600.271 Sp'13
Key to HW2

I $L_7 = \{a^i b^{i^2} \mid i \ge 1\}$.

For any value k, let $z = a^k b^{k^2}$. Note $z \in L_7$ & $|z| = k + k^2 \ge k$

For any choice $a^k b^{k^2} = uvw$ s.t. $|uv| \le k$ & $|v| > 0$, v must consist of only a's since $|uv| \le k$. Let $v = a^l$, $1 \le l \le k$.

Then $uv^2w = a^{k+l} b^{k^2} \notin L_7$. Hence L_7 is not an fa lang.

$L_{15} = \{a^i b^j c^k \mid i, j, k \ge 1, \ k \ne ij\}$

For any value k, let z = above $a^{k+1} b^{k+1} c^{(k+1)^2 + (k+1)!}$. Note $z \in L_{15}$

& $z \in L_{15}$. As above let $v = a^l$, $1 \le l \le k$. Then

$uv^iw = a^{k+1+(i-1)l} b^{k+1} c^{(k+1)^2 + (k+1)!}$. If

$i = 1 + \frac{k!}{l}$, then $uv^iw \notin L_{15}$. Hence L_{15} is not an fa lang.

$L_{26} = \{x \mid x \in \{a,b\}^*, \#_a x$ is div by $\#_b x\}$.

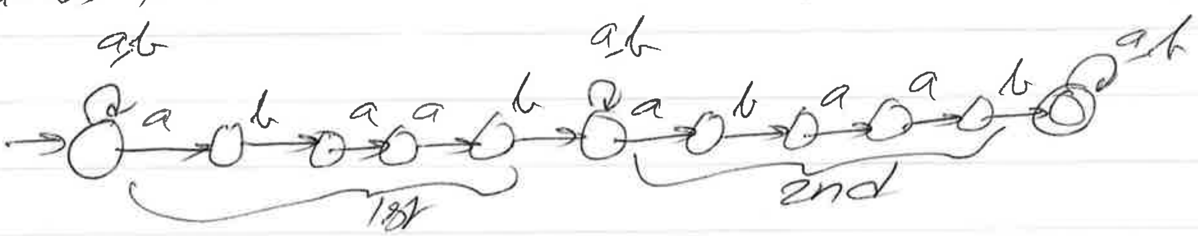
For any k, let $z = b^k a^k$. Note $z \in L_{26}$ & $|z| \ge k$.

As before, let $v = a^l$, $1 \le l \le k$. Then $uv^2w = b^{k+l} a^k \notin L_{26}$

Hence L_{26} is not an fa lang.

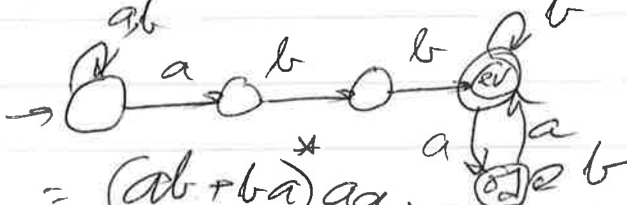
II $L_{10} = \{x \mid x$ contains abaab as substring 2 or more times$\}$.

Guess the 2 occurrences & verify.

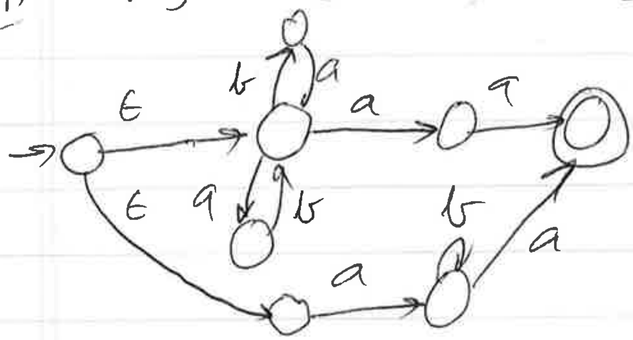


$L_{11} = \{x \mid (\exists y z) \text{ s.t. } x = yabbz \ \& \ \#_a z \text{ is even}\}$

Guess the occurrence of abb & verify.



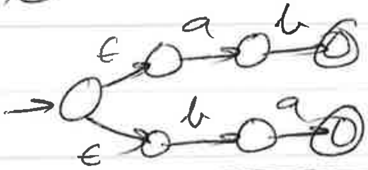
III $R_3 = (ab+ba)^* aa + aba.$

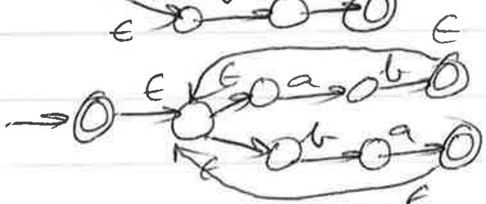


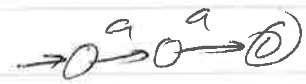
systematically:

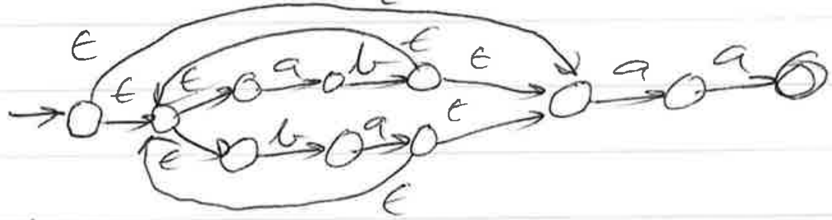
ab 

ba 

ab+ba: 

