



---

# Ray Tracing

---

Johns Hopkins Department of Computer Science  
Course 600.456: Rendering Techniques, Professor: Jonathan Cohen



# Recursive Ray Tracing

---

**Gather light from various directions by tracing rays**

**Each pixel shows light at a surface**

- trace ray from eye to surface

**Each surface illuminated by lights and other surfaces**

- trace rays from surface to other surfaces

**And so on...**



# Types of Rays

---

**Eye/pixel rays**

**Illumination/shadow rays**

**Reflection rays**

**Transmission/transparency rays**



# Eye Rays

---

**Same as in ray casting**

**Effectively determine visible surfaces**

**For perspective view, trace from eye  
through pixel**

**For orthogonal view, trace parallel rays  
through pixels in direction of projection**

**Stop at nearest intersection with surface**



# Illumination Rays

---

**From surface point towards light source**

**Intervening surfaces may block or attenuate direct illumination**

**Light reaching surface applied using local illumination model**



# Reflection Rays

---

**Gather non-local illumination reflecting (specularly) towards eye (incident ray)**

**From current point towards reflection direction**

**Contribute illumination from closest surface intersection**

**Assume perfect (sharp) specular reflection**

**Attenuated by specular coefficient**



# Transmission Rays

---

**Gather light transmitted through current surface**

**Incident ray refracted according to index of refraction and Snell's law**

**May use a single index of refraction (rather than per wavelength)**

**Attenuated by transmission coefficient**



# Trace Algorithm

---

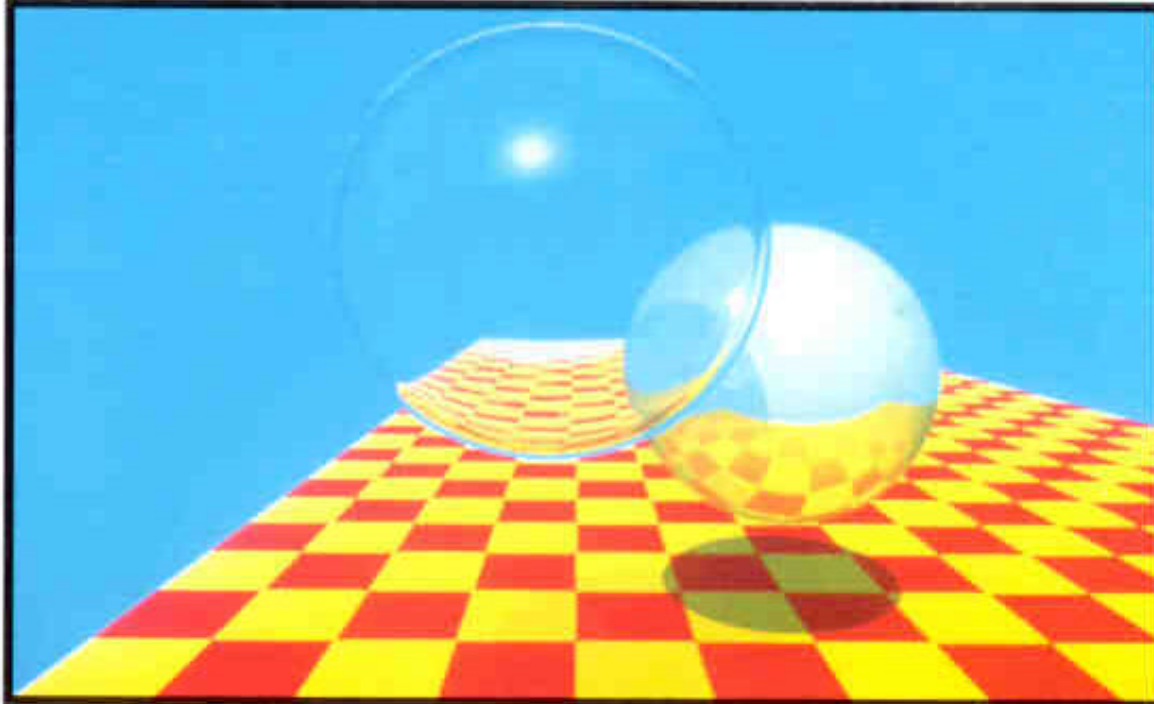
## Trace(ray)

```
Foreach object in scene
  Intersect(ray, object)
If no intersections
  return BackgroundColor
For each light
  Foreach object in scene
    Intersect(ShadowRay, object)
  Accumulate local illumination
Trace(ReflectionRay)
Trace(TransmissionRay)
Accumulate global illumination
Return illumination
```





