Lambertian model of reflectance II: harmonic analysis

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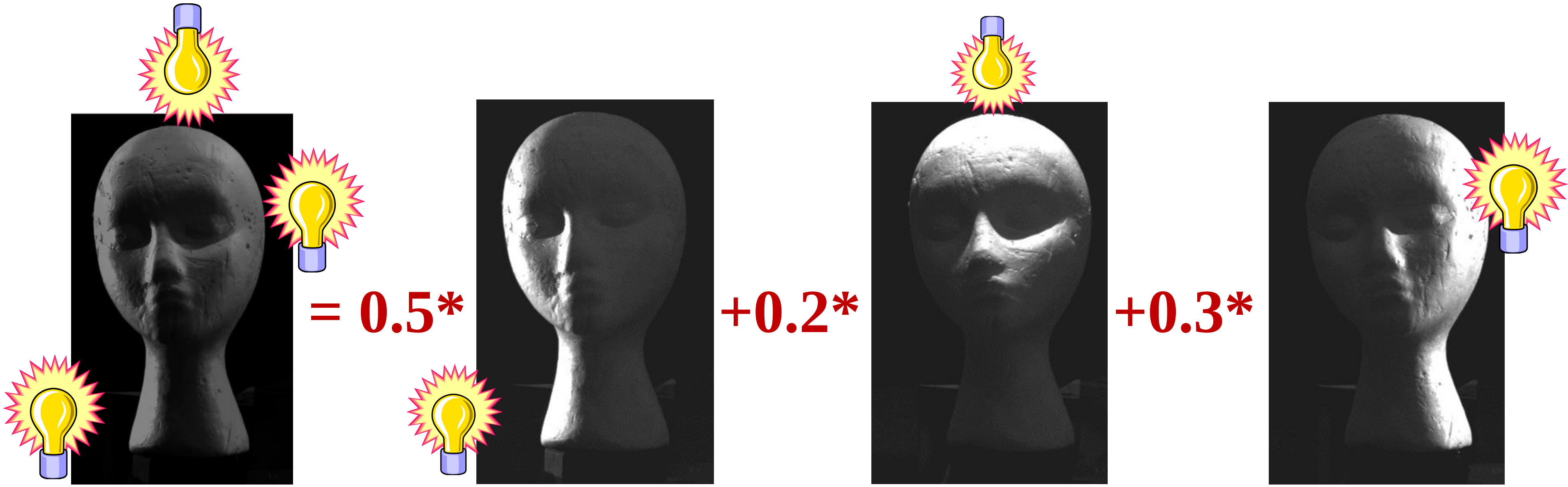
Illumination cone

- What is the set of images of an object under different lighting, with any number of sources?
- Images are additive and non-negative
- This set, therefore, forms a *convex cone* in $\mathbb{R}^p$, p number of pixels (Belhumeur & Kriegman)



Illumination cone

- Cone characterization is generic, holds also with specularities, shadows and inter-reflections
- Unfortunately, representing the cone is complicated (infinite degrees of freedom)
- Cone is "thin" for Lambertian objects; indeed the illumination cone of many objects can be represented with few PCA vectors (Yuille et al.)

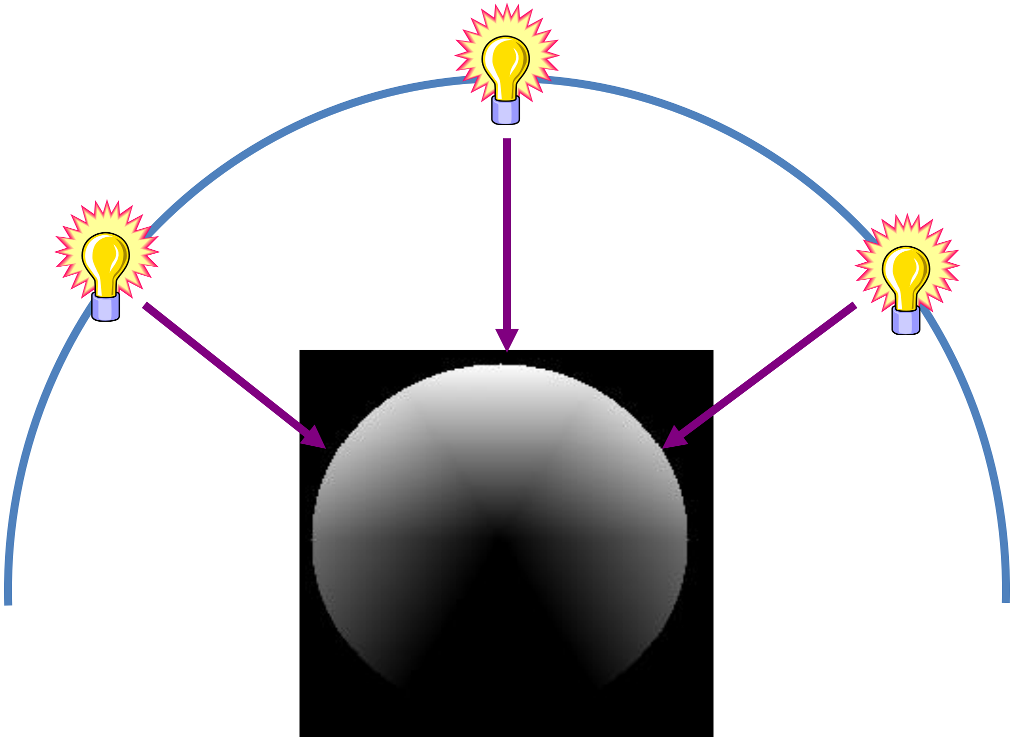
Illumination cone is often thin

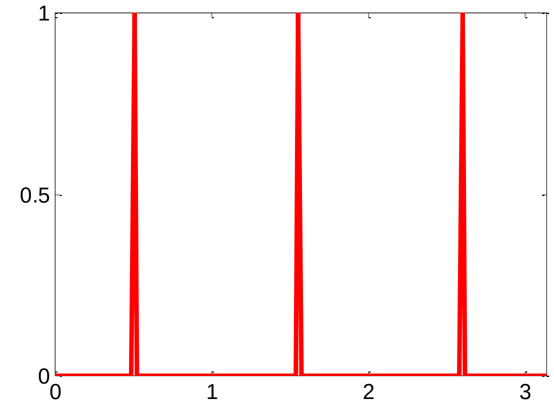
	Ball	Face	Phone	Parrot
#1	48.2	53.7	67.9	42.8
#3	94.4	90.2	88.2	76.3
#5	97.9	93.5	94.1	84.7
#7	99.1	95.3	96.3	88.5
#9	99.5	96.3	97.2	90.7

(Yuille et al.)

