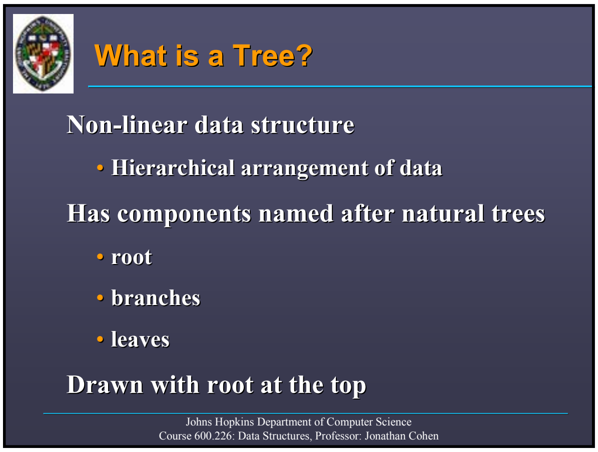
Trees



What is a Tree?

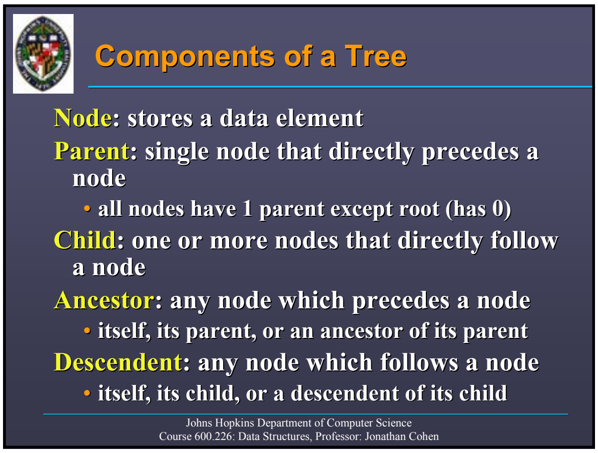
Non-linear data structure

- Hierarchical arrangement of data

Has components named after natural trees

- root
- branches
- leaves

Drawn with root at the top



Components of a Tree

Node: stores a data element

Parent: single node that directly precedes a node

- all nodes have 1 parent except root (has 0)

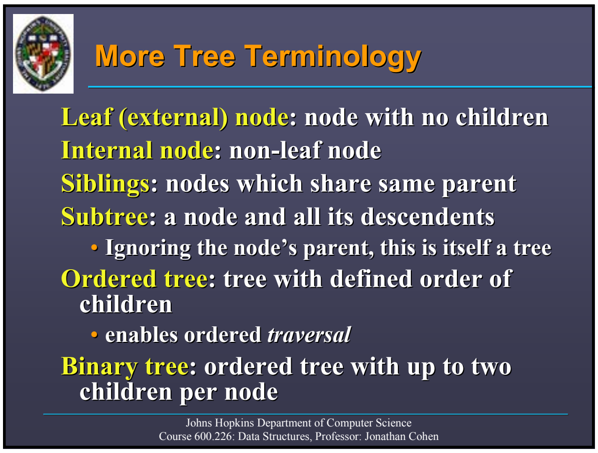
Child: one or more nodes that directly follow a node

Ancestor: any node which precedes a node

- itself, its parent, or an ancestor of its parent

Descendent: any node which follows a node

- itself, its child, or a descendent of its child



More Tree Terminology

Leaf (external) node: node with no children

Internal node: non-leaf node

Siblings: nodes which share same parent

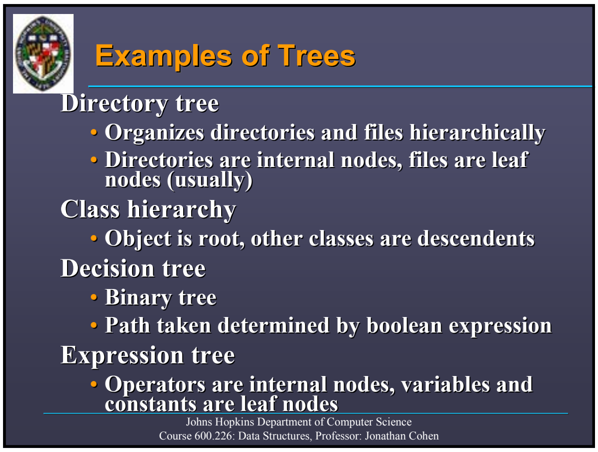
Subtree: a node and all its descendents

- Ignoring the node’s parent, this is itself a tree

Ordered tree: tree with defined order of children

- enables ordered *traversal*

Binary tree: ordered tree with up to two children per node



Examples of Trees

Directory tree

- Organizes directories and files hierarchically
- Directories are internal nodes, files are leaf nodes (usually)

Class hierarchy

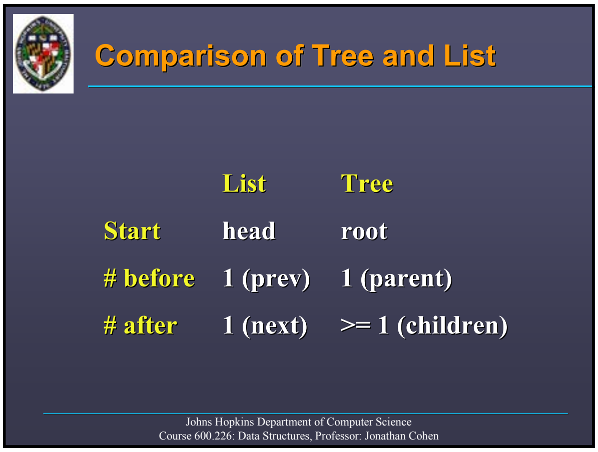
- Object is root, other classes are descendents