# Spheres and Checkerboard

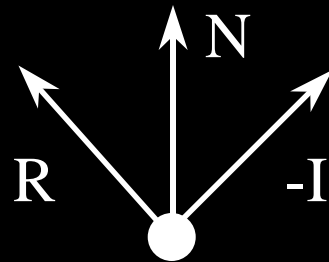


**Plate III.10** Spheres and checkerboard. An early image produced with recursive ray tracing (Section 16.12). (Courtesy of Turner Whitted, Bell Laboratories.)

**Turned Whitted, 1980 (Foley/vanDam III.10)**



# Computing Reflection Ray



I = incident ray  
N = normal vector  
R = reflected ray



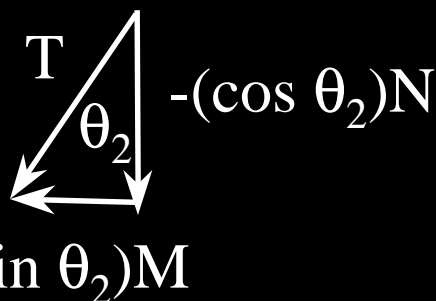
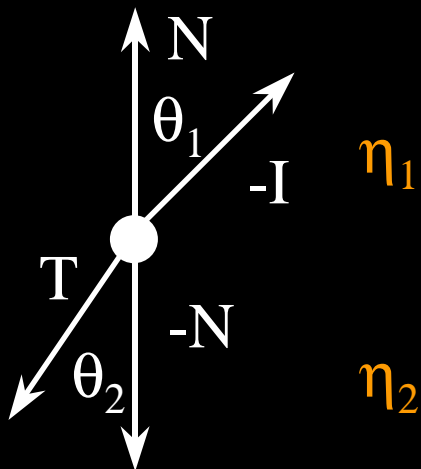
$$R = (-I.N)N + I + (-I.N)N = I - 2(I.N)N$$



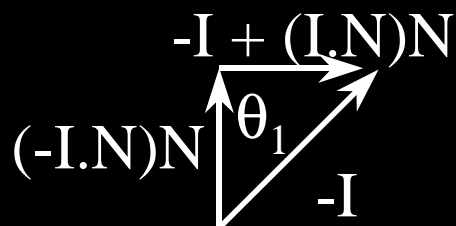
# Computing Transmission Ray

Snell's Law

$$\eta_{12} = \eta_1 / \eta_2 = \sin \theta_2 / \sin \theta_1$$



$$T = (\sin \theta_2)M - (\cos \theta_2)N$$



$$T = (\sin \theta_2 / \sin \theta_1)[I - (I.N)N] - (\cos \theta_2)N$$

$$= \eta_{12} I - [\eta_{12}(I.N) - (\cos \theta_2)]N$$

$$= \eta_{12} I - [\eta_{12}(I.N) - \text{sqrt}(1 - \sin^2 \theta_2)]N$$

$$= \eta_{12} I - [\eta_{12}(I.N) - \text{sqrt}(1 - \eta_{12}^2 \sin^2 \theta_1)]N$$

$$\sin \theta_1 = \| -I + (I.N)N \|$$

$$M = [I - (I.N)N] / \sin \theta_1$$



# Accumulating Light Contributions at a Point

---

**$I = \Sigma$  local illumination (attenuated illum. rays)**

**+ attenuated reflection ray**

**+ attenuated transmission ray**

**Illumination rays attenuated by transmission coefficients of light occluders**

**Reflection ray attenuated by specular coefficient**

**Transmission ray attenuated by trans. coefficient**

**All rays may be attenuated by  $1/r^2$**

---



# Sampling Issues

---

Currently using only a single sample for each

- Pixel
- Reflection
- Transmission
- Frame time
- Eye point

All of these can cause forms of *aliasing*



# Sampling Theory

---

**Aliasing:** a high frequency signal masquerading as a lower frequency

**Nyquist limit:** maximum frequency signal that may be adequately sampled

( $1/2$  sampling frequency)