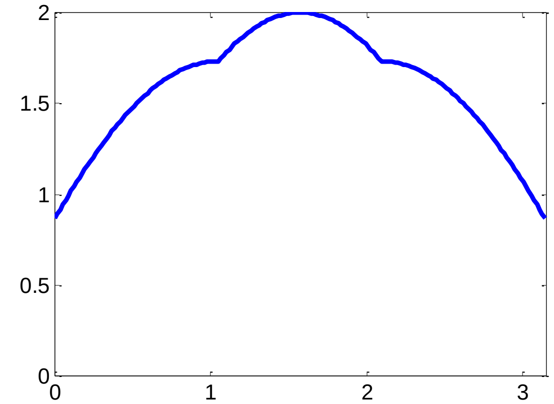
Lambertian reflectance is smooth

(Basri & Jacobs; Ramamoorthi & Hanrahan)



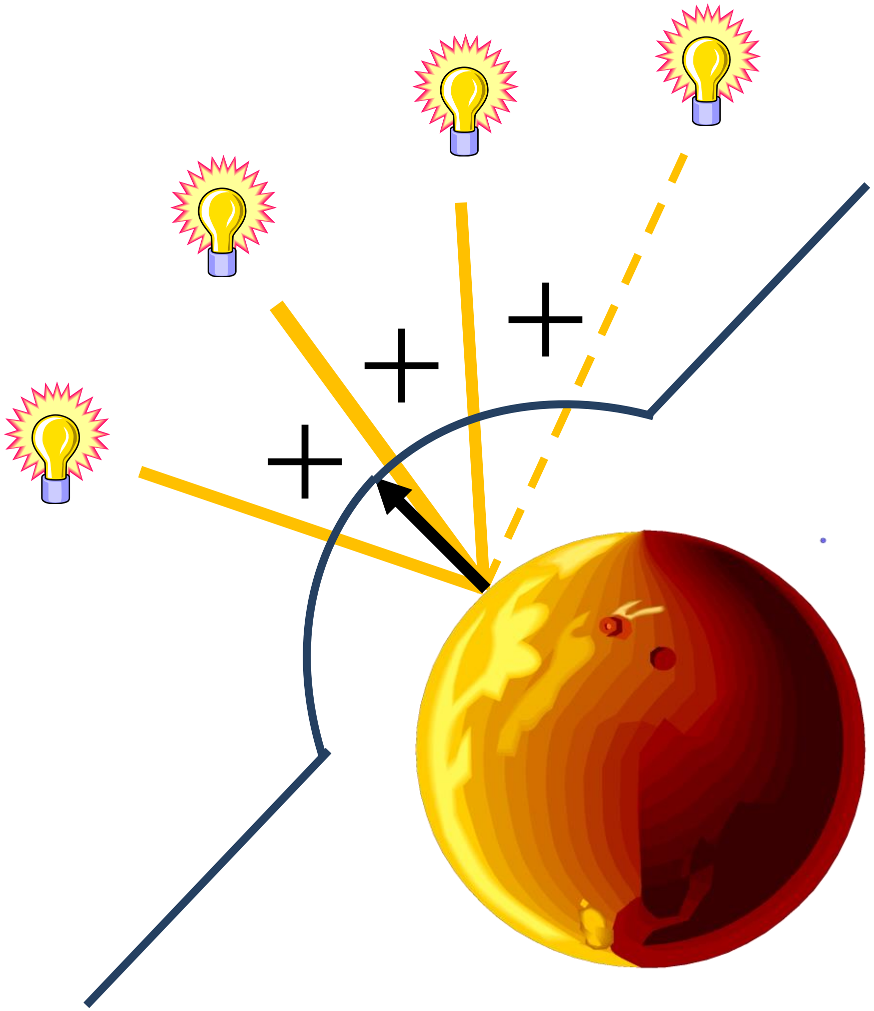


lighting



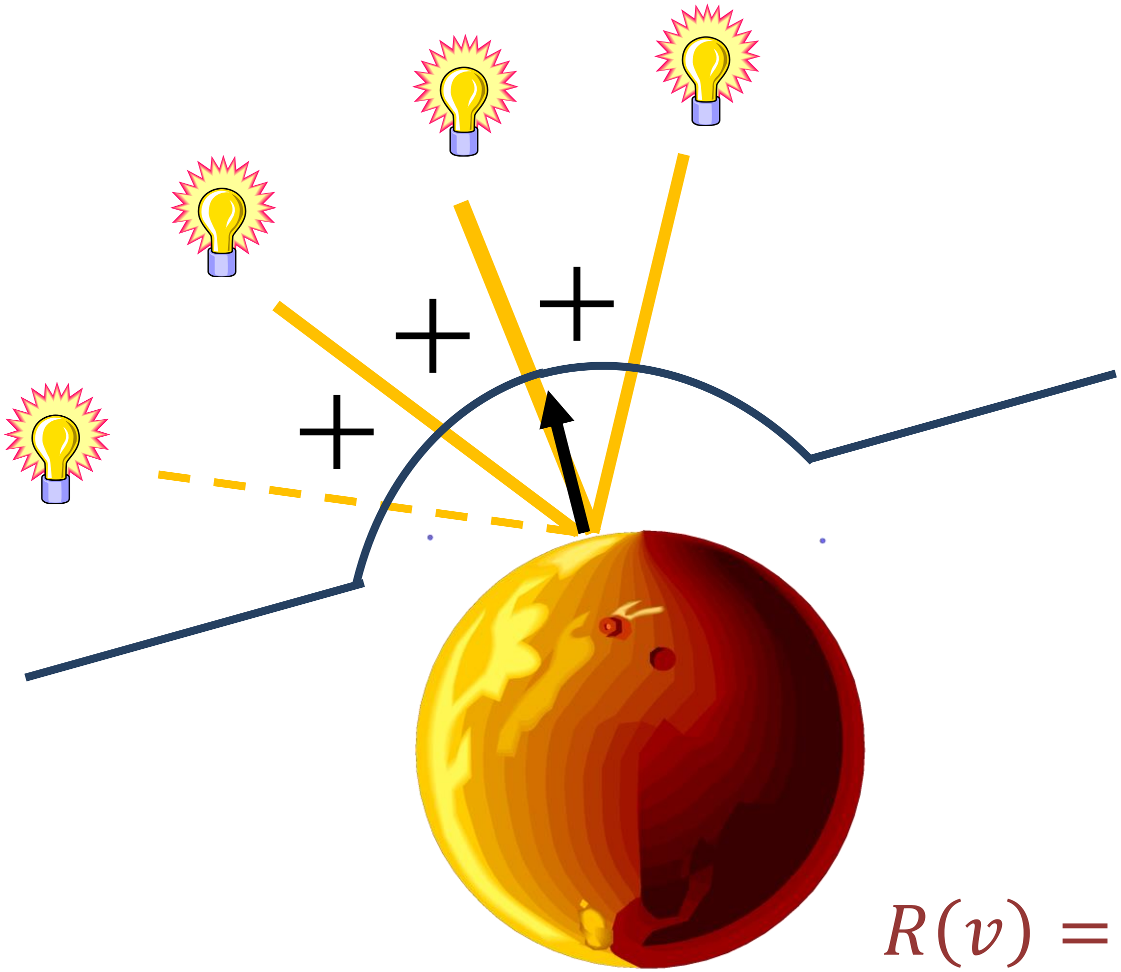
reflectance

Reflectance obtained with convolution



$$R(v) = \int_{S^2} k(u,v)l(u)du$$

Reflectance obtained with convolution



$$R(v) = \int_{S^2} k(u,v)\,l(u)\,du$$

Spherical harmonics

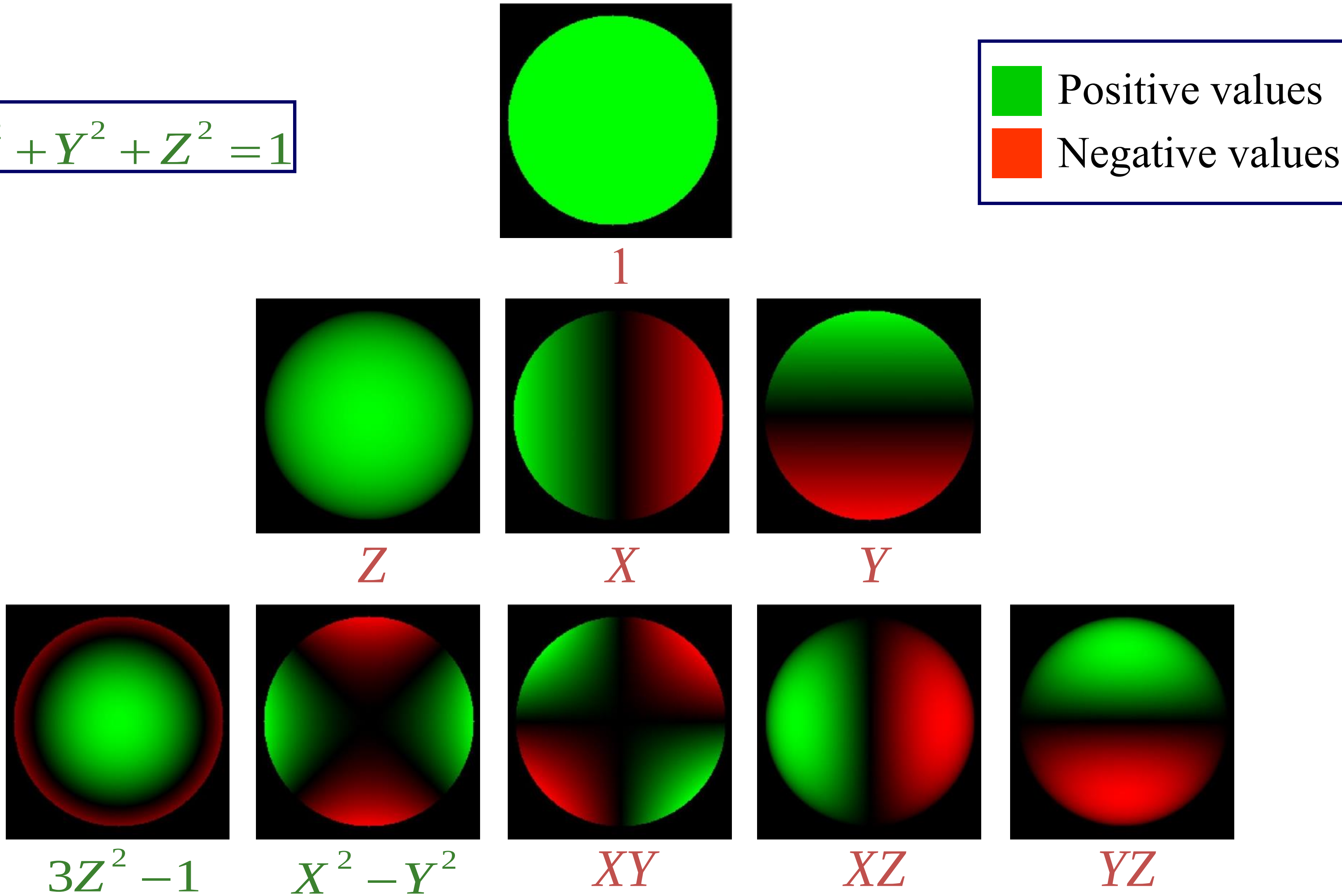
$$Y_{nm}(\theta, \emptyset) = \sqrt{\frac{(2n+1)}{4\pi} \frac{(n-|m|)!}{(n+|m|)!}} P_{nm}(\cos\theta) e^{im\emptyset}$$

$$p_{nm}(z) = \frac{(1-z^2)^{m/2}}{2^n n!} \frac{d^{n+m}}{dz^{n+m}} (z^2-1)^n$$

- Orthonormal basis for functions on the sphere
- *n*'th order harmonics have 2*n*+1 components
- Rotation = phase shift (same *n*, different *m*)
- In space coordinates: polynomials of degree *n*
- *Funk-Hecke convolution theorem*