Decision tree

- Binary tree
- Path taken determined by boolean expression

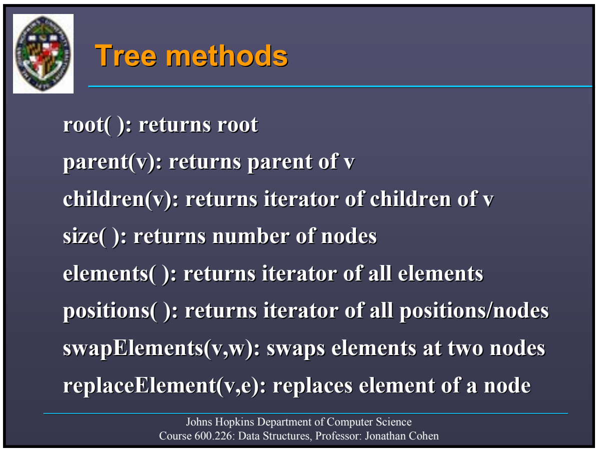
Expression tree

- Operators are internal nodes, variables and constants are leaf nodes



Comparison of Tree and List

	List	Tree
Start	head	root
# before	1 (prev)	1 (parent)
# after	1 (next)	>= 1 (children)



Tree methods

root( ): returns root

parent(v): returns parent of v

children(v): returns iterator of children of v

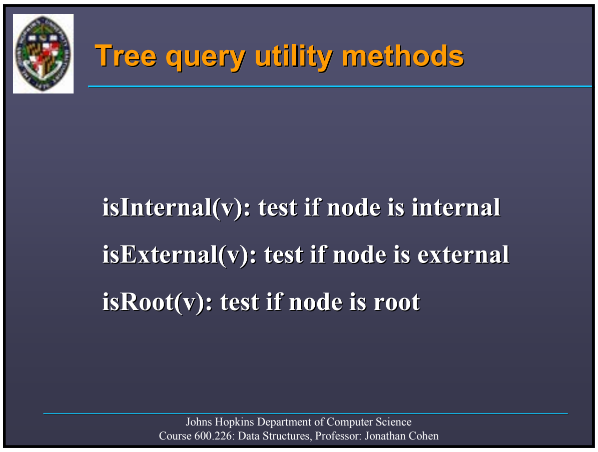
size( ): returns number of nodes

elements( ): returns iterator of all elements

positions( ): returns iterator of all positions/nodes

swapElements(v,w): swaps elements at two nodes

replaceElement(v,e): replaces element of a node

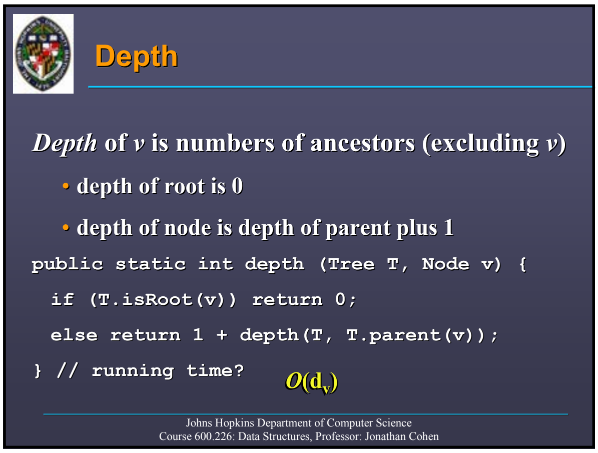


Tree query utility methods

isInternal(v): test if node is internal

isExternal(v): test if node is external

isRoot(v): test if node is root



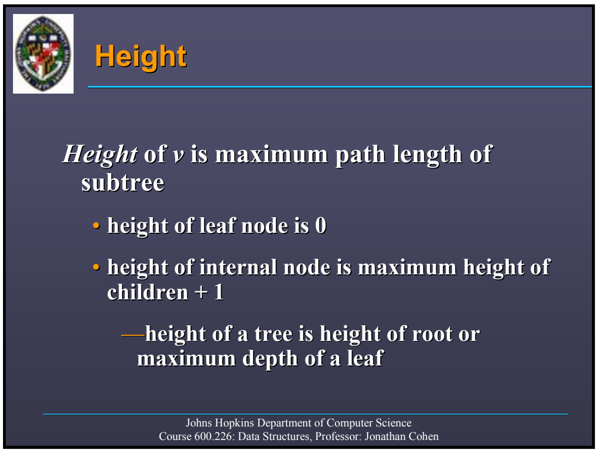
Depth

Depth of *v* is numbers of ancestors (excluding *v*)

- depth of root is 0
- depth of node is depth of parent plus 1

```
public static int depth (Tree T, Node v) {
  if (T.isRoot(v)) return 0;
  else return 1 + depth(T, T.parent(v));
} // running time?
```

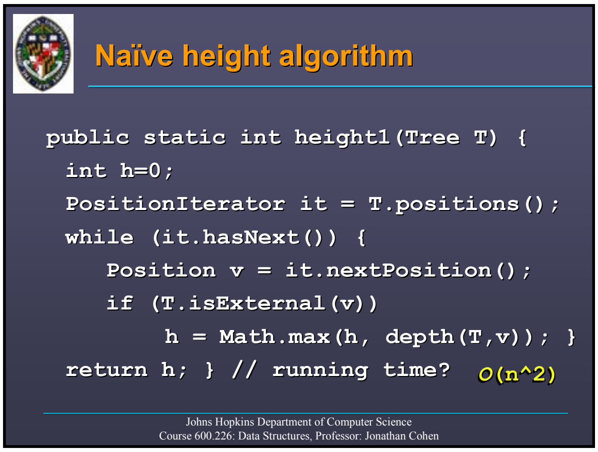
O(d_v)



Height

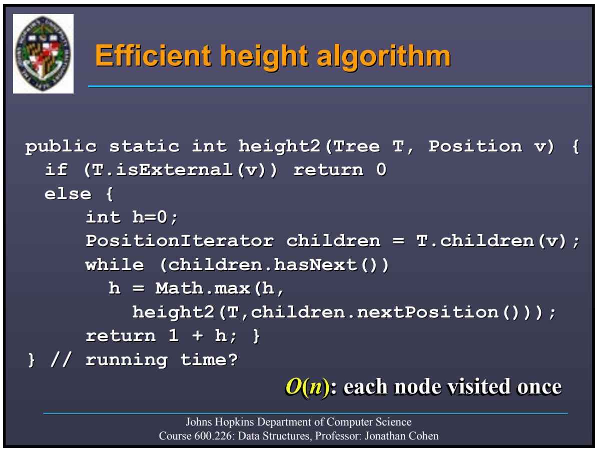
Height of *v* is maximum path length of subtree

- height of leaf node is 0
- height of internal node is maximum height of children + 1
  - height of a tree is height of root or maximum depth of a leaf