Aliasing may occur when we take regularly-spaced samples of frequencies above the Nyquist limit

---



# Examples of Aliasing

---

**Reverse-rotating wagon wheels in films**  
**(temporal aliasing)**

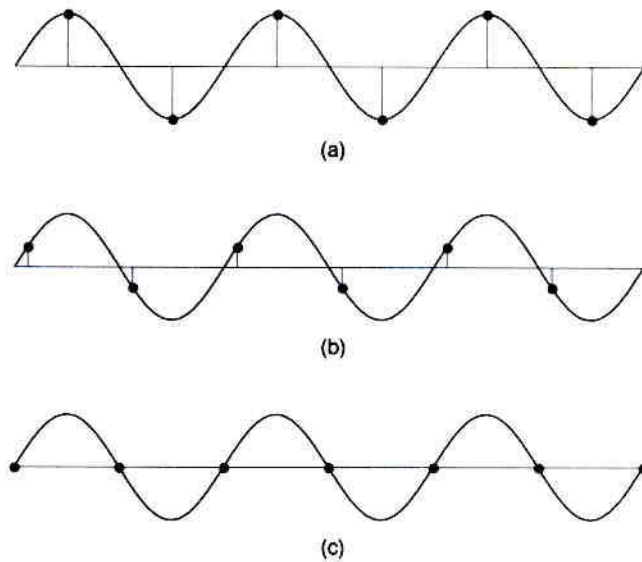
**“Jaggies” on polygon edges or specular highlights (spatial aliasing)**

**Creeping effects (jaggies in motion)**

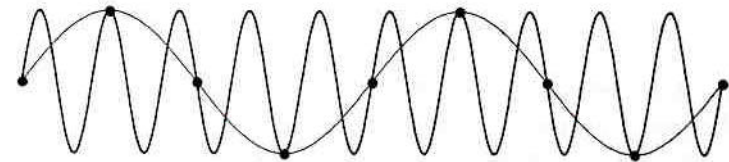
---



# Aliasing Illustration



**Fig. 14.16** Sampling at the Nyquist rate (a) at peaks, (b) between peaks, (c) at zero crossings. (Courtesy of George Wolberg, Columbia University.)



**Fig. 14.17** Sampling below the Nyquist rate. (Courtesy of George Wolberg, Columbia University.)

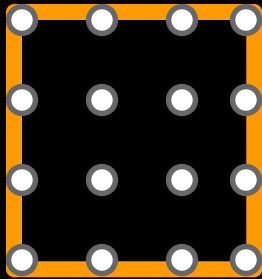
**Figures by George Wolberg, from Foley, vanDam, et al.,  
*Computer Graphics: Principle and Practice*, p. 627-628**





# Anti-aliasing - Supersampling

---



**Shoot multiple rays through each pixel**

**Aim through centers of a regular grid (e.g. 3x3 or 4x4 grid of samples)**

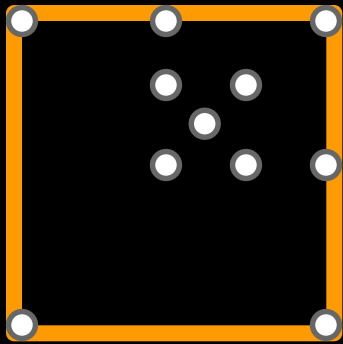
**Average resulting intensity values (box filter reconstruction)**

**Reduces spatial aliasing effects**



# Adaptive Supersampling

---



**Sample at corners and center**

**Add samples if gradient is more than some threshold**

**Add samples only in necessary regions**

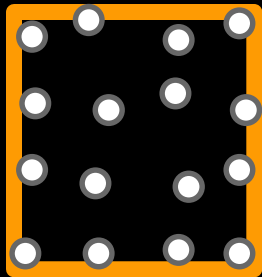
**Reduces aliasing effects more efficiently**

