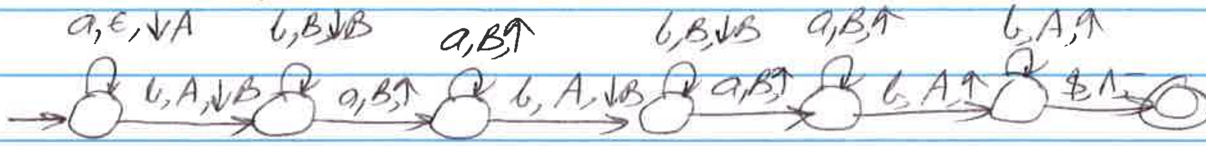
600.271 Key to HW3
Sp '13.

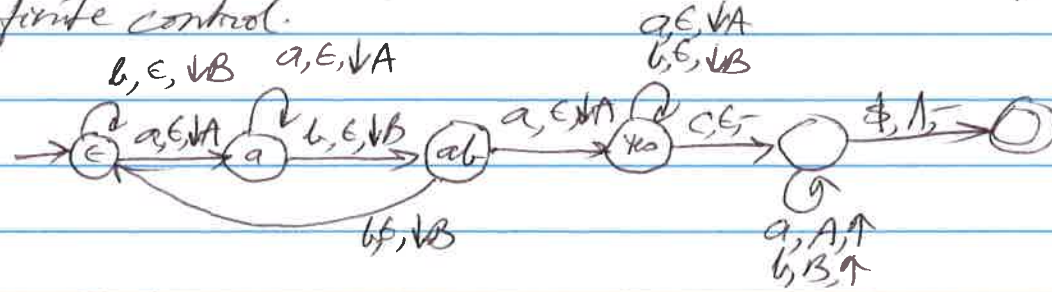
I $L_1 = \{ a^i b^j a^j b^k a^k b^i \mid i, j, k \geq 1 \}$

The nesting indicates the proper matchings.



$L_{10} = \{ x c x^R \mid aba \text{ is a substring of } x \}$

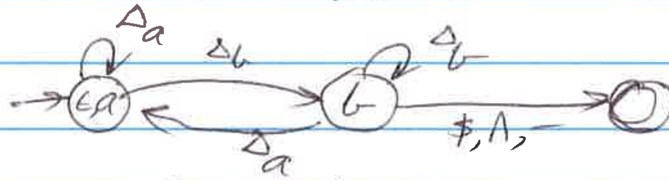
x, x^R relationship via the pushdown; substring via the finite control.



$L_{18} = \{ x \mid \#_a x = \#_b x, \text{ last symbol of } x = b \}$.

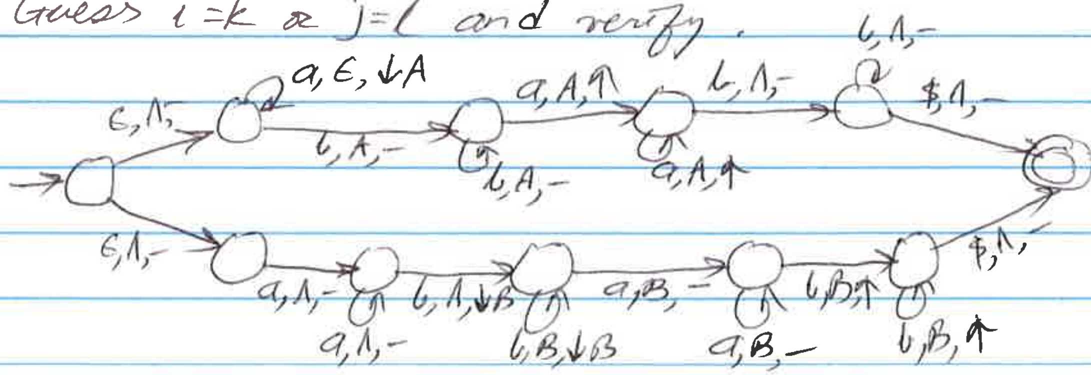
$\#_a = \#_b$ by pushdown; last symbol by the finite control.