Naïve height algorithm

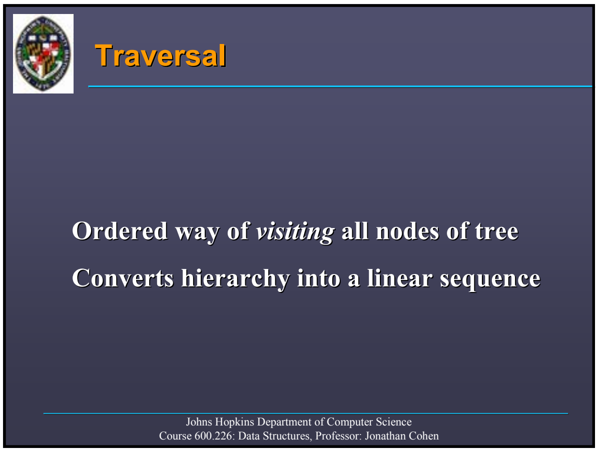
```
public static int height1(Tree T) {
  int h=0;
  PositionIterator it = T.positions();
  while (it.hasNext()) {
      Position v = it.nextPosition();
      if (T.isExternal(v))
           h = Math.max(h, depth(T,v)); }
  return h; } // running time?  O(n^2)
```



Efficient height algorithm

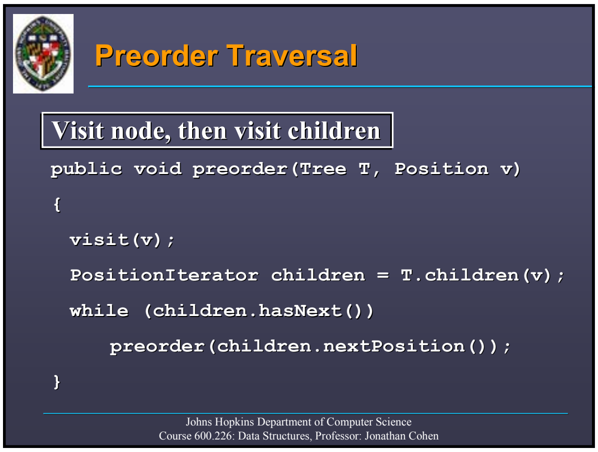
```
public static int height2(Tree T, Position v) {
  if (T.isExternal(v)) return 0
  else {
    int h=0;
    PositionIterator children = T.children(v);
    while (children.hasNext())
      h = Math.max(h,
        height2(T,children.nextPosition()));
    return 1 + h; }
} // running time?
```

O(*n*): each node visited once



Traversal

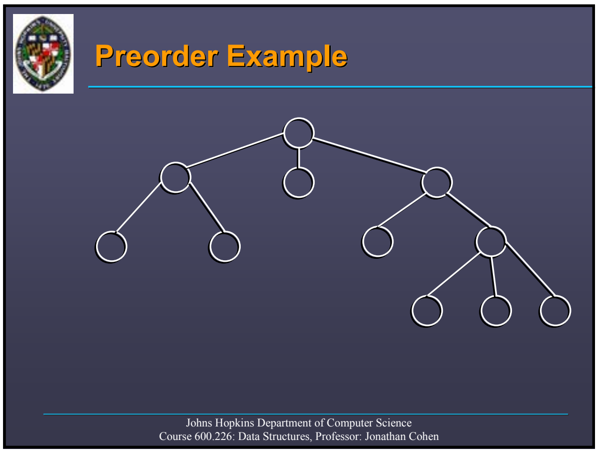
- Ordered way of *visiting* all nodes of tree
- Converts hierarchy into a linear sequence



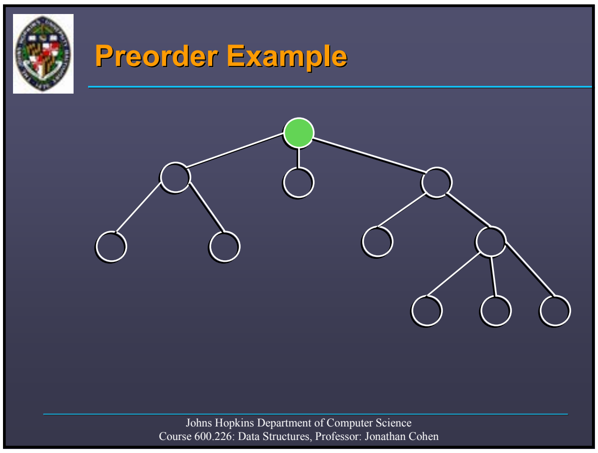
Preorder Traversal

Visit node, then visit children

```
public void preorder(Tree T, Position v)
{
  visit(v);
  PositionIterator children = T.children(v);
  while (children.hasNext())
      preorder(children.nextPosition());
}
```



Preorder Example



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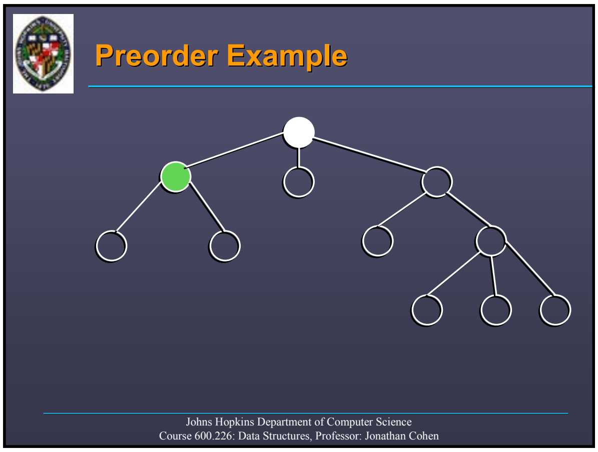

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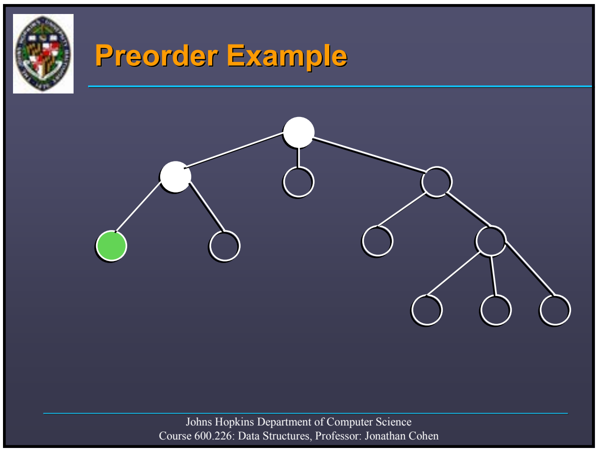