Spherical harmonics

$$X^2 + Y^2 + Z^2 = 1$$



Harmonic approximation

- Lighting, in terms of harmonics

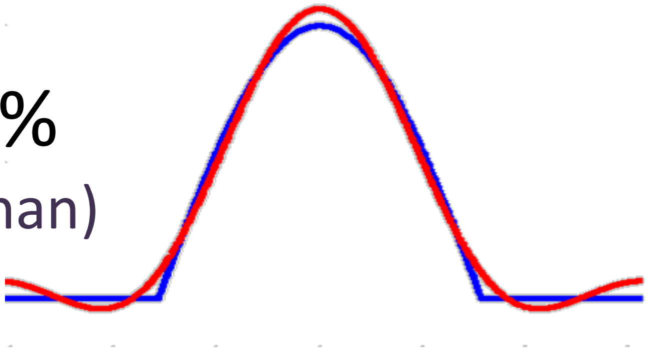
$$\ell(\theta,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} l_{nm} Y_{nm}(\theta,\phi)$$

- Reflectance

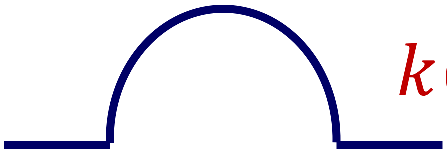
$$r(\theta,\phi) = k * \ell \approx \sum_{n=0}^{2} \sum_{m=-n}^{n} k_n l_{nm} Y_{nm}(\theta,\phi)$$

- Approximation accuracy, 99.2%

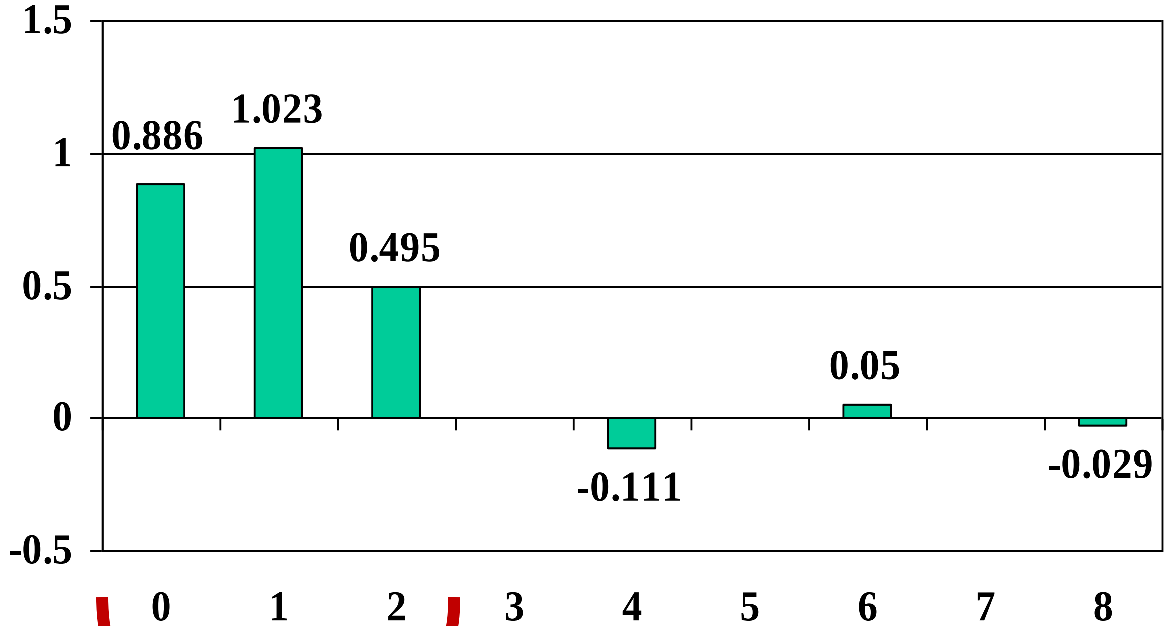
  (Basri & Jacobs; Ramamoorthi & Hanrahan)



Harmonic transform of kernel



$$k(\theta) = \max(\cos\theta, 0) = \sum_{n=0}^{\infty} k_n Y_{n0}$$



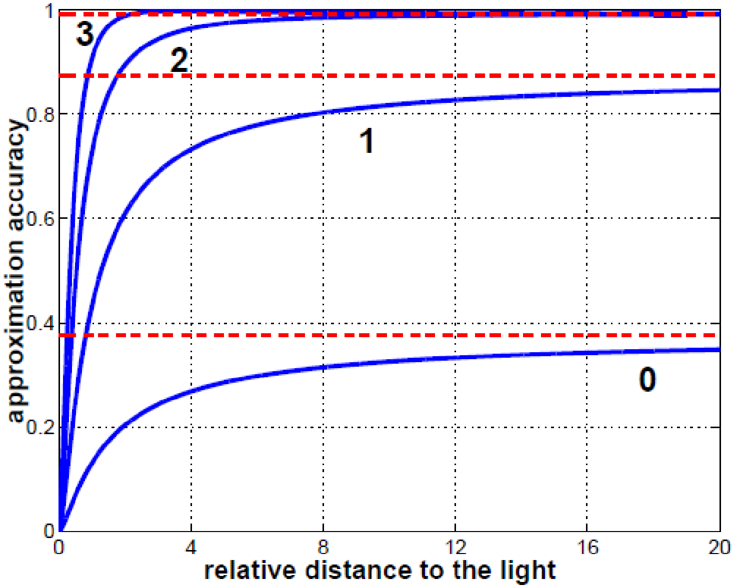
99.2%

Subspace approximation

- Up to 2nd order:
  - 9 basis images
  - Accuracy: 99.2%
- Up to 1st order:
  - 4 basis images: ambient + point source
  - Accuracy: 87.5%
- In practice, due to self occlusions ~98% can be achieved with just 6 basis images (Ramamoorthi)