Let $\Delta_a = \begin{cases} a, A, \downarrow A \\ a, \Lambda, \downarrow A \\ a, B, \uparrow \end{cases}$ and $\Delta_b = \begin{cases} b, A, \uparrow \\ b, \Lambda, \downarrow B \\ b, B, \downarrow B \end{cases}$



II $L_2 = \{ a^i b^j a^k b^l \mid i, j, k, l \geq 1, (i = k) \text{ or } (j = l) \}$

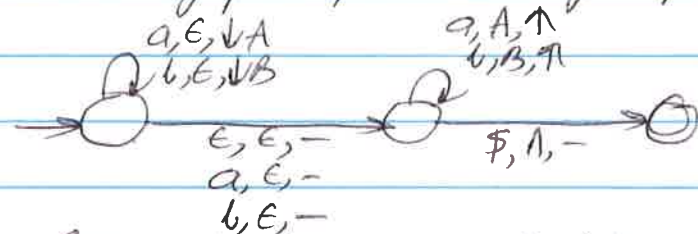
Guess $i = k$ or $j = l$ and verify.



$L_3 = \{x \mid x = x^R\}$

$x = x^R$ iff ~~$\exists y, c$~~ $(\exists y \in \{a,b\}^*, c \in \{a,b\})(x = yy^R$ or $x = ycy^R)$.

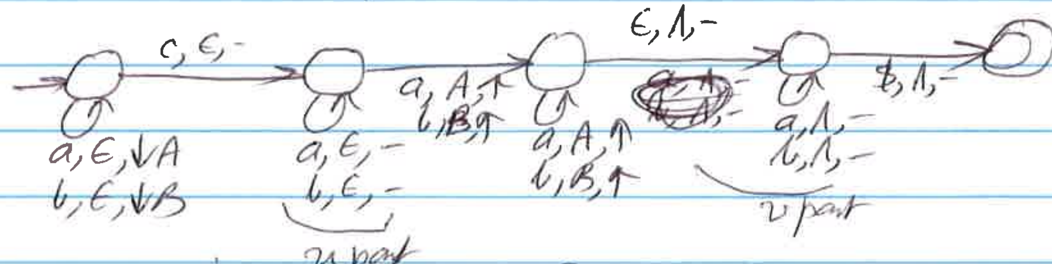
On the y part push, on y^R ~~p~~ match & pop.



$L_{13} = \{xcy \mid x^R$ is a substring of $y\}$

$y = u x^R v$

Push x, guess & through away u part, then match. After the pd becomes empty throw away the v part.



III (b) $L = \{a^i b^j c^k \mid 1 < i < j < k\}$.

For any k, let $z = a^k b^{k+1} c^{k+2}$. Note that $z \in L$ & $|z| = 3k+3 > k$.

For any way of cutting

$a^k b^{k+1} c^{k+2} = uvwxy$ st. $|vwx| \le k$ & $|vx| > 0$:

since $|vwx| \le k$, vwx and hence vx cannot contain all b's.

If vx doesn't contain c's then $uv^3wx^3y \notin L$ since $i<j<k$ cannot be maintained.

If vx doesn't contain a's then $uv^0wx^0y \notin L$ since $i<j<k$ cannot be maintained.

Hence L is not an npda lang.

(m) $L = \{x \mid \#_a x = \#_b x \geq \#_c x\}$

Let $z = a^{k+1} b^{k+1} c^{k+1}$. As before vx ~~can~~ cannot contain all b's.

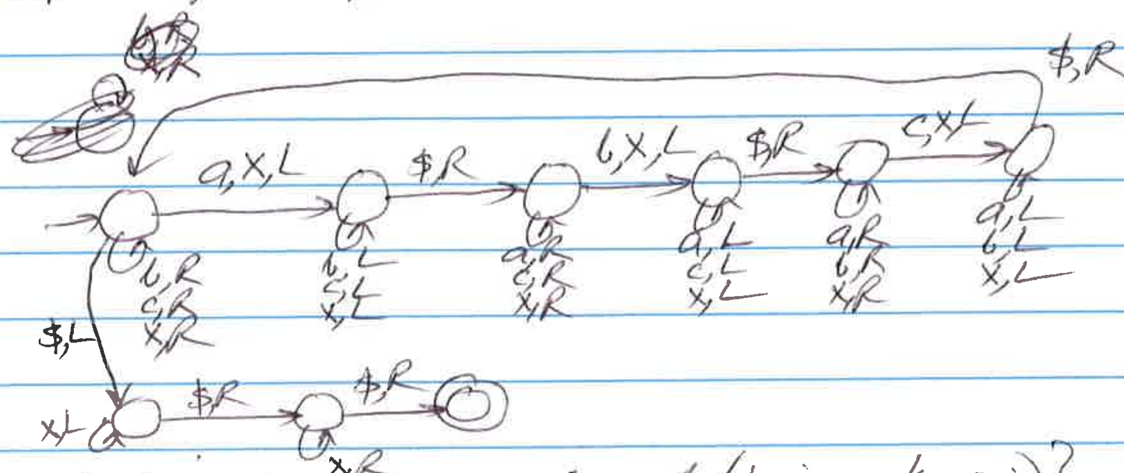
If vx doesn't contain c's, then $uv^0wx^0y \notin L$ since a's and/or b's will be reduced & $\#_a x \geq \#_c x$, $\#_b \geq \#_c x$ cannot hold.