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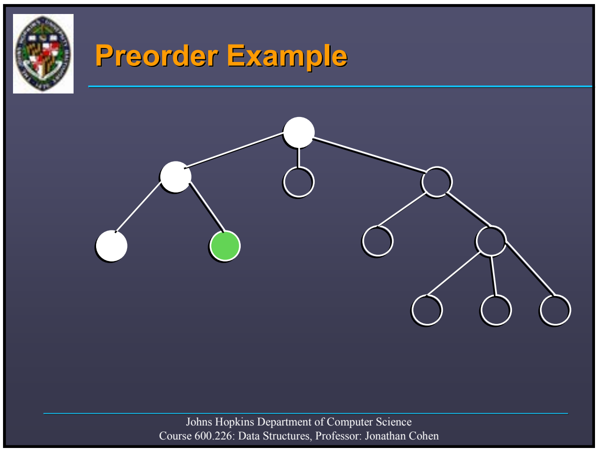

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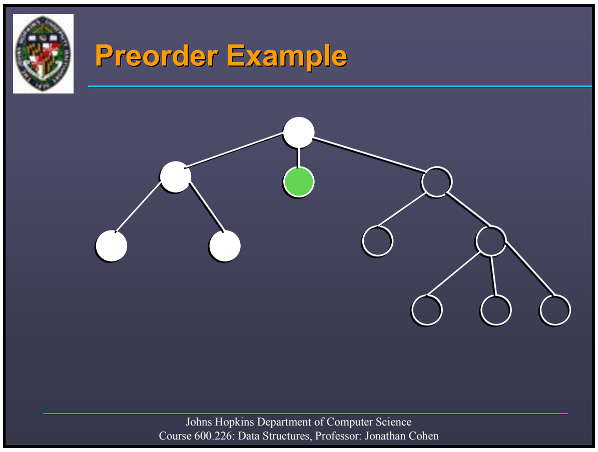

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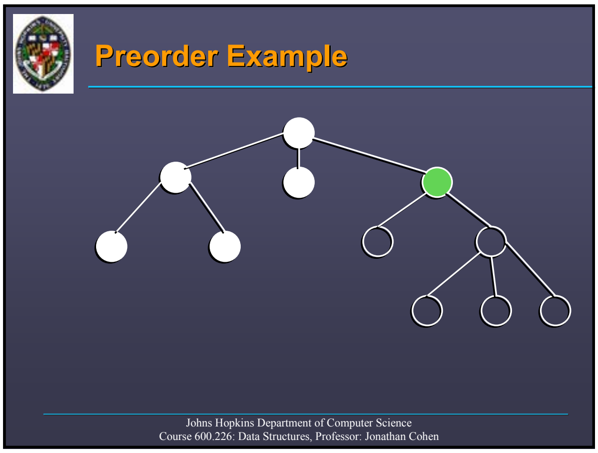

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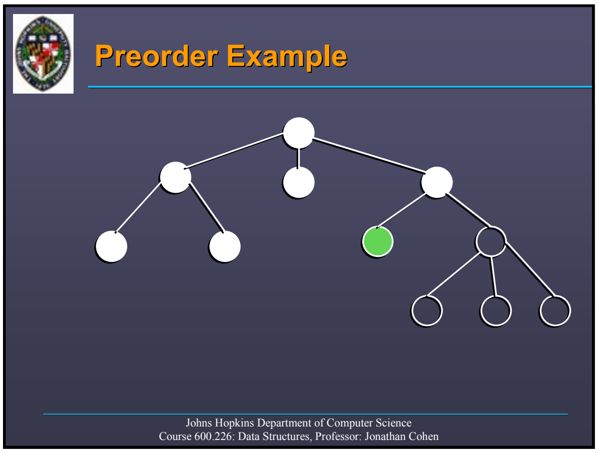

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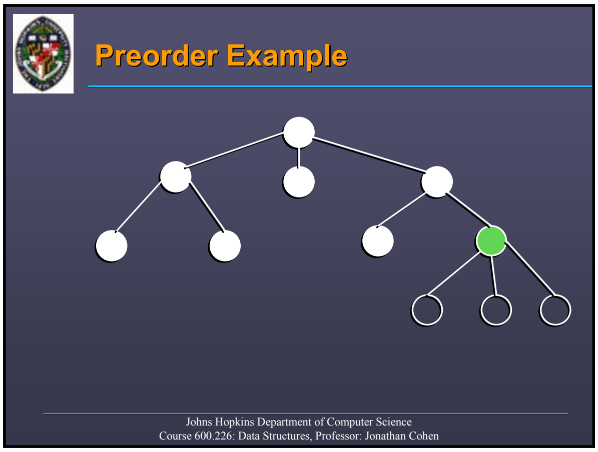

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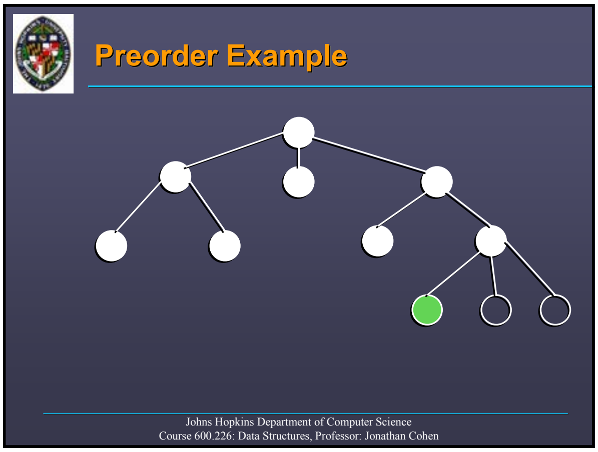
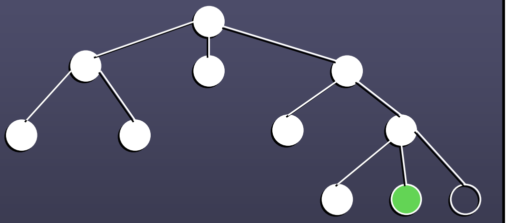


