Particle Filters

Note Title 4/23/2006

Prediction

(1) $p(x_{t+1}|Y_t) = \sum_{x_t} p(x_{t+1}|x_t)\, p(x_t|Y_t)$

Correction.

(2) $p(x_{t+1}|Y_{t+1}) = \dfrac{p(y_{t+1}|x_{t+1})\, p(x_{t+1}|Y_t)}{p(Y_{t+1})}$

with $p(Y_{t+1}) = \sum_{x_{t+1}} p(y_{t+1}|x_{t+1})\, p(x_{t+1}|Y_t)$

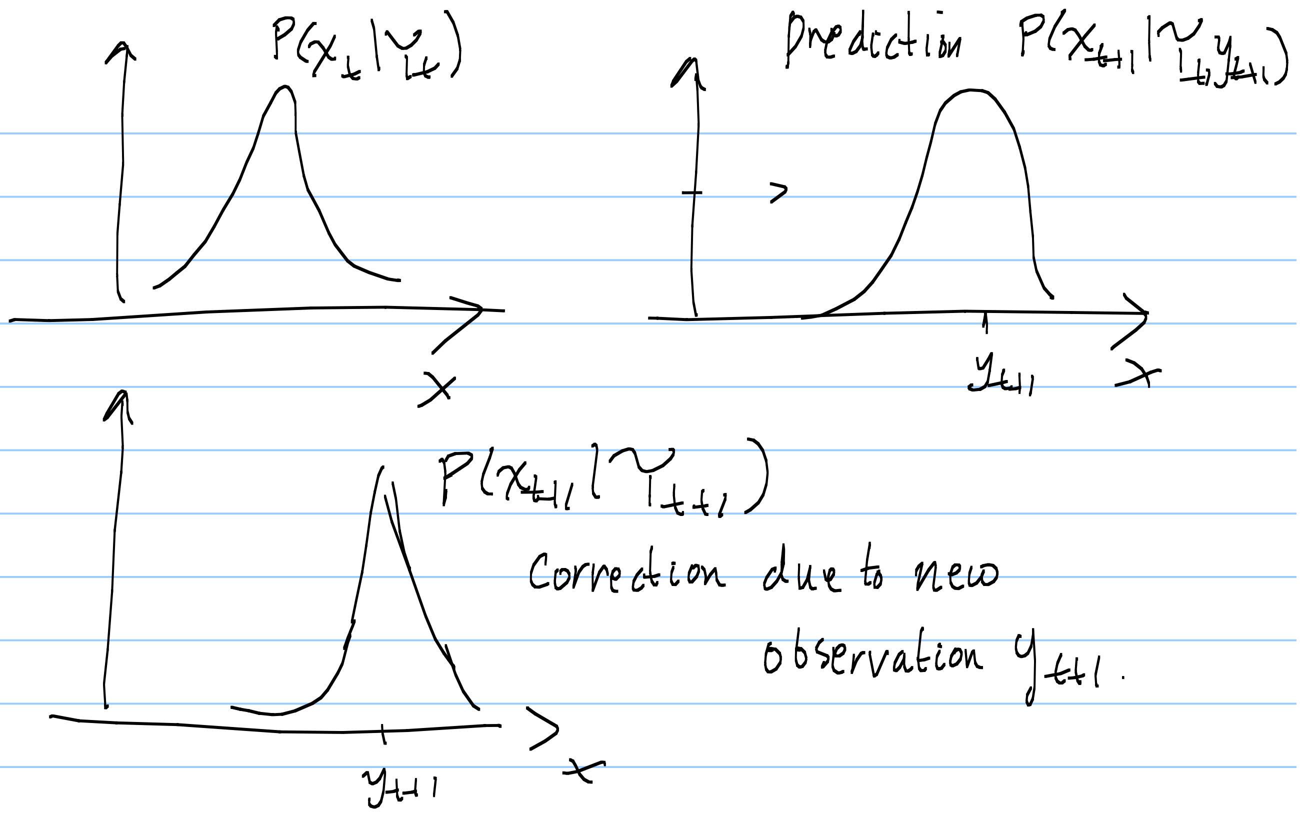
normalization constant.

Problems:

It may be difficult to compute these two stages.

If the distributions are Gaussians — prior and likelihood — then both stages can be reduced to algebra, previous lecture

(2)



Kalman Filter is very efficient, but requires the distributions $p(x_{t+1}|x_t)$ and $p(y_t|x_t)$ to be Gaussian.

Breakdown:

(1.) Tracking an object, and another object appears nearby.

(2.) x_t, y_t take finite set of values (e.g. Biology Applications)

(3) Section 3.3 Jun Liu.

Motivates Sampling Approach.

Particle Filters or Bootstrap Filters.

Represents the Distributions by Samples - Particles

At time $t \to$ random samples $\{x_t^1, \ldots, x_t^m\}$ from $p(x_t \mid \tilde{Y}_t)$.

(a) Draw samples x_{t+1}^{*j} (predict) from $p(x_{t+1} \mid x_t^{(j)})$. for $j = 1 \ldots m$.

(b) Weight each sample by $\omega^{(j)} \propto p(y_{t+1} \mid x_{t+1}^{j*})$ (agreement with observation).

(c) Resample from $\{x_{t+1}^{*1}, \ldots, x_{t+1}^{*m}\}$ with probability proportional to $\omega^{(j)}$ to produce random sample $\{x_{t+1}^1, \ldots, x_{t+1}^{(m)}\}$ for time $t+1$.

Claim: It can be shown that if the $\{x_t^1 \ldots x_t^m\}$ follow $P(x_t \mid Y_t)$ and if m is suff. big, then the $\{x_{t+1}^1 \ldots x_{t+1}^m\}$ follow $P(x_{t+1} \mid Y_{t+1})$

(4)

<u>Note</u>: this theory can be generalized to cases where the distributions $p(x_{t+1}|x_t)$ & $p(y_t|x_t)$ change over time

So we can write them as

$p_t(x_{t+1}|x_t)$ & $p_t(y_t|x_t)$.

Particle Filter / Bootstrap were developed in the past 10 years. They have had considerable success.

Limitation:

(a) They do not use the current available information y_{t+1} in the sampling step. (b) The use of resampling may cause inefficiency.

(5)

Bootstrap / Particle Filters are a special case of Sequential Monte Carlo.

(Chp. 3 of Jun Liu).

Justification for Claim

1. Prediction. $P(b,a) = P(b|a)P(a)$

samples $\{(b^i, a^i)\}$ from $p(b,a)$

sample a^i from $P(a)$

b^i from $P(b^i|a^i)$

Then $\{b^i\}$ are samples from $P(b)$

2. Correction.
$$p(a|b) = \frac{p(b|a)P(a)}{P(b)}$$

sample $\{a^i\}$ from $P(a)$

accept each sample with prob $\propto p(b|a^i)$

gives new samples $\{a^{i*}\}$ from $P(a|b)$