Scope

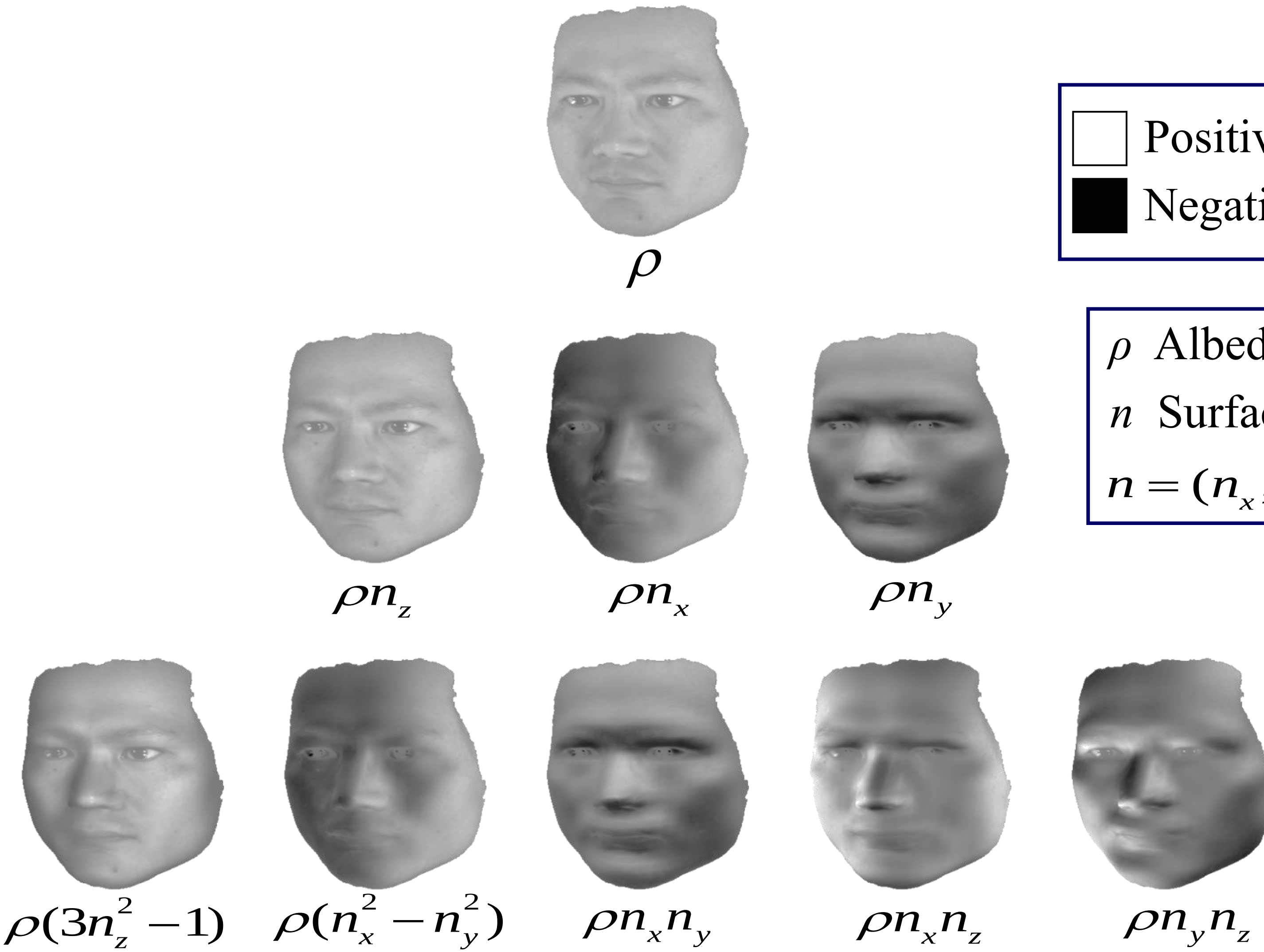
- Harmonic representations handle convex, lambertian objects with multiple light sources (including attached shadows)
- Harmonic representations do not model cast shadows and inter-reflections
- Accuracy is maintained for fairly close light sources
- Representing specular objects may require a very large basis



Applications

- We can use this theory to predict novel appearances under new lighting
- Harmonic lighting theory has led to applications in
  - Face recognition
  - Photometric stereo
  - 3D reconstruction with prior
  - Motion analysis

“Harmonic faces”



Positive values

Negative values

ρ Albedo

n Surface normal

$n = (n_x, n_y, n_z)$

Non-negative light

- We can enforce in addition that light is non-negative, by projecting the illumination cone onto the harmonic space
- Closed-form constraints for 1st order approximation
- Sampling method, or Toeplitz matrix (Shirdhonkar & Jacobs) for higher orders

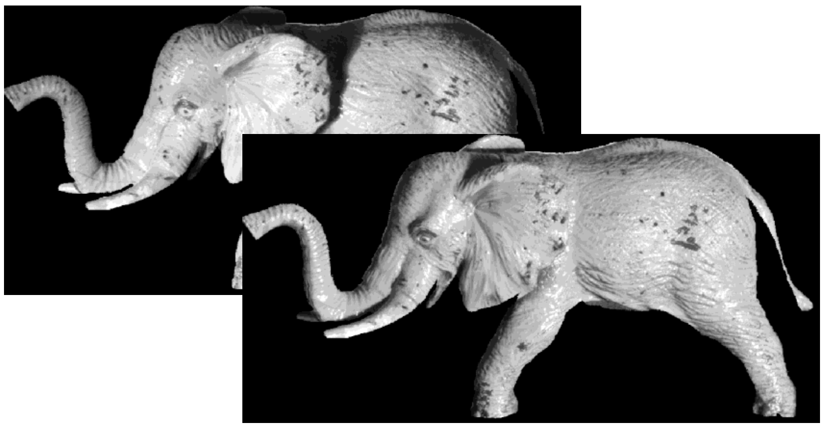
Photometric stereo

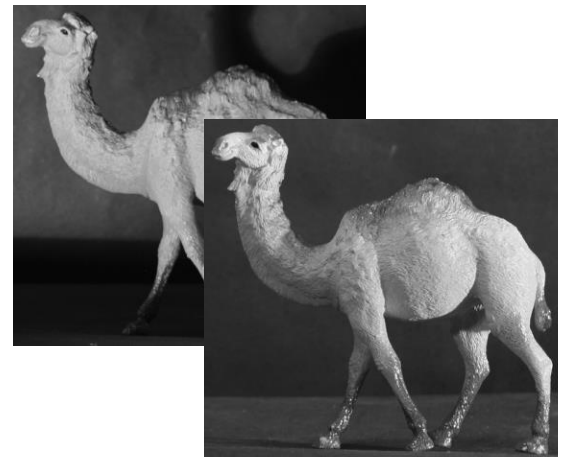
(Basri, Jacobs & Kemelmacher)