Preorder Example

Preorder Example



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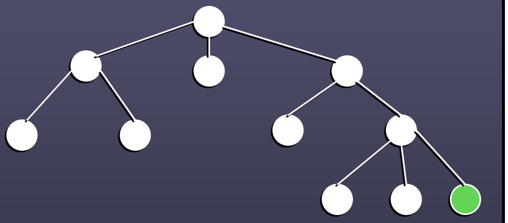

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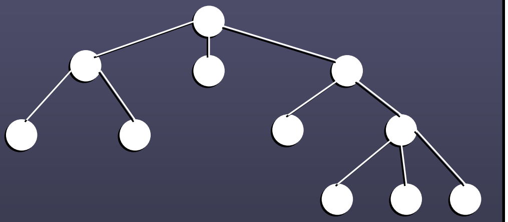
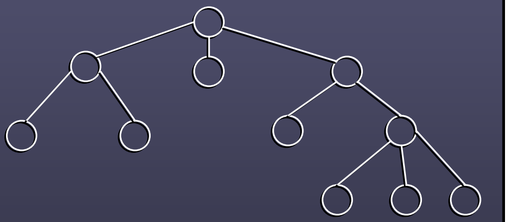


Postorder Traversal

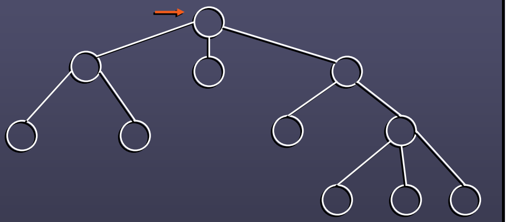
Visit children, then visit node

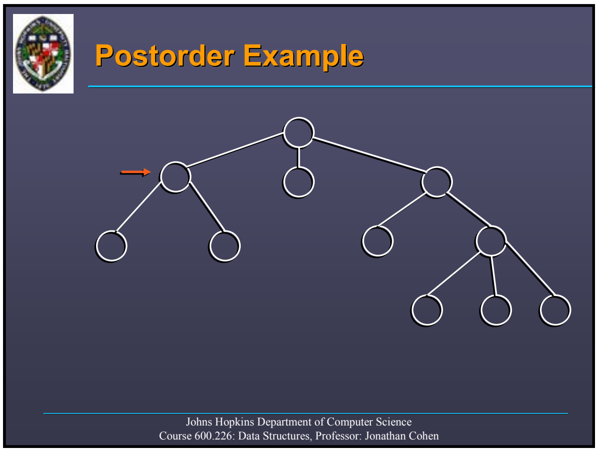
```
public void postorder(Tree T, Position v)
{
  PositionIterator children = T.children(v);
  while (children.hasNext())
      postorder(children.nextPosition());
  visit(v);
}
```

Postorder Example



Postorder Example





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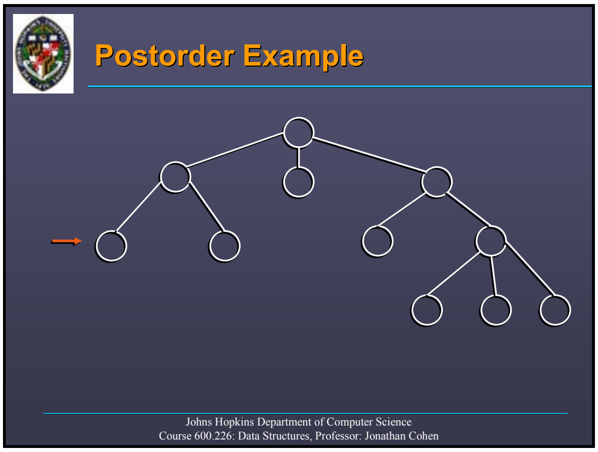

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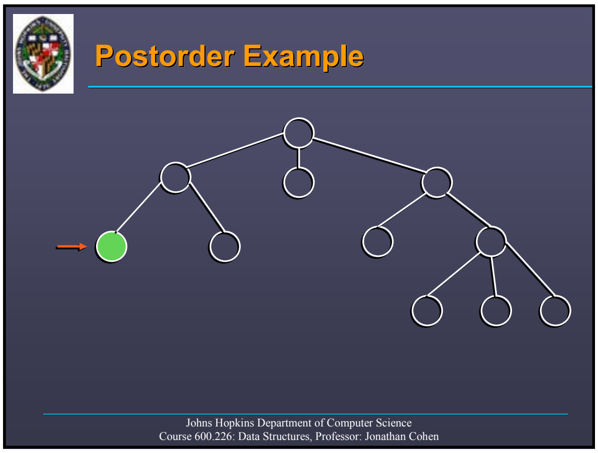

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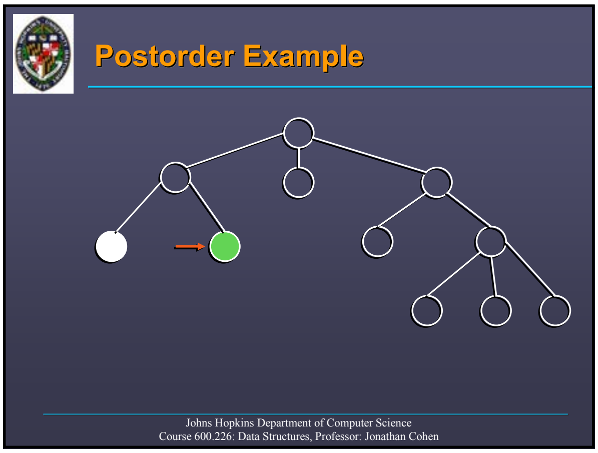

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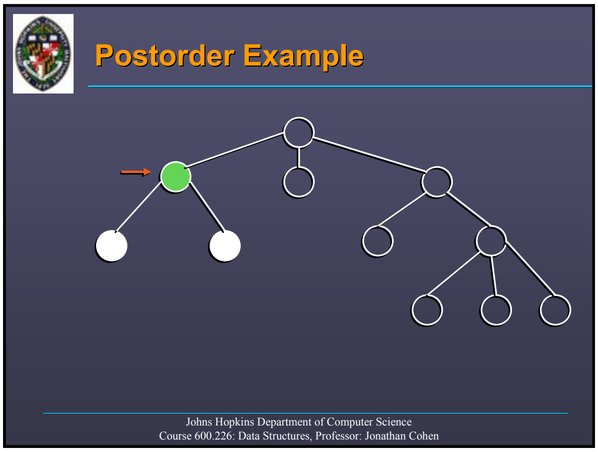

