600.271 Automata & Computation Theory
Final Examination
May 10, 2012
In-class, Closed Book
Time: 2 hrs 30 mins.

All the subproblems carry equal weight.

I. Design a nondeterministic pda for the language:
$\{xcycz \mid x, y, z \in \{a, b\}^*, y = x^R \text{ or } (z \neq y^R \text{ and } \#_a yz \text{ is an odd integer})\}$.

II. Design a CFG (i.e. type 2 grammar) for the language:
$\{a^i b^j a^k a^\ell \mid i, j, k, \ell \geq 1, i = k \text{ or } i + j = \ell\}$.

III. Prove that the following problem is decidable.

1. Given $[M_1]$ and $[M_2]$, M_1 a dfa and M_2 a dlba, is $L(M_1)$ a finite set and $L(M_1) \cap L(M_2) \neq \phi$?

IV. Prove that the following set S is recursively enumerable by designing an appropriate enumeration algorithm:

$S = \{[M] |$ Turing machine M halts on the inputs 1, 10, 100$\}$.

V. Prove the undecidability of the following problem.

1. Given $[M_1], [M_2], [M_3]$, do there exist x_1, x_2, x_3 such that $x_1 < x_2 < x_3$ and for $i = 1, 2, 3$, Turing machine M_i halts on input x_i? (Hint: Make use of the undecidability of the blank tape halting problem.)

VI. Reduce the Post correspondence problem to the following problem. Given a dlba recognizer M over the alphabet $\{a, b\}$, is $L(M) = \{x|\ x \in \{a, b\}^*,\ \text{and } |x| \text{ is even}\}$?

VII. Design a P algorithm for one of the following problems. Estimate its speed.

1. Given a directed graph G with n vertices, is there an ordering of its vertices into $v_{k_1}, v_{k_2}, \cdots, v_{k_n}$ such that for every $1 \leq i < j \leq n$ there is an edge from v_{k_i} to v_{k_j}?

2. Given an nfa M and a string x, is $x \in L(M)$? (Conversion of M into dfa is not possible since this can take an exponential number of steps.)

VIII. In the set cover problem, given sets $S_1, S_2, \cdots, S_n$ and a positive integer k, we ask whether there are indices $i_1, i_2, \cdots, i_k$ such that $S_{i_1} \cup S_{i_2} \cup \cdots \cup S_{i_k} = S_1 \cup S_2 \cup \cdots \cup S_n$. We know that the set cover problem is NP-complete. Prove that the following variant of the set cover problem is NP-complete.

Let $\Omega = S_1 \cup S_2 \cup \cdots \cup S_n$. The variant has the additional requirement that for every $1 \leq i \leq n$, $|S_i| \geq \frac{|\Omega|}{2}$.