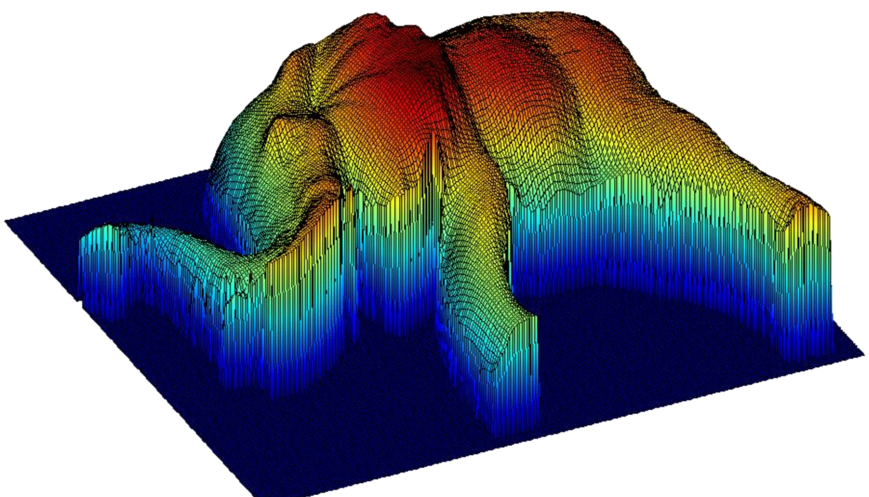
$$\underset{M}{\begin{bmatrix} \text{Image } 1 \\ \vdots \\ \text{Image } n \end{bmatrix}} \approx \underset{L}{\begin{bmatrix} \text{Light } 1 \\ \vdots \\ \text{Light } n \end{bmatrix}} \overset{S}{\begin{bmatrix} \rho \\ \rho n_z \\ \rho n_z \\ \rho n_y \\ \rho(3n_z^2-1) \\ \rho(n_x^2-n_y^2) \\ \rho n_x n_y \\ \rho n_x n_z \\ \rho n_y n_z \end{bmatrix}}$$

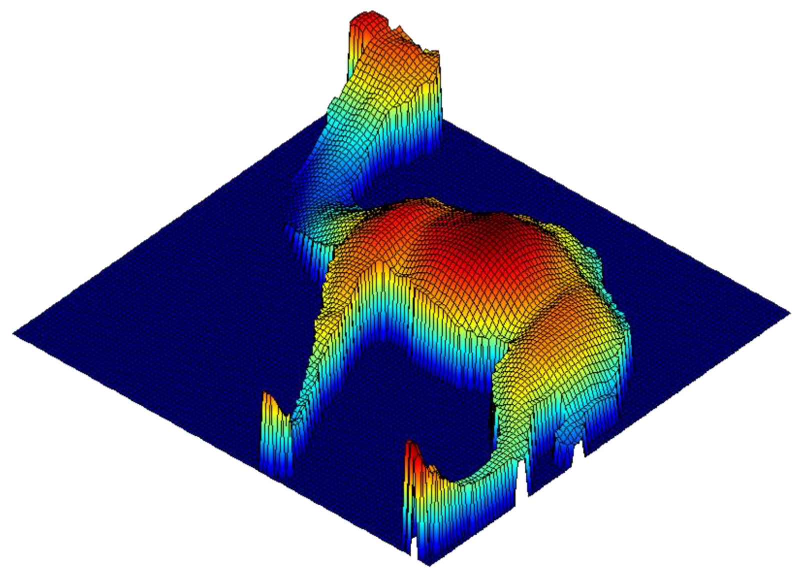
SVD recovers L and S up to an $(r \times r)$ ambiguity

Photometric stereo









Reconstruction with a prior

(Kemelmacher & Basri)

- Given just one image SFS is impractical
- Reconstruction is possible when a prior is available
- Energy

$$\min_{l,\rho,Z} \iint_{\Omega} D + S$$

- Data term

$$D = \left(I - \rho l^T Y(\hat{n})\right)^2$$

- Regularization

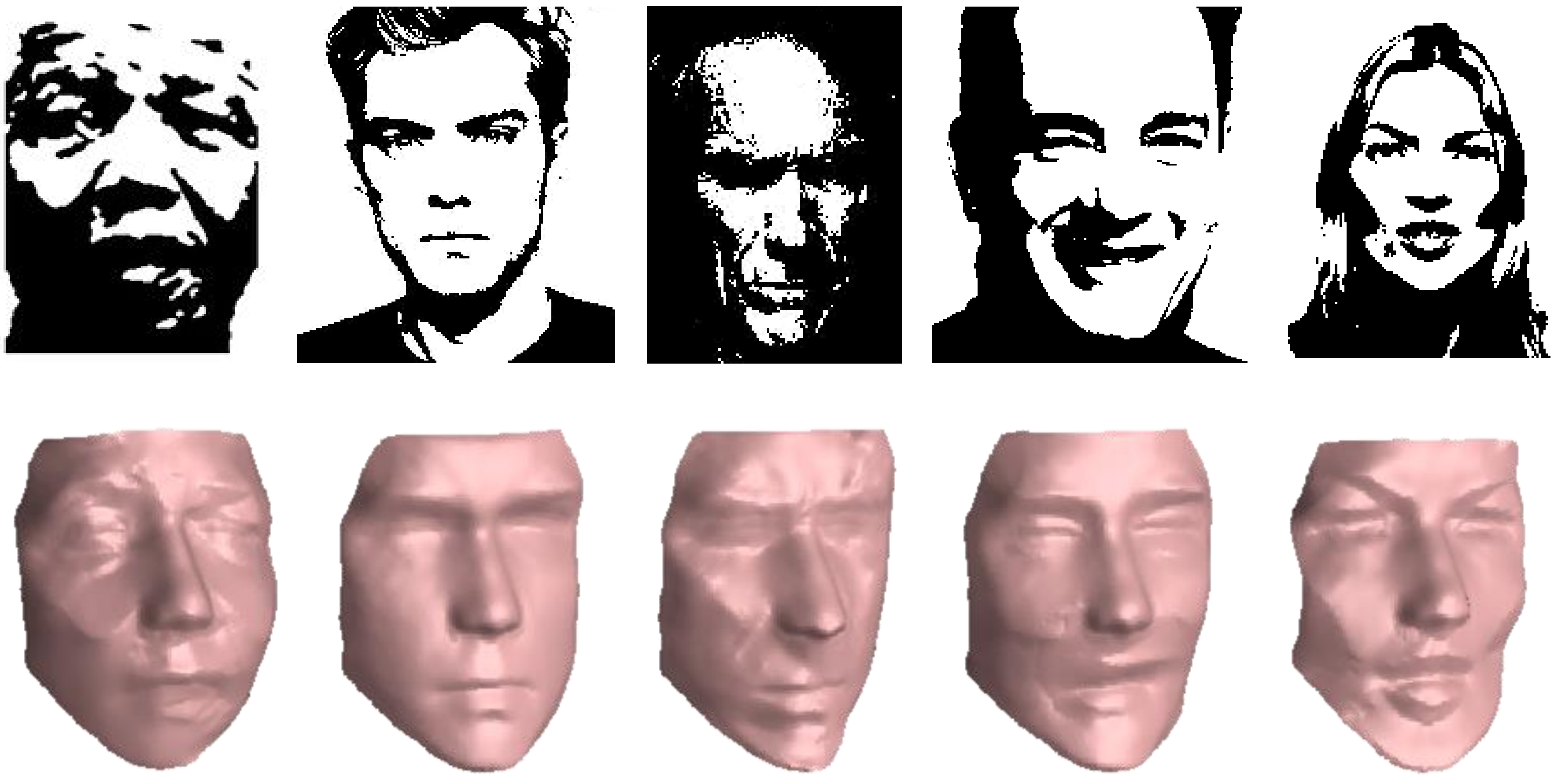
$$S = \lambda_1 \left(\Delta\left(Z - Z_{prior}\right)\right)^2 + \lambda_2 \left(\Delta\left(\rho - \rho_{prior}\right)\right)^2$$