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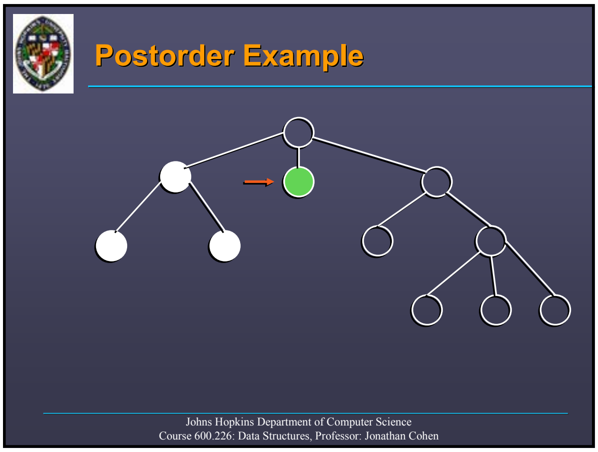

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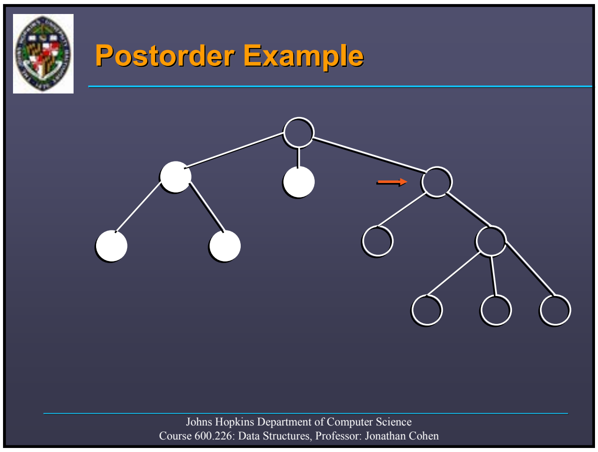

Postorder Example

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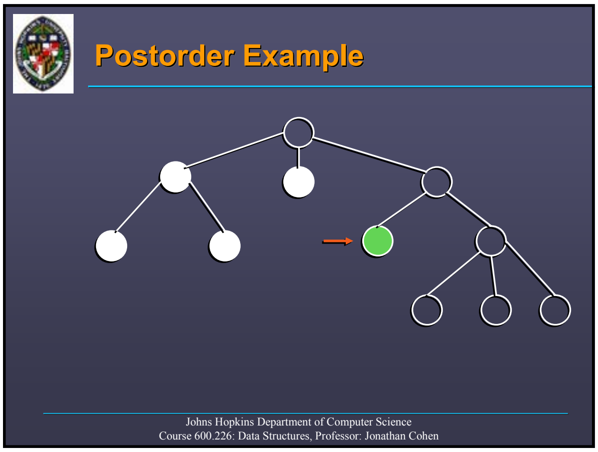
Course 600.226: Data Structures, Professor: Jonathan Cohen



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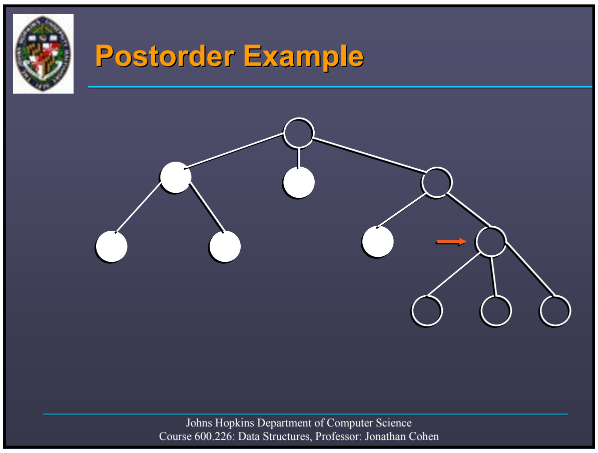

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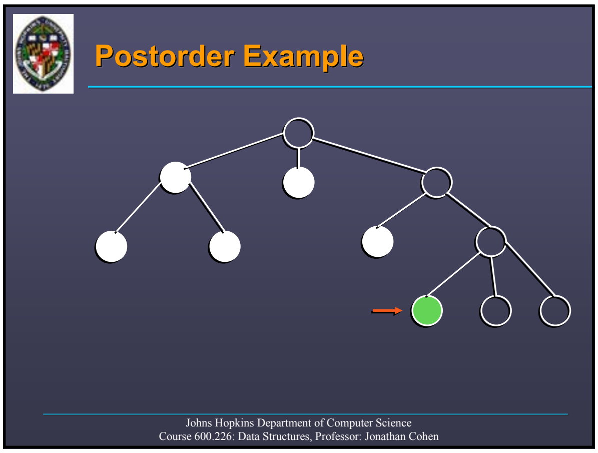

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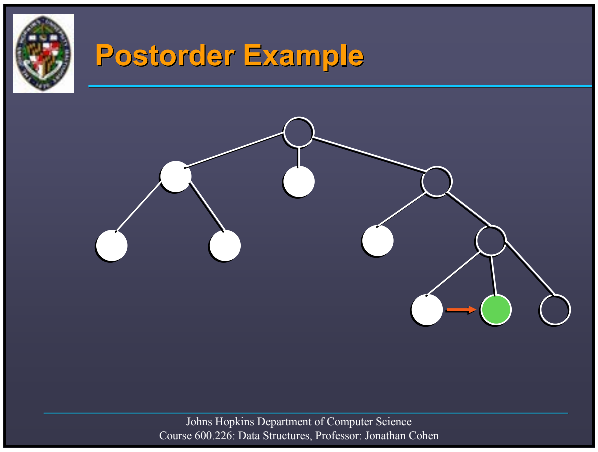

