600.271 Automata & Computation Theory
Final Examination
June 26, 2009
In-class, Closed Book
Time: 2 hrs

All the problems carry equal weight.

I. Design an nfa for the language:
$\{xabby \mid x, y \in \{a, b\}^*,\ \#_a x \pmod 2 = \#_a y \pmod 2, \text{ and } \#_b x \pmod 2 \neq \#_b y \pmod 2\}$.

II. Design a CFG (i.e. type 2 grammar) for the language:
$\{a^i b^j c^k \mid i, j, k \geq 1, (i = k \text{ and } i \pmod 2 = j \pmod 2) \text{ or } (i = j \text{ and } k \text{ is odd}))\}$.

III. Prove that the following problem is computable.

$$f(x,y) = \begin{cases} 1 & \text{if there exist TMs } M_1, M_2 \text{ s.t. } x = [M_1],\ y = [M_2], \text{ and } L(M_1) \cap L(M_2) \neq \phi \\ undefined & \text{otherwise} \end{cases}$$

IV. Given TMs $M_1, M_2, \cdots, M_n$, prove that the following set is recursively enumerable:
$\{x|$ every M_i halts on input $x\}$.

V. Prove that the following problem is undecidable:

Given dlbas M_1 and M_2, test $L(M_1) = L(M_2) \neq \phi$?

VI. Prove that the following problem is undecidable.

Given a TM M, does there exist a value n s.t. M halts on any input of length no more than n, and doesn't halt on any input of length greater than n?

VII. Design a P algorithm for the following problem. Also, estimate the speed of the algorithm.

Given a digraph G, a vertex u, and a value k, are there k distinct vertices $v_1, v_2, \cdots, v_k$ s.t. for every $i \in \{1, \cdots, k\}$, there is a path from u to v_i and there is a path from v_i to u?

VIII. Design an NP algorithm for the following problem. Prove its correctness and estimate its speed.

Given n digraphs, $G_1, G_2, \cdots, G_n$, each having n vertices, does there exist a $k \geq n/2$ s.t. at least $n/2$ of the digraphs contain a simple cycle of length k?

IX. Design an NP algorithm for the following problem. Prove its correctness and estimate its speed. Given 2 acyclic nfas M_1 and M_2, is $L(M_1) \neq L(M_2)$?

X. In an undirected graph, a set of vertices is *independent* if there is no edge between every pair of vertices in the set. Prove that the following problem is NP-complete: Given G and k, does G contain an independent set of size k? (Hint: Vertex cover problem and Clique problem are known to be NP-complete)