**Still may not sample adequately**



# Jittering

---



Apply random perturbations  
to sample positions of  
uniform or adaptive grid

Does not increase sampling  
rate, but removes regularity

Converts aliasing to *noise*,  
which is perceptually more  
tolerable



# Aliasing vs. Noise

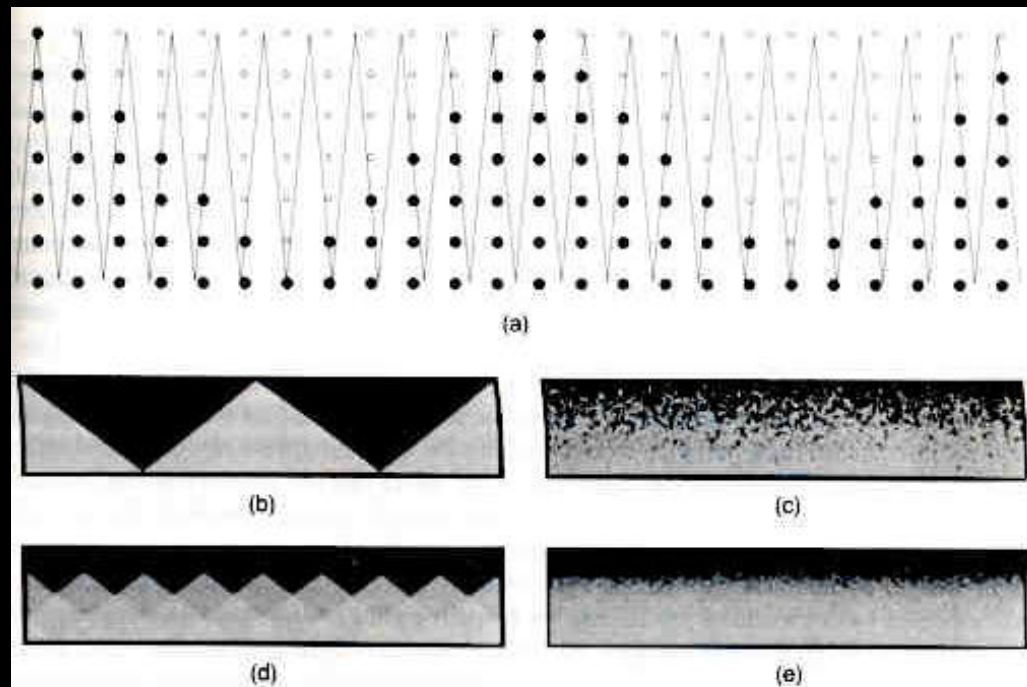


Fig. 16.62 Aliasing vs. noise. (a) A comb with regularly spaced triangles, each  $(n + 1)/n$  pixels wide, sampled with one sample per pixel.  $\circ$  = samples that fall outside comb;  $\bullet$  = samples that fall inside comb. (b) A comb with 200 triangles, each 1.01 pixels wide and 50 pixels high. 1 sample/pixel, regular grid. (c) 1 sample/pixel, jittered  $\pm \frac{1}{2}$  pixel. (d) 16 samples/pixel, regular grid. (e) 16 samples/pixel, jittered  $\pm \frac{1}{8}$  pixel. (Images (b)–(e) by Robert Cook, Lucasfilm Ltd.)

Figures by Robert Cook, from Foley, vanDam, et al.,  
*Computer Graphics: Principles and Practice*, p. 791.



# Distributed Ray Tracing

---

**Apply distribution-based sampling to many parts of the ray-tracing algorithm**

**Fuzzy reflection/transmission**

- **perturb directions reflection/transmission, with distribution based on angle from ideal ray**

