All the subproblems carry equal weight. There are 5 subproblems in this examination.

I. Design the specified automaton for every one of the following languages.

1. An nfa for the language \( L_1 = \{ xcy \mid x, y \in \{ a, b \}^*, \text{aba is a substring of both } x \text{ and } y, \text{ and the last symbol of } x=\text{last symbol of } y = b \} \).

2. An npda for the language \( L_2 = \{ a^{i+j} b^j a^k b^\ell \mid i, j, k, \ell \geq 1, \text{ and } (k = i \text{ or } \ell = 2i \text{ or } k = \ell) \} \).
3. A dIba for the language $L_3 = \{xcxyxcy\mid x, y \in \{a, b\}^*, |x| = |y|\}$.

4. A CFG for the language $L_2$, which is respecified:
$L_2 = \{a^i b^j a^k b^\ell \mid i, j, k, \ell \geq 1, \text{ and } (k = i \text{ or } \ell = 2i \text{ or } k = \ell)\}$. 
II. Prove that the following language is not an FA language by applying the pumping lemma.

$L_5 = \{a^i b^j | i, j \geq 1, \ (i \geq j) \text{ or } (i < j \text{ and } j \text{ is a multiple of } i)\}.$