

**600.271 Automata & Computation Theory  
Mid-Semester Examination**

**March 10, 2011**

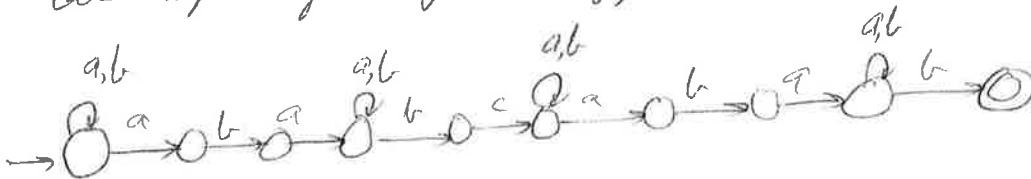
**In-class, Closed Book, Time: 1 hr, 10 mins**

All the subproblems carry equal weight. There are 5 subproblems in this examination.

I. Design the specified automaton for every one of the following languages.

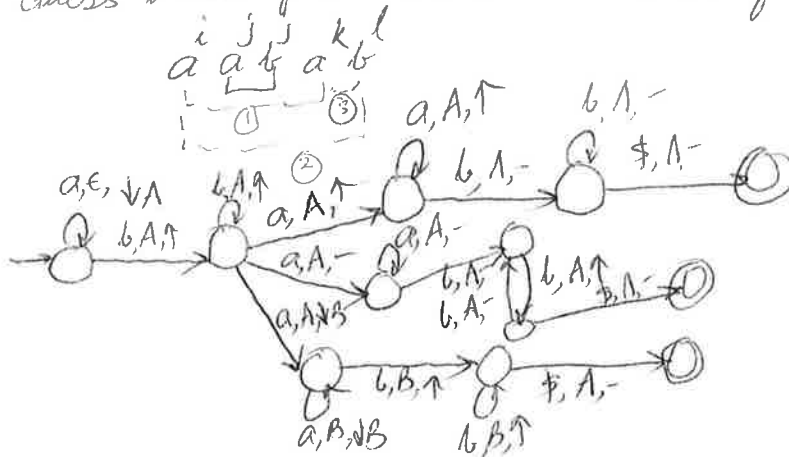
1. An nfa for the language  $L_1 = \{xycy \mid x, y \in \{a, b\}^*, \text{aba is a substring of both } x \text{ and } y, \text{ and the last symbol of } x = \text{last symbol of } y = b\}$ .

*Guess the occurrences of aba in x & y & verify. Guess the last symbols of x & y & verify.*



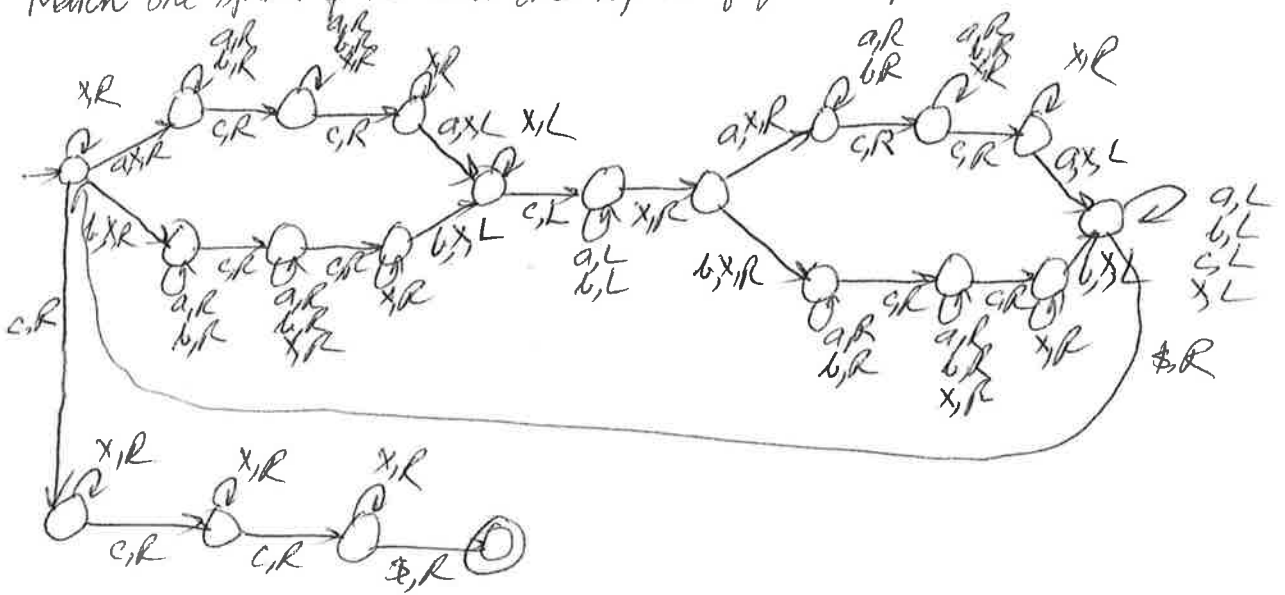
2. An npda for the language  $L_2 = \{a^{i+j}b^j a^k b^l \mid i, j, k, l \geq 1, \text{ and } (k = i \text{ or } l = 2i \text{ or } k = l)\}$ .

*Guess which of the 3 conditions is satisfied & verify.*



3. A dlba for the language  $L_3 = \{xycyxcy \mid x, y \in \{a, b\}^*, |x| = |y|\}$ .

Match one symbol of x's then one symbol of y's. Repeat



4. A CFG for the language  $L_2$ , which is specified:

$L_2 = \{a^{i+j}b^j a^k b^l \mid i, j, k, l \geq 1, \text{ and } (k = i \text{ or } l = 2i \text{ or } k = l)\}$ .

$a^i b^j a^k b^l$

$S \rightarrow X \mid Y \mid Z$

$\begin{cases} X \rightarrow xb \mid Ub \\ U \rightarrow aUa \mid aVa \\ V \rightarrow avb \mid ab \end{cases}$

$\begin{cases} Y \rightarrow aybb \mid aWbb \\ W \rightarrow wa \mid Pa \\ P \rightarrow aPb \mid ab \end{cases}$

$\begin{cases} Z \rightarrow QR \\ R \rightarrow arb \mid ab \\ Q \rightarrow aQ \mid aM \\ M \rightarrow amb \mid ab \end{cases}$

$x: k=i$   
 $y: l=2i$   
 $z: k=l$

II. Prove that the following language is not an FA language by applying the pumping lemma.

$$L_5 = \{a^i b^j \mid i, j \geq 1, (i \geq j) \text{ or } (i < j \text{ and } j \text{ is a multiple of } i)\}.$$

Let  $z = a^{3k} b^{3k}$ . Note that  $|z| = 6k \geq k$ , and  $z \in L_5$  ( $i=j$  condition is satisfied).

For any way of writing  $a^{3k} b^{3k} = uv^l w$  s.t.  $|uv| \leq k$  &  $|v| > 0$ :

since  $|uv| \leq k$ ;  $uv$  & hence  $v$  contains only  $a$ 's.

Let  $v = a^l$ ,  $1 \leq l \leq k$ .

Then  $uv^0 w = a^{3k-l} b^{3k} \notin L_5$  since  $3k$  is not a multiple of  $3k-l$ .

Hence  $L_5$  is not an FA language.