- Solve as a linear PDE

Reconstruction with a prior

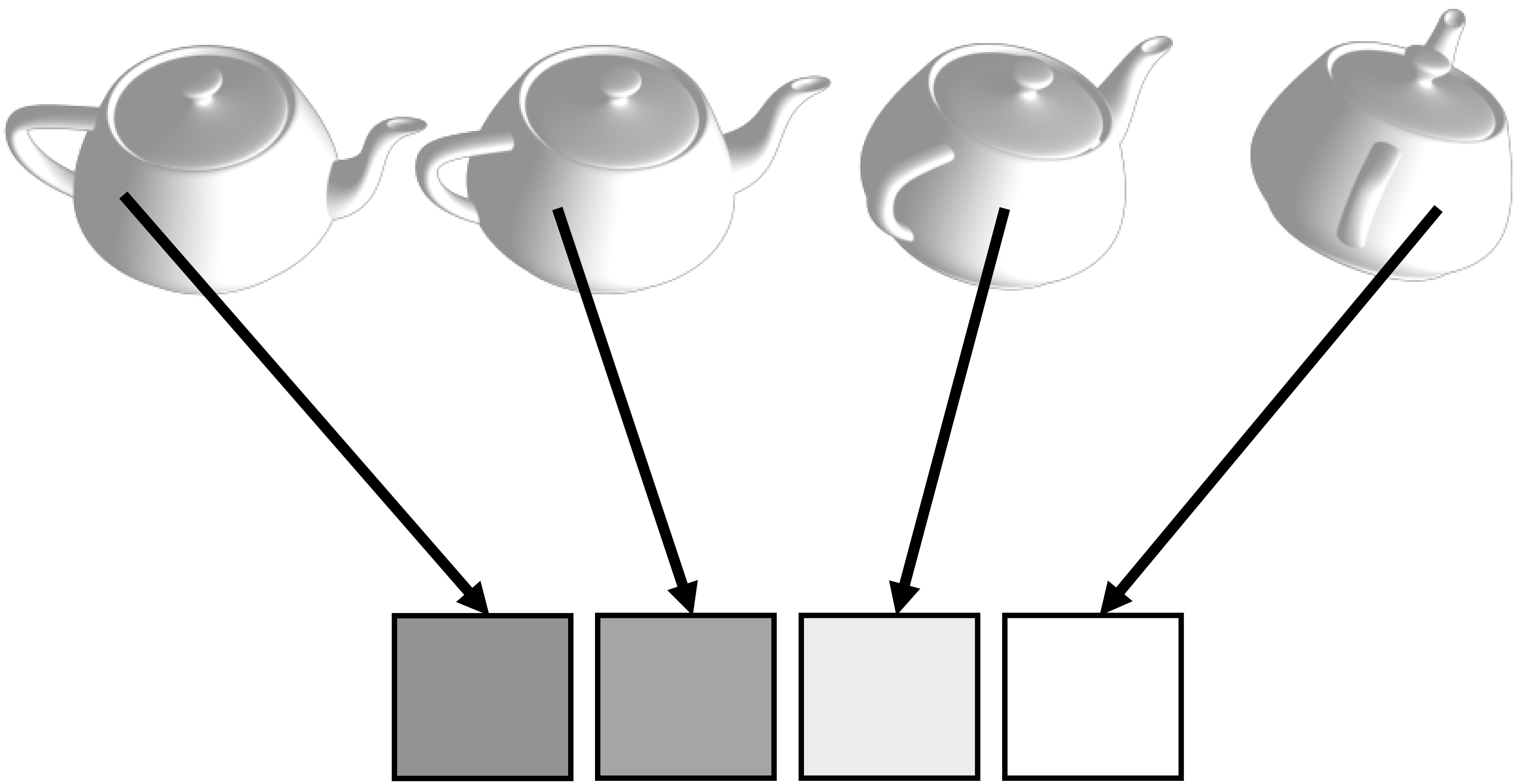
More…

Mooney faces



(Kemelmacher, Nadler & B, CVPR 2008)

Motion + lighting



Motion + lighting

(Basri & Frolova)

- Given 2 images

$$I(p) = \rho l^T \hat{n} \qquad J(p') = \rho l^T R\hat{n}$$

- Take ratio to eliminate albedo

$$\frac{J(p')}{I(p)} = \frac{l^T R n}{l^T n}$$

- If motion is small we can represent $J(p')$ using a Taylor expansion around p

Small motion

- We obtain a PDE that is quasi linear in z

$$az_x + bz_y = c$$

- Where

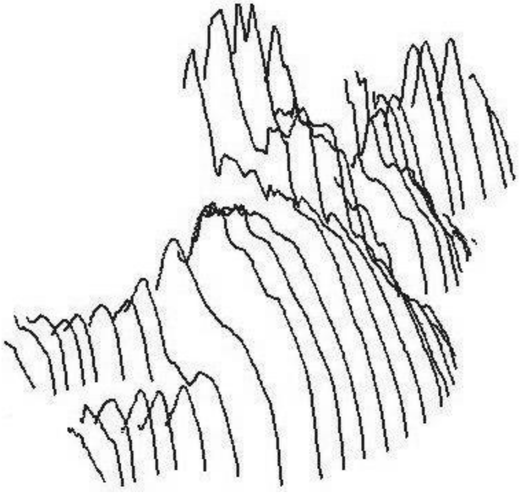
$$a(x, y, z) = l_1(I_\theta - zJ_x) - l_3 I$$
$$b(x, y, z) = l_2(I_\theta - zJ_x)$$
$$c(x, y, z) = -l_3(I_\theta - zJ_x) - l_1 I$$