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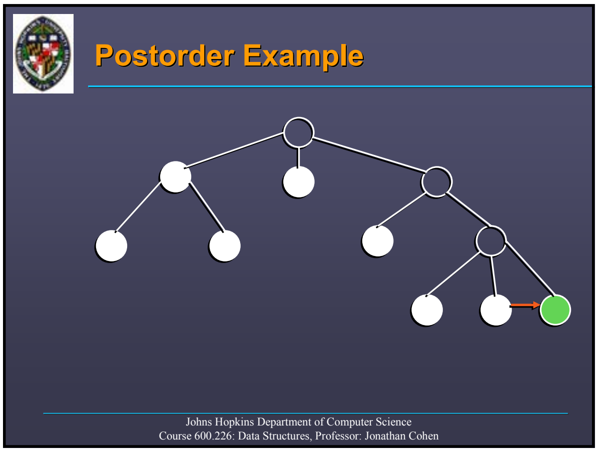

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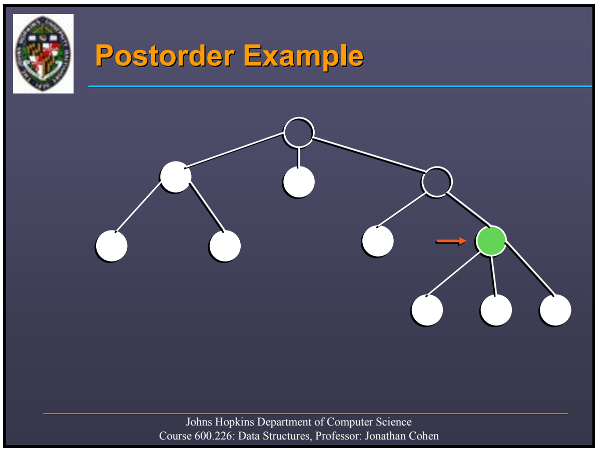

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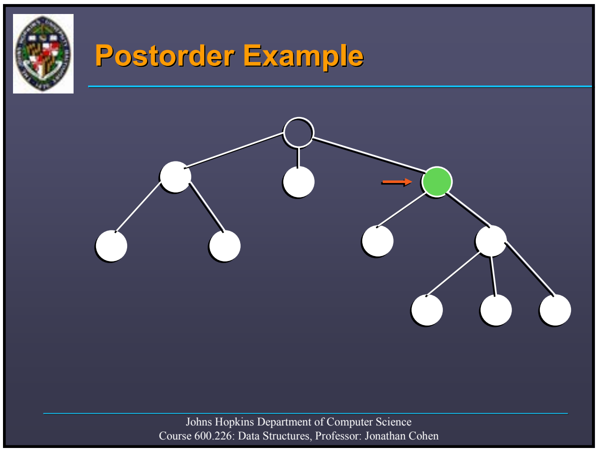

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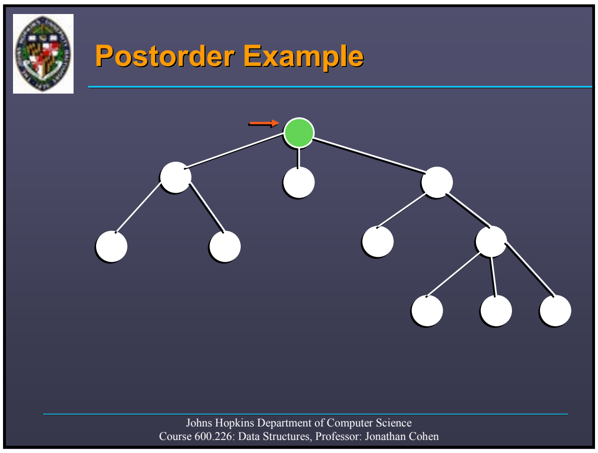

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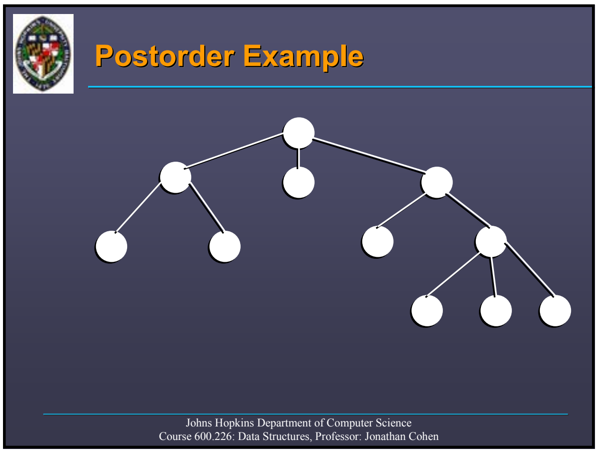

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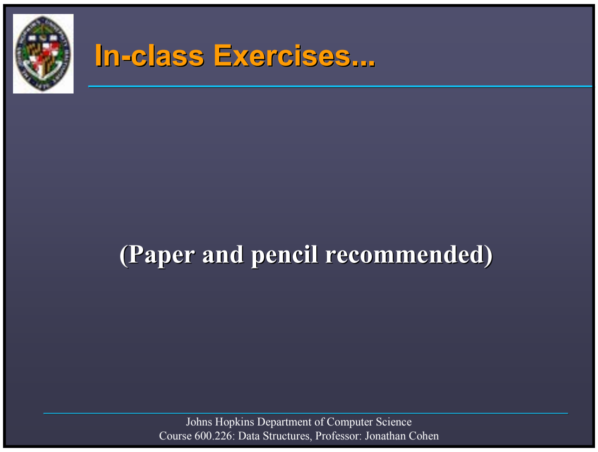
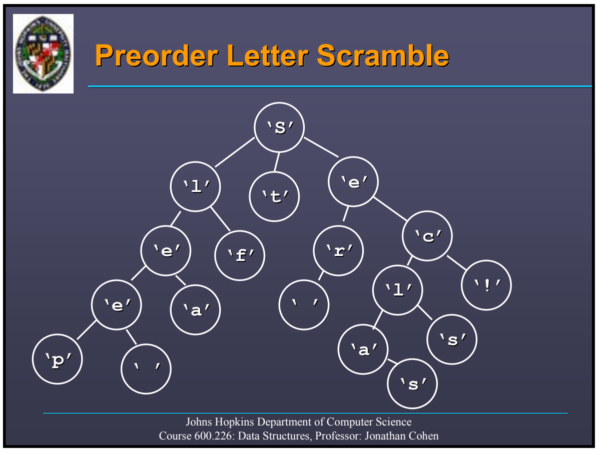


