Statistics 202C. Spring 2012. Homework 1.

Due: Monday 23/April. 2012.

Please also hand in code. Code is preferably in R, but other languages are okay. Electronic submission is preferred. If so, put your name and student ID in the email header and on the homework.

Question 1. Rejection Sampling.

Describe how to use rejection sampling to sample from a distribution $\pi(x)$ where $\pi(x)$ is only known up to a normalization constant. What is the rejection rate of the sampler?

Suppose $\pi(x) = (1/Z)e^{-x^4}$ defined over $-1 \leq x \leq 1$.

Select a sampling distribution $g(x)$ and implement a rejection sampler. Hence estimate $\sum_x x\pi(x), \sum_x x^2\pi(x)$ and the variance of $\pi(x)$. Plot this for $m = 10, 20, 30$ samples.

What is the rejection rate?

Question 2. Rao-Blackwellization and Exact Sampling.

Explain the basic properties of Rao-Blackwellization. Suppose we write $\pi(x_1, x_2) = \pi(x_1|x_2)\pi(x_2)$. Describe two sampling methods for estimating $\sum_{x_1,x_2} h(x_1, x_2)\pi(x_1, x_2)$.

Let $\pi(x_1|x_2) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x_2-x_1)^2/(2\sigma^2)} : -\infty < x_1 < \infty$ and $\pi(x_2) = 1 : 0 \leq x_2 \leq 1$. Let $\sigma = 5$.

Implement both sampling methods to estimate $\sum_{x_1,x_2} x_1^2\pi(x_1, x_2)$. Do this for $m = 5, 10, 20$ samples. Estimate the mean and variance of each method. Which sampler is better?

Question 3. Importance Sampling.

Describe two methods to do importance sampling. What are the differences between these approaches?

Use importance sampling to estimate $\sum_x x^2\pi(x)$ where $\pi(x) = (1/Z)e^{-x^4}$ defined over $-1 \leq x \leq 1$. Do this for $m = 5, 10, 20$ samples.

Calculate the coefficient of variation of the weights. Hence estimate the effective sample size of your importance sampler (see Liu, page 34).

Question 4. Rejection Control.

Describe the rejection control algorithm to sample from a distribution $\pi(x)$ to estimate $\sum_x \pi(x)h(x)$ when the normalization of $\pi(x)$ is unknown.

Implement rejection control for distribution $\pi(x) = \frac{1}{Z}\{e^{-x^4} + 0.5e^{-(x-0.5)^2}\}$ defined in the range $0 \leq x \leq 1$. Estimate $\sum_x \pi(x)h(x)$, for $h(x) = x$ and $h(x) = x^2$. For $m = 5, 10, 20$ samples.