600.271 Automata & Computation Theory
Second Mid-Semester Examination
In-class, Closed Book
April 11, 2012
Time: 1 hr, 10 minutes

I. (10 pts.) Design a context-free grammar for the language
$\{xcy|\ x, y \in \{a, b\}^*,\ |x| = |y|,\ x \neq y^R,$ and $|x|$ is an odd integer$\}$.

II. (10 pts.) Specify a decision procedure for the following problem:

- Given n dfas, $M_1, M_2, \cdots, M_n$, over the alphabet Σ, do there exist 2 dfas M_k and M_ℓ, $k \neq \ell$, such that $L(M_k) \cup L(M_\ell) = \Sigma^*$?

III. (10 pts.) Design a Turing machine for computing the following function:

$$f(x, y, z) = \begin{cases} 2x + y - 2z & \text{if } 2x + y - 2z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

IV. (10 pts.) Solve one of the following problems.

- Assume that testing whether a given TM halts on exactly one input is undecidable. Prove that testing whether a given TM halts on exactly two inputs is undecidable.

- Recall that a given R-PCP $(x_0, y_0),\ (x_1, y_1),\ \cdots,\ (x_n, y_n)$ has a solution if there is a sequence of indices $i_1, i_2, \cdots, i_m,\ m \geq 1$, such that $x_0 x_{i_1} x_{i_2} \cdots x_{i_m} = y_0 y_{i_1} y_{i_2} \cdots y_{i_m}$. Prove that the R-PCP problem is undecidable assuming that the PCP problem is undecidable.