Note Title 5/15/2006

(*) Enables bigger jumps in one step.

Multiple-Try-Metropolis

There are many ways to extent Metropolis → here is one.

Define $\omega(\underline{x}, \underline{y}) = \pi(\underline{x})\, T(\underline{y} \mid \underline{x})\, \lambda(\underline{x}, \underline{y})$

$\lambda(\underline{x}, \underline{y})$ non-negative symmetric function of $\underline{x}$ & $\underline{y}$

Current state $\underline{x}^{t}$.

MTM: Draw k indep trial proposals $\underline{y}_1 \dots \underline{y}_k$ from $T(\ \mid \underline{x})$

Compute $\omega(\underline{y}_j, \underline{x})$

- Select $\underline{y}$ from $(\underline{y}_1 \dots \underline{y}_k)$ with probability prop to $\omega(\underline{y}_j, \underline{x})$

Special Case
$T(\underline{y}|\underline{x})$ symmetric.
$\lambda(\underline{x},\underline{y}) = T^{-1}(\underline{y}|\underline{x})$

Produce reference set $\underline{x}^*_1, \dots \underline{x}^*_{k-1}$ from $T(\ \mid \underline{y})$

$\underline{x}^*_k = \underline{x}$

- Accept $\underline{y}$ with prob

$$r_g = \min\left\{1, \frac{\omega(\underline{y}_1, \underline{x}) + \dots + \omega(\underline{y}_k, \underline{x})}{\omega(\underline{x}^*_1, \underline{y}) + \dots + \omega(\underline{x}^*_k, \underline{y})}\right\}$$

r_g = generalized M-H ratio.

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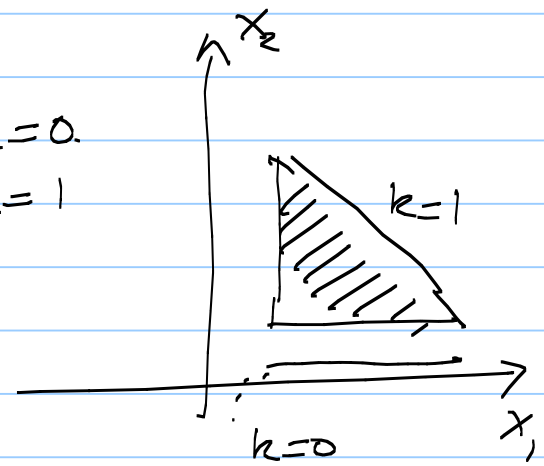
Reversible Jumps:

changing the dimension of the space.

Example

1 dim space $k=0$

2 dim space $k=1$



states $\left\{ \begin{array}{l} x, k=0 \\ (x_1, x_2), k=1 \end{array} \right\}$

π_0 - uniform in triangle

π_1 - uniform in $[0,1]$

$p \rightarrow$ prob that data is generated by model π_0

$1-p \rightarrow$ prob that data is generated by model π_1

Full distribution $\quad p\pi_0 + (1-p)\pi_1$

To sample from this distribution is simple

with prob p, sample from π_0

$1-p$, Sample from π_1

But to design an MCMC, we must be able to jump between space $k=0$ & space $k=1$

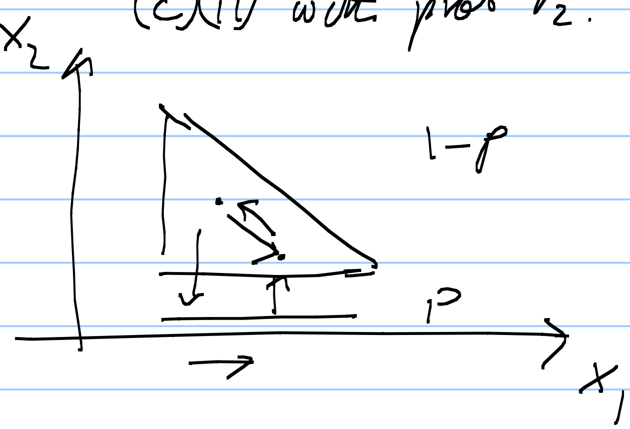
Design three moves: ✗ conditions at end?