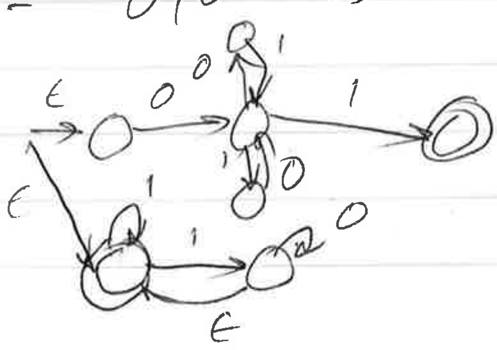
$(ab+ba)^*$: 

aa 

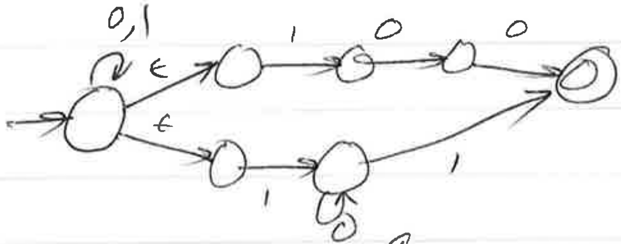
$(ab+ba)^* aa$ 

etc.

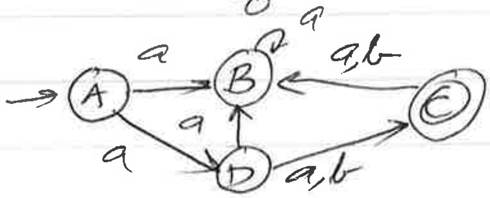
$R_{10} = 0(01+10)^* 1 + (1+10^*)^*$



$R_{13} = 0(0+1)^*(100+10^*1)$

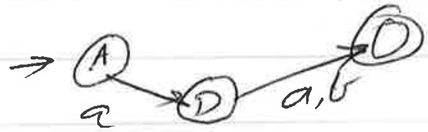


IV (b)



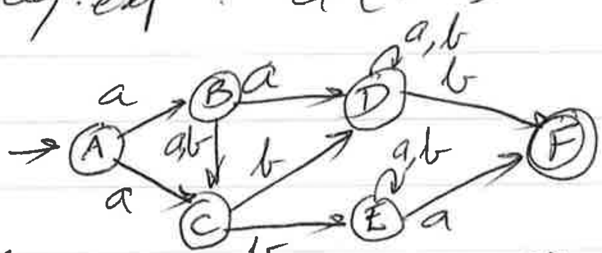
There are no paths from A to C going through B. Hence B ~~be~~ can be removed. Alternatively, removing B by the standard procedure adds no additional labels.

Hence:



Reg. exp: $a(a+b)$.

(c)



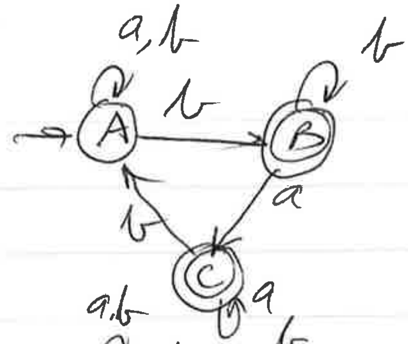
Through D: $(aa+(a+a(a+b))b)(a+b)^*b$ let it be R_1.

Through E: $(a+a(a+b))b(a+b)^*a$ let it be R_2.

Then: R_1+R_2.

Alternatively, let delete ~~B, C~~, D, E one after the other

(e)



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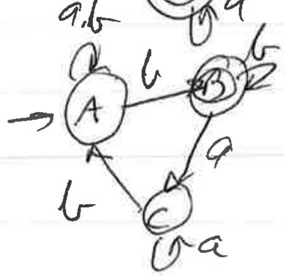

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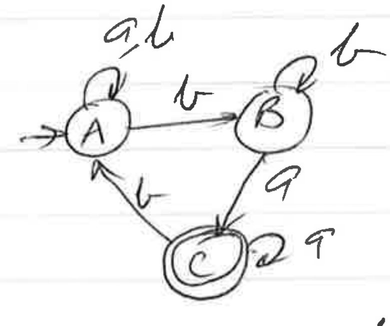
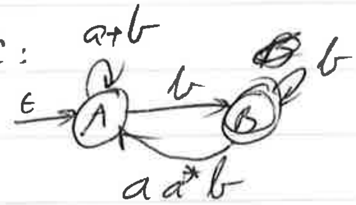
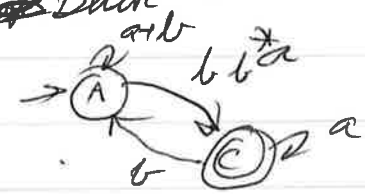


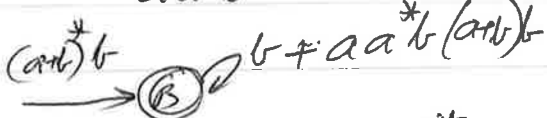
Delete C:



Delete B:



Delete A:



$R_1 = (a+b)^* b \left(b + a a^* b (a+b) b\right)^*$

$R_2 = (a+b)^* b b^* a \left(a + b(a+b)^* b b^* a\right.$

Finally: $R_1 + R_2$