600.271 Automata & Computation Theory
Mid-Semester Examination
March 10, 2011
In-class, Closed Book, Time: 1 hr, 10 mins

All the subproblems carry equal weight. There are 5 subproblems in this examination.

I. Design the specified automaton for every one of the following languages.

1. An nfa for the language $L_1 = \{xcy|\ x, y \in \{a, b\}^*$, aba is a substring of both x and y, and the last symbol of x=last symbol of $y = b\}$.

2. An npda for the language $L_2 = \{a^{i+j}b^j a^k b^\ell|\ i, j, k, \ell \geq 1,$ and $(k = i$ or $\ell = 2i$ or $k = \ell\}$.

3. A dlba for the language $L_3 = \{xcycxcy|\ x, y \in \{a, b\}^*, |x| = |y|\}$.

4. A CFG for the language L_2, which is respecified:
$L_2 = \{a^{i+j}b^j a^k b^\ell|\ i, j, k, \ell \geq 1,\ \text{and}\ (k = i\ \text{or}\ \ell = 2i\ \text{or}\ k = \ell\}$.

II. Prove that the following language is not an fa language by applying the pumping lemma.

$L_5 = \{a^i b^j \mid i, j \geq 1,\ (i \geq j) \text{ or } (i < j \text{ and } j \text{ is a multiple of } i\}$.