All the subproblems carry equal weight.

1. Design a nondeterministic pda for the language:
   \[\{xcycz | x, y, z \in \{a, b\}^*, y = x^R \text{ or } (z \neq y^R \text{ and } \#_ayz \text{ is an odd integer})\}\].
II. Design a CFG (i.e. type 2 grammar) for the language:
\[ \{a^i b^j a^k a^\ell \mid i, j, k, \ell \geq 1, i = k \text{ or } i + j = \ell \} \].
III. Prove that the following problem is decidable.

1. Given \([M_1]\) and \([M_2]\), \(M_1\) a dfa and \(M_2\) a dtda, is \(L(M_1)\) a finite set and \(L(M_1) \cap L(M_2) \neq \phi\)?
IV. Prove that the following set $S$ is recursively enumerable by designing an appropriate enumeration algorithm:

$S = \{[M] | \text{Turing machine } M \text{ halts on the inputs 1, 10, 100}\}$. 
V. Prove the undecidability of the following problem.

1. Given $[M_1], [M_2], [M_3]$, do there exist $x_1, x_2, x_3$ such that $x_1 < x_2 < x_3$ and for $i = 1, 2, 3$, Turing machine $M_i$ halts on input $x_i$? (Hint: Make use of the undecidability of the blank tape halting problem.)
VI. Reduce the Post correspondence problem to the following problem. Given a diba recognizer $M$ over the alphabet \{a, b\}, is $L(M) = \{x \mid x \in \{a, b\}^*, \text{ and } |x| \text{ is even}\}$?
VII. Design a $P$ algorithm for one of the following problems. Estimate its speed.

1. Given a directed graph $G$ with $n$ vertices, is there an ordering of its vertices into $v_{k_1}, v_{k_2}, \ldots, v_{k_n}$ such that for every $1 \leq i < j \leq n$ there is an edge from $v_{k_i}$ to $v_{k_j}$?

2. Given an nfa $M$ and a string $x$, is $x \in L(M)$? (Conversion of $M$ into dfa is not possible since this can take an exponential number of steps.)
VIII. In the set cover problem, given sets $S_1, S_2, \cdots, S_n$ and a positive integer $k$, we ask whether there are indices $i_1, i_2, \cdots, i_k$ such that $S_{i_1} \cup S_{i_2} \cup \cdots \cup S_{i_k} = S_1 \cup S_2 \cup \cdots \cup S_n$. We know that the set cover problem is NP-complete. Prove that the following variant of the set cover problem is NP-complete.

Let $\Omega = S_1 \cup S_2 \cup \cdots \cup S_n$. The variant has the additional requirement that for every $1 \leq i \leq n$, $|S_i| \geq \frac{|\Omega|}{2}$. 