

I $L_5 = \{a^i b^j \mid i, j \geq 1 \text{ and } |i-j| \leq 3\}$

$S \rightarrow aSb \mid aXb$ (generates $a^{\min\{i,j\}} X b^{\min\{i,j\}}$)

$X \rightarrow aaa \mid aa \mid a \mid \epsilon \mid b \mid bb \mid bbb$

$L_6 = \{a^i b^j c^k d^l \mid i, j, k \geq 1 \text{ and } k \geq 2i\}$

$S \rightarrow sd \mid X$ generates $k-2i$ d's.

$X \rightarrow aXdd \mid aYdd$

$Y \rightarrow bbYc \mid bbc$

II (d) $\{x \mid x \in \{a,b\}^*, abb, aab \text{ are substrings of } x\}$

X generates $\dots abb \dots aab \dots$ Y generates $\dots aab \dots abb \dots$

$S \rightarrow X \mid Y$

$X \rightarrow Xa \mid Xb \mid Pa$
 $P \rightarrow Qa$
 $Q \rightarrow Ra$
 $R \rightarrow Ra \mid Rb \mid Tb$
 $T \rightarrow Ub$
 $U \rightarrow Va$

$V \rightarrow Va \mid Vb \mid \epsilon$

$Y \rightarrow Ya \mid Yb \mid Ab$
 $A \rightarrow Bb$
 $B \rightarrow Ca$
 $C \rightarrow Ca \mid cb \mid Db$
 $D \rightarrow Ea$
 $E \rightarrow \epsilon \mid Va$

(f) $\{a^i b^j \mid i, j \geq 0 \text{ and } i \neq j\}$

$S \rightarrow aSb \mid A \mid B$

$A \rightarrow Aa \mid a$

$B \rightarrow Bb \mid b$

(j) $\{xcy \mid xy \in \{a,b\}^*, x^R \text{ is a prefix of } y\}$

$y = x^R u, u \in \{a,b\}^*$

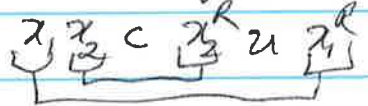
strings: $xcx^R u$

$S \rightarrow Sa \mid Sb \mid x$ generates u parts

$x \rightarrow axa \mid bxb \mid c$

(k) $\{x_1 x_2 c y \mid x_1, x_2, y \in \{a,b\}^*, (\exists z \in \{a,b\}^*) (z, z^R = y^R)\}$

$y = x_2^R z^R x_1^R = z^R u x_1^R, u \in \{a,b\}^*$



$S \rightarrow aSa \mid bSa \mid x$

generates x_1, \dots, x_1^R

$x \rightarrow xa \mid xb \mid y$

generates u

$y \rightarrow aYa \mid bYb \mid c$

generates x_2, c, x_2^R

(p) $\{a^i b^j c^k \mid i, j, k \geq 0\}$

First generate a string in $A_1 A^+ AB (AB)^+ (ABC)^+ A_1, AB (A_1 AB + ABC)^+$

Permute. verify in $A^+ B^+ C^*$.

$S \rightarrow SABC \mid SAB \mid SA \mid A_1, AB$

$AB \rightarrow BA, AC \rightarrow CA, BA \rightarrow AB, BC \rightarrow CB, CA \rightarrow AC, CB \rightarrow BC$

$A_1 A \rightarrow A_1 A_1, A_1 B \rightarrow A_1 B, B_1 B \rightarrow B_1 B_1, B_1 C \rightarrow B_1 C_1, C_1 C \rightarrow C_1 C_1,$

$A_1 \rightarrow a, B_1 \rightarrow b, C_1 \rightarrow c.$

iii

$L^R = \{x^R \mid x \in L\}$

Replace each production Let G be the CFG for L .

In G , replace each production $x \rightarrow \beta$ by $x \rightarrow \beta^R$.

Argue that the new grammar generates L^R . Hence L^R is a CFL.