600.271 Key to HW4
Sp '13

I $L_5 = \{a^i b^j \mid i,j \geq 1 \text{ and } |i-j| \leq 3\}$

$S \to aSb \mid aXb$ [generates $a^{\min\{i,j\}} X b^{\min\{i,j\}}$]

$X \to aaa \mid aa \mid a \mid \epsilon \mid b \mid bb \mid bbb$

$L_6 = \{a^i b^{2j} c^j d^k \mid i,j,k \geq 1 \text{ and } k \geq 2i\}$

$S \to Sd \mid X$ generates $k-2i$ d's.

$X \to aXdd \mid aYdd$

$Y \to bbYc \mid bbc$

II (d) $\{x \mid x \in \{a,b\}^*,\ abb, aab \text{ are substrings of } x\}$

X generates ...abb...aab... Y generates ...aab...abb...

$S \to X \mid Y$

$X \to Xa \mid Xb \mid Pb$
$P \to Qa$
$Q \to Ra$
$R \to Ra \mid Rb \mid Tb$
$T \to Ub$
$U \to Va$

$V \to Va \mid Vb \mid \epsilon$

$Y \to Ya \mid Yb \mid Ab$
$A \to Bb$
$B \to Ca$
$C \to Ca \mid Cb \mid Db$
$D \to Ea$
$E \to Va$

(f) $\{a^i b^j \mid i,j \geq 0 \text{ and } i \neq j\}$

$S \to aSb \mid A \mid B$
$A \to Aa \mid a$
$B \to Bb \mid b$

(j) $\{xcy \mid x,y \in \{a,b\}^*,\ x^R \text{ is a prefix of } y\}$

$y = x^R u \quad , u \in \{a,b\}^*$

Strings: $x c x^R u$

$S \to Sa \mid Sb \mid X$ generates u parts

$X \to aXa \mid bXb \mid c$

(k) $\{x_1 x_2 c y \mid x_1, x_2, y \in \{a,b\}^*, (\exists x \in \{a,b\}^*)(x_1 x x_2 = y^R)\}$

$y = x_2^R x^R x_1^R = x_2^R u x_1^R \quad u \in \{a,b\}^*$

$x_1 x_2 c\, x_2^R u\, x_1^R$

$S \to aSa \mid bSa \mid X$ generates $x_1 \ldots x_1^R$

$X \to Xa \mid Xb \mid Y$ generates u

$Y \to aYa \mid bYb \mid c$ generates $x_2 c x_2^R$

(p) $\{a^i b^j c^k \mid i > j > k \geq 0\}$

First generate a string in $A_1 A^*$ ~~$AB\,(AB)^*(ABC)^*$~~ $A_1 AB(A+AB+ABC)^*$

Permute. verify in $A^+ B^+ C^*$.

$S \to SABC \mid SAB \mid SA \mid A_1 AB$

$AB \to BA,\ AC \to CA,\ BA \to AB,\ BC \to CB,\ CA \to AC,\ CB \to BC$

$A_1 A \to A_1 A_1,\ A_1 B \to A_1 B_1,\ B_1 B \to B_1 B_1,\ B_1 C \to B_1 C_1,\ C_1 C \to C_1 C_1,$

$A_1 \to a,\ B_1 \to b,\ C_1 \to c.$

III) $L^R = \{x^R \mid x \in L\}$.

~~Replace each producte~~ Let G be the CFG for L.

In G, replace each production $X \to \beta$ by $X \to \beta^R$.

Argue that the new grammar generates L^R. Hence L^R is a CFL.