All the problems carry equal weight.

I. Design an NFA for the language:
   \( \{ x abb y | x, y \in \{a, b\}^*, \#_a x \pmod{2} = \#_a y \pmod{2}, \text{ and } \#_b x \pmod{2} \neq \#_b y \pmod{2} \} \).

II. Design a CFG (i.e. type 2 grammar) for the language:
   \( \{ a^i b^j c^k | i, j, k \geq 1, (i = k \text{ and } i \pmod{2} = j \pmod{2}) \text{ or } (i = j \text{ and } k \text{ is odd}) \} \).
III. Prove that the following problem is computable. 

\[ f(x, y) = \begin{cases} 
1 & \text{if there exist TMs } M_1, M_2 \text{ s.t. } x = [M_1], \ y = [M_2], \text{ and } L(M_1) \cap L(M_2) \neq \emptyset \\
\text{undefined} & \text{otherwise} 
\end{cases} \]

IV. Given TMs \( M_1, M_2, \ldots, M_n \), prove that the following set is recursively enumerable: 
\[ \{ x \mid \text{every } M_i \text{ halts on input } x \}. \]
V. Prove that the following problem is undecidable:
Given dlbas $M_1$ and $M_2$, test $L(M_1) = L(M_2) \neq \phi$?

VI. Prove that the following problem is undecidable.
Given a TM $M$, does there exist a value $n$ s.t. $M$ halts on any input of length no more than $n$, and doesn’t halt on any input of length greater than $n$?
VII. Design a $P$ algorithm for the following problem. Also, estimate the speed of the algorithm.

Given a digraph $G$, a vertex $u$, and a value $k$, are there $k$ distinct vertices $v_1, v_2, \cdots, v_k$ s.t. for every $i \in \{1, \cdots, k\}$, there is a path from $u$ to $v_i$ and there is a path from $v_i$ to $u$?

VIII. Design an NP algorithm for the following problem. Prove its correctness and estimate its speed.

Given $n$ digraphs, $G_1, G_2, \cdots, G_n$, each having $n$ vertices, does there exist a $k \geq n/2$ s.t. at least $n/2$ of the digraphs contain a simple cycle of length $k$?
IX. Design an NP algorithm for the following problem. Prove its correctness and estimate its speed.
Given 2 acyclic nfas $M_1$ and $M_2$, is $L(M_1) \neq L(M_2)$?

X. In an undirected graph, a set of vertices is independent if there is no edge between every pair of vertices in the set. Prove that the following problem is NP-complete: Given $G$ and $k$, does $G$ contain an independent set of size $k$? (Hint: Vertex cover problem and Clique problem are known to be NP-complete)