(a) For $k=0$, $x \to U(x-\epsilon, x+\epsilon)$
(b) For $k=1$, swap x_1 & x_2.
(c) Jump between $k=0$ & $k=1$ by
(i) for $k=0$, choose u from $U(0,1)$
and propose $(x_1, x_2) = (x, u)$
(ii) for $k=1$, set $x = x_1$

At each step:
For $k=0$, choose (a) with prob $1-r_1$
(c)(i) with prob r_1

For $k=1$, choose (b) with prob $1-r_2$
(c)(ii) with prob r_2.

Move within triangle along diagonal.
From triangle to bar
Within bar along arrow 4.



More Generally

Green Reversible Jumps.

$\mathcal{X}$, $\mathcal{Y}$ is a subspace of $\mathcal{X}$ with lower dimension.

$\pi(\underline{x}) \propto q_0(\underline{x}) 1_{\underline{x} \in \mathcal{Y}} + q_1(\underline{x})$

with $q_0(\underline{x})$ & $q_1(\underline{x})$ as unnormalized distributions.

Need jumps $\mathcal{X} \to \mathcal{Y}$ & $\mathcal{Y} \to \mathcal{X}$.

Requiring "matching space" $\mathcal{Z}$, so that $\mathcal{Y} \times \mathcal{Z}$ has same dimension as $\mathcal{X}$, and matching proposal.

$g(\underline{z}|\underline{y})$

Special Case $\mathcal{X} = \mathcal{Y} \times \mathcal{Z}$

$\underline{x} = (\underline{y}, \underline{z})$, $\mathcal{Y} \sim \mathcal{Y} \times \{z_0\}$ for some $z_0 \in \mathcal{Z}$

To jump from $\mathcal{Y}$ to $\mathcal{X}$,

expansion transition:
- first propose $\underline{y} \to \underline{y}'$ from $T_1(\underline{y}'|\underline{y})$
- propose $\underline{z}'$ from $g(\underline{z}'|\underline{y}')$
- let $\underline{x}' = (\underline{y}', \underline{z}')$

from $\mathcal{X}$ to $\mathcal{Y}$,

contraction proposal:
- drop $\underline{z}$ component of $\underline{x}$
- propose $\underline{y}'$ from $T_2(\underline{y}'|\underline{y})$

Accept, expansion proposal $\underline{y} \to \underline{x}'$ with probability

$$\alpha = \min\left\{1, \frac{q_1(\underline{y}', \underline{z}')\, T_2(\underline{y}|\underline{y}')}{q_0(\underline{y})\, T_1(\underline{y}'|\underline{y})\, g(\underline{z}'|\underline{y}')}\right\}$$

Accept, contraction proposal $\underline{x} \to \underline{y}'$ with prob.

$$\beta = \min\left\{1, \frac{q_0(\underline{y}')\, T_1(\underline{y}|\underline{y}')\, g(\underline{z}|\underline{y})}{q_1(\underline{y}, \underline{z})\, T_2(\underline{y}'|\underline{y})}\right\}$$

Expansion & Contraction together satisfy "detailed balance. ((Check)

The Expansion Proposal involves first "proposing" and then "lifting" (e.g. increase dim of space)

Alternatively, can "lift" first and then "propose".

More precisely,

to get proposal $\underline{y} \to \underline{x}'$

draw $\underline{z} \sim g(\cdot|\underline{y})$ and then draw

$\underline{x}'$ from $S_1[\,\cdot\,|(\underline{y},\underline{z})]$

Contraction, propose $\underline{x} \to \underline{x}' = (\underline{y}',\underline{z}')$

from $S_2(\underline{x}'|\underline{x})$

then drop $\underline{z}'$

Acceptance probabilities are:

$$\alpha' = \min\left\{1, \frac{q_1(\underline{x}')\, S_2((\underline{y},\underline{z})|\underline{x}')}{q_0(\underline{y})\, g(\underline{z}|\underline{y})\, S_1(\underline{x}'|(\underline{y},\underline{z}))}\right\}$$

$$\beta' = \min\left\{1, \frac{q_0(\underline{y}')\, g(\underline{z}'|\underline{y}')\, S_1(\underline{x}|(\underline{y}',\underline{z}'))}{q_1(\underline{x})\, S_2((\underline{y}',\underline{z}')|\underline{x})}\right\}$$

Both S_1 & S_2 are proposals in the higher-dim space $\mathcal{X}$,

but T_1 & T_2 are proposals in lower-dim space $\mathcal{Y}$.

Other things being equal, prefer lifting after proposing...