**Motion blur**

- **perturb eye ray samples in time**

**Depth of field**

- **perturb eye position on lens**

**Fuzzy shadows/penumbra**

- **sample illumination rays across area light**
-



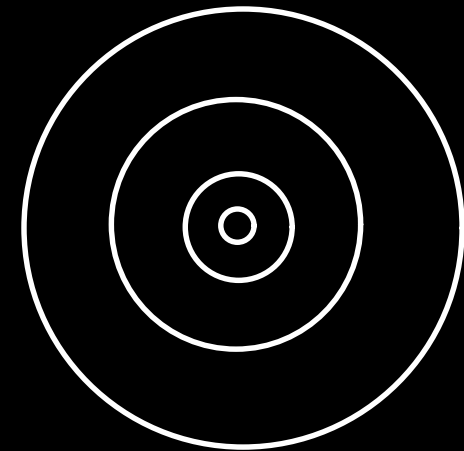
# Importance Sampling

---

**Divide sample space into blocks of equal area under weighting function**

**Assign each pixel sample a different (random) block**

**Perturb randomly within block (ideally, points in block have nearly equal weight...)**



possible block arrangement for reflection ray direction



# Motion Blur Illustration

**Plate III.16** 1984. Rendered using distributed ray tracing (Section 16.12.4) at  $4096 \times 3550$  pixels with 16 samples per pixel. Note the motion-blurred reflections and shadows with penumbrae cast by extended light sources. (By Thomas Porter. © Pixar 1984. All Rights Reserved.)



**“1984,”** by Thomas Porter, Pixar. Excerpted from Foley, vanDam, et al., *Computer Graphics: Principles and Practice*, plate III.16





# Depth of Field Illustration

---



Fig. 17. Example of depth of field from *Young Sherlock Holmes*. Copyright 1985, Paramount Pictures Corp.

**from *Young Sherlock Holmes*, Paramount Pictures Corp,  
1985. Excerpted from *An Introduction to Ray Tracing*,  
Andrew Glassner, ed., p. 195**





# Penumbras and Blurry Reflection

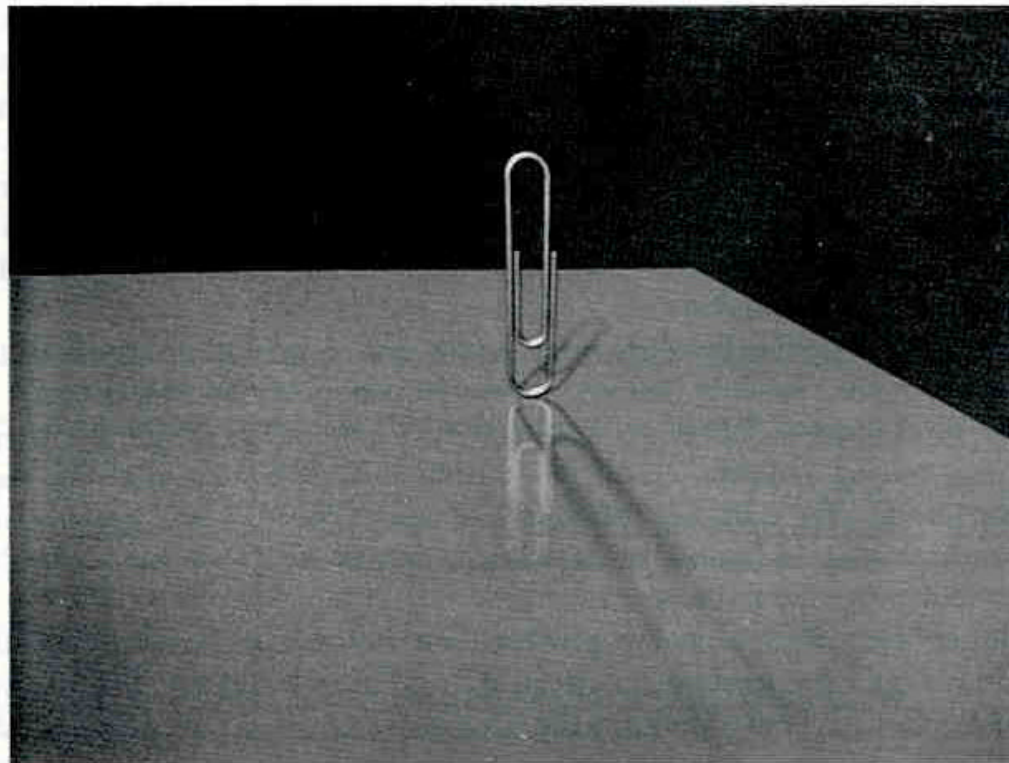


Fig. 18. Example of penumbras and blurry reflection.

**Excerpted from “Stochastic Sampling and Distributed Ray Tracing,” by Robert L. Cook, in *An Introduction to Ray Tracing*, Andrew Glassner, ed., p. 195**