If vx doesn't contain a's, then $uv^2wx^2y \notin L$ since $\#_b x = \#_c x$ and/or ~~#~~ $\#_a x \geq \#_c x$ cannot hold.

Hence L is not an npda lang.

IV (a) $\{x \mid \#_a x = \#_b x = \#_c c\}$.

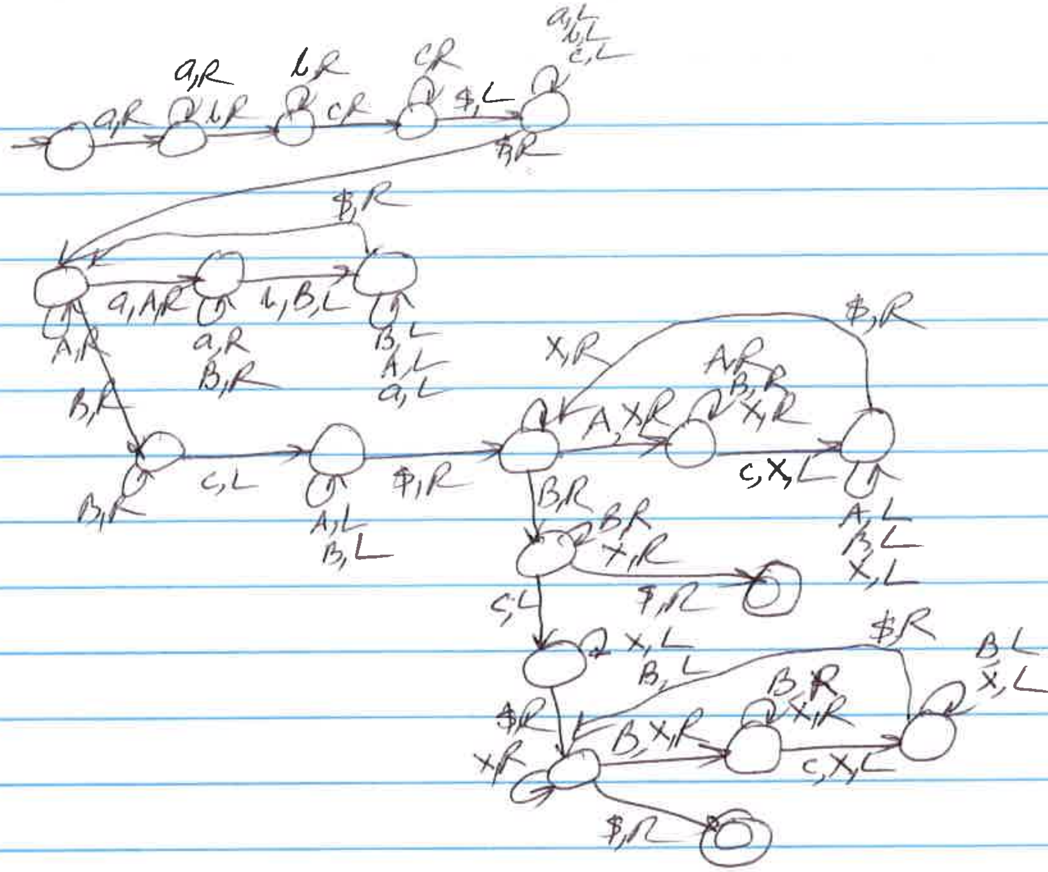
Repeatedly cross off one a, one b, one c.



(e) $\{a^i b^j c^k \mid i,j,k \geq 1,\ i=j \text{ and } (k=i \text{ or } k=2i)\}$

check form $\in a^+b^+c^+$, check $i=j$; then $\#_c = \#_b$ or $\#_c = \#_a + \#_b$.

See next page.


a,R
b,R
c,R
$,L
a,L
b,L
c,L
$,R
a,A,R
b,B,L
A,R
B,R
B,L
A,L
a,L
c,L
$,R
X,R
A,X,R
c,X,L
X,L
X,R
B,X,R