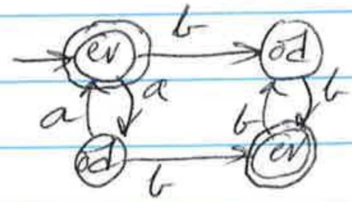
600.271 Auto. & Comp. Th

Key to HW1

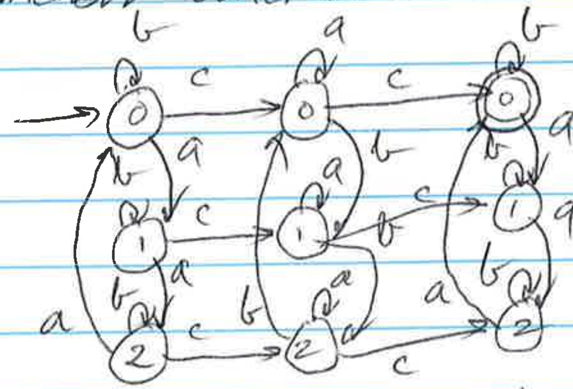
I $L_1 = \{a^i b^j \mid i, j \geq 0,\ i+j \text{ is even}\}$

At any stage, ~~remember~~ remember $i+j$ even or odd. Also remember whether we are in a's or b's.



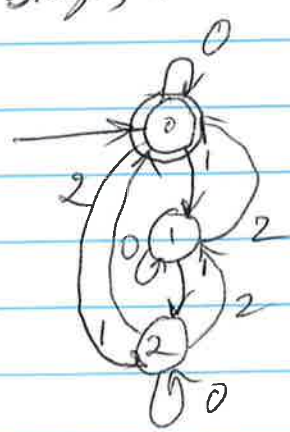
$L_4 = \{ xcycz \mid x,y,z \in \{a,b\}^*,\ \#_a x + \#_b y + \#_a z \text{ is div by } 3\}$

At any stage, remember $\#_a x + \#_b y + \#_a z \pmod 3$. Also remember whether we are in x's, y's or z's.



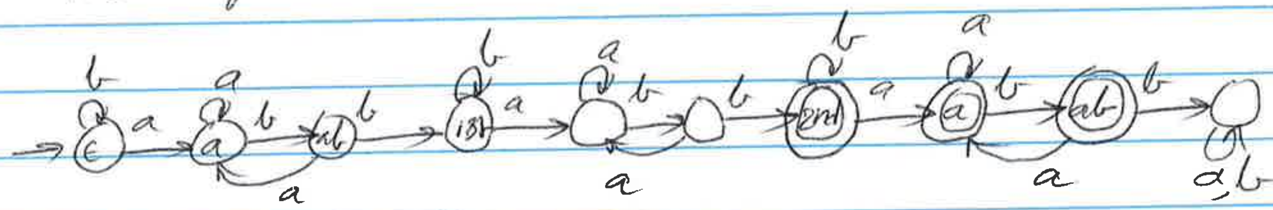
$L_{11} = \{ x \mid x \in \{0,1,2\}^*,\ \#_1 x + 2\#_2 x \text{ is div by } 3\}$

At any stage, remember $\#_1 x + 2\#_2 x \pmod 3$.



$L_{20} = \{x \mid x \in \{a,b\}^*,$ abb occurs exactly 2 times$\}$

Remember whether in 1st or 2nd occurrence. Afterwards track for 3rd occurrence



II $L_1 = \{a^i b^{2i} \mid i \geq 1\}$.

Assume that L_1 can be accepted by a dfa, & let it have s states. Consider the set

$$S = \{a, a^2, \ldots, a^{s+1}\}$$

of $s+1$ distinct strings. By the rt. cong lemma, two of the strings are rt. cong. Let they be

$$a^l, a^m, \; l \neq m.$$

By the definition of rt cong, for every $z \in \{a,b\}^*$,

$$a^l z \in L_1 \Leftrightarrow a^m z \in L_1.$$

Let $z = a^l$. Then $a^l b^{2l} \in L_1$ but $a^m b^{2l} \notin L_1$.

Hence L_1 cannot be accepted by a dfa.

$L_8 = \{a^i b^j \mid i, j \geq 1$ and $j \neq i^2\}$.

Proceed as above with same S.

Let a^l, a^m, $l \neq m$ be rt cong.

Then $a^l z \in L_8 \Leftrightarrow a^m z \in L_1$.

let $z = b^{l^2}$. Then $a^l b^{l^2} \notin L_8$ but $a^m b^{l^2} \in L_8$.
Hence L_8 is not a dfa lang.

$L_9 = \{a^i b^j \mid i, j \geq 1, i \leq j \leq i^2\}$.

By mistake I solved L_9 instead of L_{19}. L_{19} is solved below.

L_9 ↑ (Extra by mistake)

Proceed as above with the same S.

let a^l, a^m, $l < m$, be rt cong.

Then $a^l z \in L_9 \Leftrightarrow a^m z \in L_9$

let $z = b^{m^2}$. Then $a^l b^{m^2} \notin L_9$ but $a^m b^{m^2} \in L_9$.
Hence L_9 is not a dfa lang.

($z = b^{l^2}$ is incorrect since $a^l b^{l^2} \in L_{19}$ but $a^m b^{l^2}$ is not guaranteed not to be in L_{19}.)

11) $L = \{a b^n a^{2n} \mid n \geq 1\}$.
Proceed as above with $S = \{ab, ab^2, \ldots, ab^{s+1}\}$

let ab^l and ab^m, $l \neq m$, be rt cong.

Then for every $z \in \{a,b\}^*$, $ab^l z \in L \Leftrightarrow ab^m z \in L$

chose $l = a^{2l}$. Then $ab^l a^{2l} \in L$ but $ab^m a^{2l} \notin L$.

Hence L is not a dfa lang.

$L_{19} = \{xcy \mid x, y \in \{a,b\}^*$ and $x \neq y$.

Proceed as above with the same S.

Let a^l, a^m, $l \neq m$, be rt. cong.

Then $a^l z \in L_{19} \Leftrightarrow a^m z \in L_{19}$.

Let $z = ca^l$.

Then $a^l c a^l \notin L_{19}$, but $a^m c a^l \in L_{19}$.

Hence L_{19} is not a dfa lang.