

Statistics 202C. Spring 2012. Homework 4.

Due: Monday 4/June. 2012.

Question 1. Swendsen-Wang

Describe the Swendsen-Wang algorithm. Implement it on the Ising model $\pi(\vec{x}) = (1/Z)e^{(x_1x_2+\dots+x_9x_{10})/T}$ for temperatures $T = 1$ and $T = 10$ to estimate the expected magnetization $\frac{\sum_{i=1}^{10} x_i}{10}$. How does the result and the convergence rate depend on the temperature?

Question 2. Hybrid Monte Carlo

Describe the Hybrid Monte Carlo (HMC) algorithm. What is the physical motivation for the algorithm? What are the properties of the algorithm which ensure that it converges to the target distribution? What are the important properties of the leapfrog algorithm?

Implement the HMC in one spatial dimension when the target distribution is $\pi(x) = (1/Z)e^{-U(x)}$, where $U(x) = -(1/2)x^2 + (1/4)x^4$. Use sampling to estimate the mean and variance of the distribution. How does the result depend on the number of steps L used by the Leapfrog algorithm?

Question 3. Genetic Algorithms

Describe Genetic Algorithms. What are schemata? Why are they important? What is the schema theorem?

Implement a Genetic Algorithm to search a three-dimensional space and maximize the function $F(x, y, z) = 5x + y + 10z - 4xy - 5xz$. Here x, y, z take values 0, 1. Use a population of size 10. Use 2-point crossover. Set $p_m = 0.01$ and $p_c = 0.01$. Plot the average of the population as a function of number of iterations. Do you observe any schemata?