{4, 2, 1\}$.

(h) dfa M, is $L(M) \neq \phi$?

Method 1: Let dfa M have s states.

Claim: If M accepts any string, it must accept a string of length $\leq s$.

Pf: If not, let the shortest string be $a_1 \ldots a_m$ s.t. $m > s$. In the sequence of states visited there must exist a repetition. If we delete the substring corresponding to any such repetition, the shorter string must be accepted too - contradicting the minimality of m.

Then check whether M accepts at least one of strings in $\{\epsilon\} \cup \Sigma \cup \Sigma^2 \cup \ldots \cup \Sigma^s$; i.e. all the strings of length $\leq s$. If so, respond 'yes' else respond 'no' & halt.

Method 2: Compute all the states of M that can be reached from its initial state by a process analogous to the spread of a contagion. Technically it is known as breadth-first scan.

Start with the set $\{q_0\}$.

At any stage let the set be A.

For ~~a~~ every $q \in A$, if there is a transition to state q', include q' in A.

If this process increases the size of A, Repeat the process.

~~If the p~~ Else, the set A gives all the states that can be reached from $\{q_0\}$. Since the fa has s states & since repetition increases the size of the set, the max. number of repetitions is $s-1$. Hence the process is finite.

Check whether the ~~set~~ resulting set A contains at least one final state. If so, respond 'yes' else respond 'no' & halt.

(ii) Let M_1 & M_2 have s_1 & s_2 states, respectively.

Method 1: If M_1 & M_2 accepts some common strings, then they must accept a string of length $\le s_1 s_2$.

Proof is ~~similar~~ ~~analogous to~~ a simple extension of the above proof.

Then check whether some string of length $\le s_1 s_2$ is accepted by both M_1 & M_2.

Method 2: Construct an fa for the language

$L(M_1) \cap L(M_2)$.

~~This involves the~~

This involves running M_1 & M_2 in parallel; i.e. keeping track states of the form (q, q') in which q & q' are states of M_1 & M_2, respectively. At the end, check that both the states are final states.

On the intersection ~~sa~~ fa, apply the methods ~~it~~ of problem (h).

$\underline{\text{II}}$ (b) M_1, M_2, $L(M_1) = L(M_2)$?

We show BTHP $\leq_m$ This problem

Typical instances M $\quad$ M_1, M_2

Given M, we want to construct M_1 and M_2 s.t. M halts on BT iff $L(M_1) = L(M_2)$.

~~Let~~ Construct M_1 s.t. $L(M_1) = \{a,b\}^*$. — Easy.

M_2: On any string x of a's & b's, M_2 ~~erases~~ simulates M on BT. If M halts, then M_2 accepts x.

Thus If M halts on BT then $L(M_2) = \{a,b\}^*$

If M doesn't halt on BT then $L(M_2) = \phi$.

Hence M halts on BT iff $L(M_1) = L(M_2)$.

Note the transformation of $[M]$ to $[M_1]$ $[M_2]$ is a computable function.

II(e) CFGs G_1, G_2 is $L(G_1) \cap L(G_2) \neq \phi$

we show that PCP $\leq_m$ this prob

Typical instances $(x_1,y_1), \cdots (x_n,y_n)$ G_1, G_2

Given $E = (x_1,y_1), \cdots, (x_n,y_n)$ we want to specify G_1 & G_2

s.t. E has a solution iff $L(G_1) \cap L(G_2) \neq \phi$.

G_1 : $S \to x_1 S c_1 \mid x_2 S c_2 \mid \cdots \mid x_n S c_n \mid x_1 c_1 \mid \cdots \mid x_n c_n$

G_2 : $S \to y_1 S c_1 \mid y_2 S c_2 \mid \cdots \mid y_n S c_n \mid y_1 c_1 \mid \cdots \mid y_n c_n$

If $x_{i_1} \cdots x_{i_m} = y_{i_1} \cdots y_{i_m}$

then in G_1 : $S \overset{*}{\Rightarrow} x_{i_1} \cdots x_{i_m} c_{i_m} \cdots c_{i_1}$ $\parallel$
in G_2 : $S \overset{*}{\Rightarrow} y_{i_1} \cdots y_{i_m} c_{i_m} \cdots c_{i_1}$

Hence there is string in $L(G_1) \cap L(G_2)$.

~~the~~ If there is a common string z, then z is of the form uw, $u \in \{0,1\}^+$ & $w \in \{c_1, \cdots, c_n\}^+$.

If $w = c_{i_m} \cdots c_{i_1}$ then $u = x_{i_1} \cdots x_{i_m}$ since $z \in L(G_1)$.

$u = y_{i_1} \cdots y_{i_m}$ since $z \in L(G_2)$.

Hence $x_{i_1} \cdots x_{i_m} = y_{i_1} \cdots y_{i_m}$

Note that the transformation from E to G_1 & G_2 is a computable function.

III ~~Given~~ Will show BTHP $\leq_m$ this prob.

Typical inst [M] [M']

Given [M], we transform it [M'] s.t.

TM M halts on BT iff TM M' computes $f(n)=n^2$.

M': Given any n, on a separate part of the tape, M' simulates M on BT. If M halts, then M' erases the computation part of the tape, then compute n^2 & halt.

M halts on BT $\Rightarrow$ M' computes $f(n)=n^2$.

M doesn't halt on BT $\Rightarrow$ M' computes $f(n)=$ undefined for every n.

Hence our goal is achieved.

Note also that the transformation from [M] to [M'] is a computable function.