I \quad L_1 = \{ a^i b^j c^k \mid i \geq j \geq k \}.

For any value \( k \), let \( z = a^k b^k c^k \). Note \( z \in L_1 \) and \( |z| = k^2 \).

For any choice \( a^k b^k = uvw \), let \( |u| \leq k \) and \( |v| > 0 \). \( u \) must consist of only a's since \( |uv| = k \). Let \( u = a^l \), \( l \leq k \).

Then \( u v^2 w = a^{k+l} b^k c^k \notin L_1 \). Hence \( L_1 \) is not an FA language.

\[ L_{15} = \{ a^i b^j c^k \mid i, j, k \geq 1, \frac{i}{j} = \frac{k}{i} \} \quad \text{(Note:} \quad z \notin L_{15} \).

For any value \( k \), let \( z = a^k b^k c^k \).

Let \( x, y \in L_{15} \). As before, let \( v = a^l \), \( 1 \leq l \leq k \). Then \( u v^i w = a^{k+1+i} b^{k+i} c^{k+i} \).

If \( i = 1 + \frac{k+1}{l} \), then \( u v^i w \notin L_{15} \). Hence \( L_{15} \) is not an FA language.

\[ L_{26} = \{ z \mid z \in L_6 \} \quad \text{where} \quad z \text{ is divisible by } 6^2 \).

For any \( k \), let \( z = b^k c^k \). Note \( z \in L_{26} \) and \( |z| \geq k \).

As before, let \( u v = a^l \), \( 1 \leq l \leq k \). Then \( u v^2 w = a^{k+1} b^k c^k \).

Hence \( L_{26} \) is not an FA language.

II \quad L_{10} = \{ \text{\#a, \#b, \#c contained exactly once or at least two more times} \}.

Guess the occurrences and verify.
\( L_n = \{ x \mid (xy)_n \text{ s.t. } \# y = abn \geq \# y \text{ is even} \} \)

Guess the occurrence of \( ab \) and verify.

\[ R_3 = (ab + ba)^* aa + aba. \]

Systematically:

\( ab \):

\( ab + ba \):

\((ab + ba)^* \):

\((ab + ba)^*aa \):

etc.

\[ R_{10} = (0(01 + 10)^* 1 + (1 + 10)^*) \]
\[ R_{13} = \emptyset (0^+)^* (100 + 10^+1) \]

There are no paths from A to C going through B. Hence, B can be removed. Alternatively, removing B by the standard procedure adds no additional labels.

Here: \( N(b) \rightarrow A \rightarrow B \rightarrow C \)

Reg. exp: \( a(a+b) \)

Through D: \( (aa+(a+a(a+b))b)(a+b)^*b \) let it \( \in R \).

Through E: \( (a+a(a+b))b(a+b)^*a \) let it \( \in R' \).

Then: \( R + R' \)

Alternatively, let delete B, D, E one after the other.
\[(e)\]

\[R_1 = (a+b) b (b + a a^* b (a+b) b^*)\]

\[R_2 = (a+b) b b^* a (a+b)(a+b) b^* b\]

**Finally:** \[R_1 + R_2\]