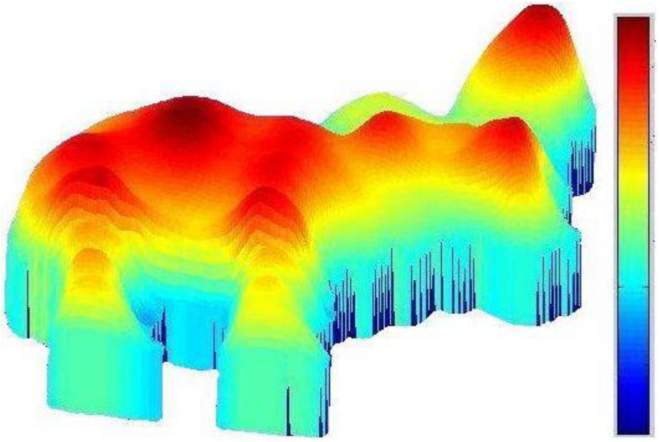
with

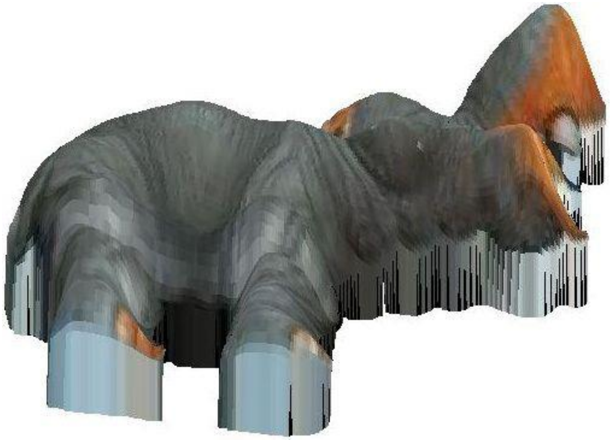
$$I_\theta = \frac{J - I}{\theta}$$

- Can be solved with continuation (characteristics)

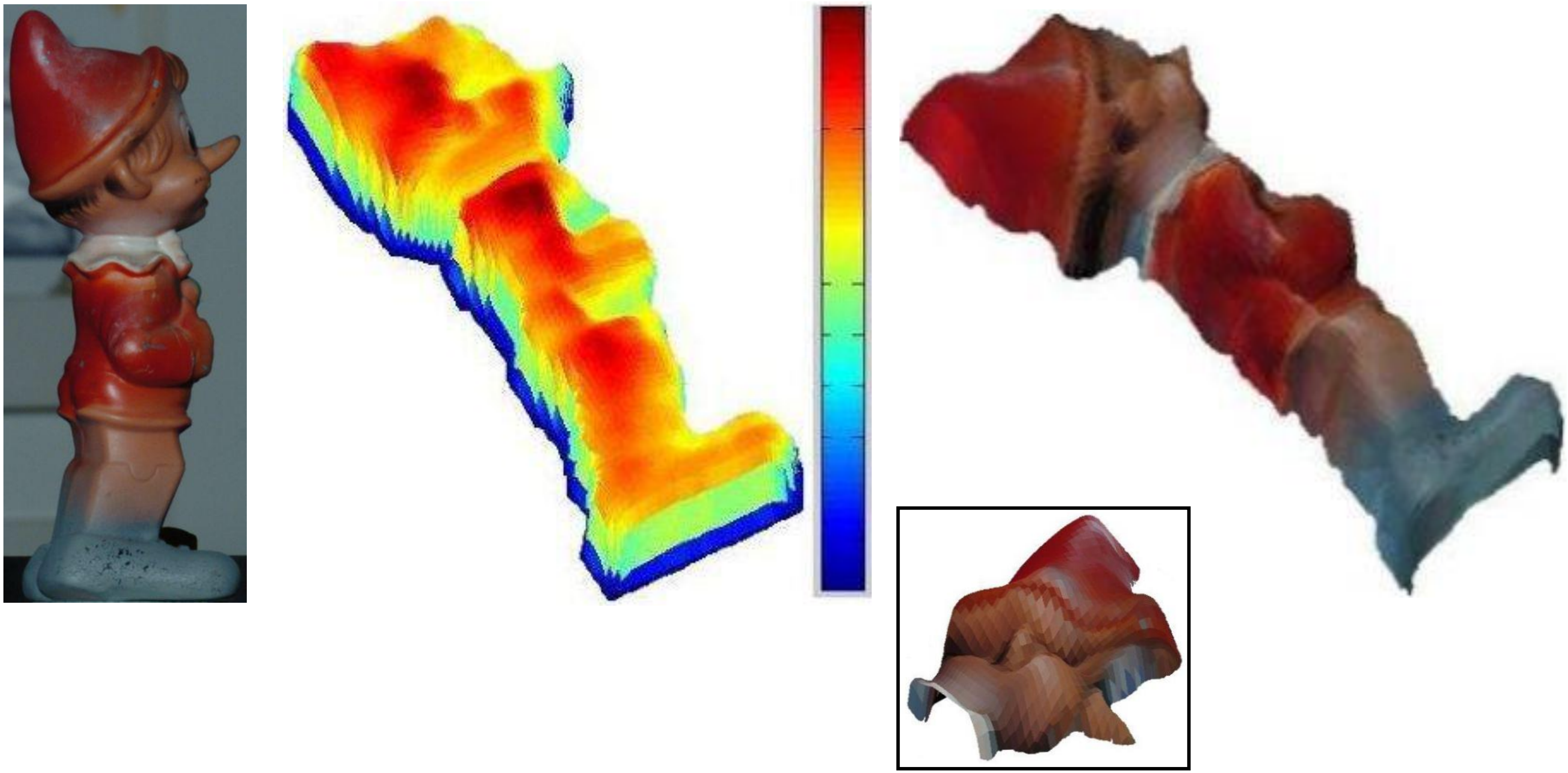
Reconstruction







More reconstructions



Conclusion

- Understanding the effect of lighting on images is challenging, but can lead to better interpretation of images
- Harmonic analysis allows to model complex lighting in a linear model
- Various applications in recognition and reconstruction
- We only looked at Lambertian objects…