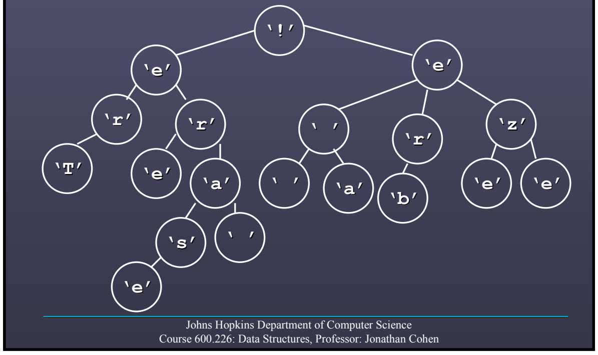
In-class Exercises…

(Paper and pencil recommended)



Preorder Letter Scramble

Postorder Letter Scramble



Binary Tree

Each node has no more than 2 children

- *Proper* binary tree: each node has either 0 or 2 children

Binary Tree ADT

leftChild(v): returns left child of v

rightChild(v): returns right child of v

sibling(v): returns sibling of v

Binary Tree Traversal

Preorder: node, left, right

Postorder: left, right, node

Inorder: left, node, right

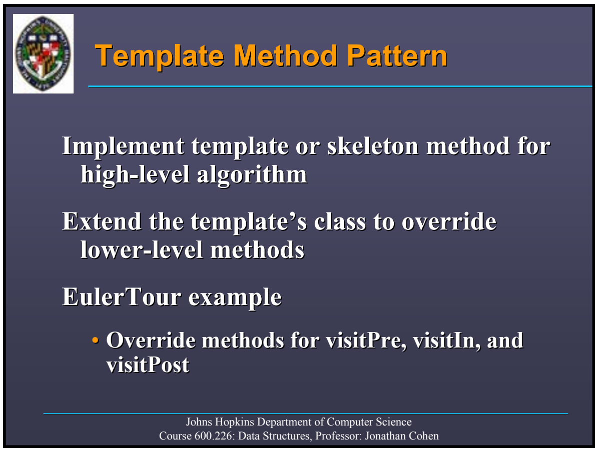
Euler Tour Traversal

Generalizes preorder, inorder, and postorder

Visit each internal node 3 times

Euler Tour

```
public void eulerTour(Tree T,
                      position v) {
 visitPre(T,v);
 if (T.hasLeft(v)
    eulerTour(T, T.leftChild(v));
 visitIn(T,v);
 if (T.hasRight(v)
    eulerTour(T, T.rightChild(v));
 visitPost(T,v); return; }
```



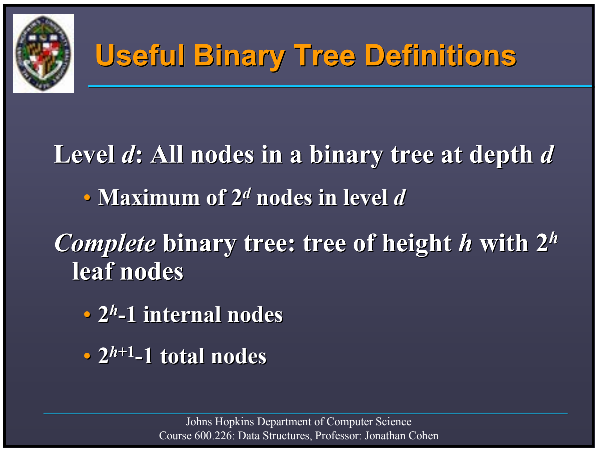
Template Method Pattern

Implement template or skeleton method for high-level algorithm

Extend the template's class to override lower-level methods

EulerTour example

- Override methods for visitPre, visitIn, and visitPost



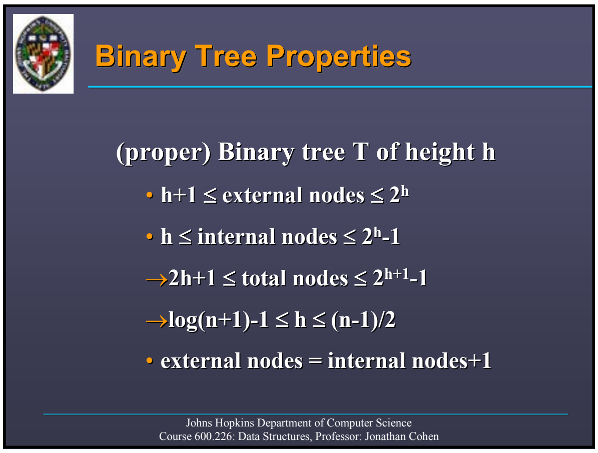
Useful Binary Tree Definitions

Level d: All nodes in a binary tree at depth d

- Maximum of 2^d nodes in level d

Complete binary tree: tree of height h with 2^h leaf nodes

- 2^h-1 internal nodes
- 2^{h+1}-1 total nodes



Binary Tree Properties

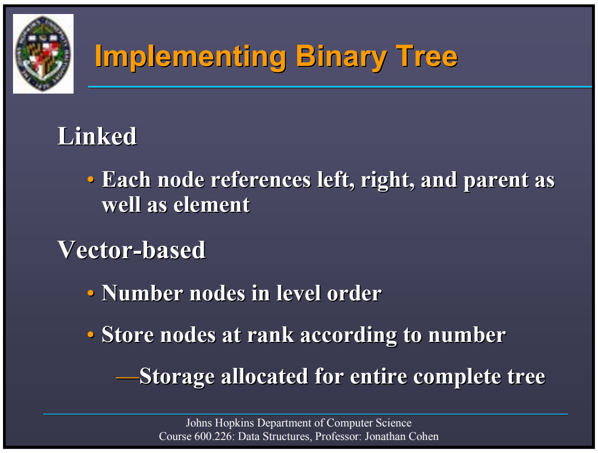
(proper) Binary tree T of height h

- $h+1 \le$ external nodes $\le 2^h$
- $h \le$ internal nodes $\le 2^h-1$

→ $2h+1 \le$ total nodes $\le 2^{h+1}-1$

→ $\log(n+1)-1 \le h \le (n-1)/2$

- external nodes = internal nodes+1



Implementing Binary Tree

Linked

- Each node references left, right, and parent as well as element

Vector-based

- Number nodes in level order
- Store nodes at rank according to number
  - —Storage allocated for entire complete tree