Models of Lighting.

Note Title 3/7/2006

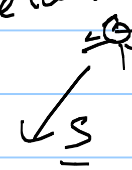
(1.) The appearance of objects will depend on the lighting conditions

Effects — shading, shadows & specularities.

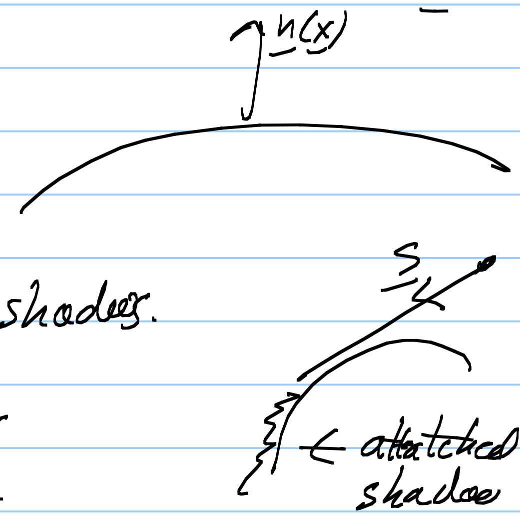
Linear Lambertian Model.

$$I(x) = a(x)\, \underline{n}(\underline{x}) \cdot \underline{s}$$

albedo, surface normal, light source direction + strength.



Non-linear Lambertian



$$I(x) = \max\{a(x)\,\underline{n}(\underline{x}) \cdot \underline{s}, 0\},$$

deals with attached shadows.

The linear model implies that the set of images of an object lies in a three dimensional space. (Sha'shua & Moses).

Non-linear model is harder to analyze — until Ramamoorthi & Hanrahan, Basri & Jacobs.

(2) Empirical Analysis.

Harvard scaffold

Yale geodesic dome.

Take many images of the same object under different lighting conditions.

Do PCA analysis of the images (separate PCA for each object).

$$\bar{I}(x) = \frac{1}{N} \sum_{\mu=1}^{N} I^{\mu}(\underline{x})$$

$$K(\underline{x}, \underline{y}) = \frac{1}{N} \sum_{\mu=1}^{N} \{I^{\mu}(\underline{x}) - \bar{I}(x)\}\{I^{\mu}(\underline{y}) - \bar{I}(\underline{y})\}$$

Results. 5 ± 2 eigenvectors capture about 95% of the variation.

In spite of specularities and shadows.

These are often perceptually salient, but contribute little to the variation energy.

Justifies the Lambertian models as a good approximation — linear Lambertian would predict 3 eigenvalues. Ramamoorthi predicts 5.

(3) To justify Lambertian further.

$$\underline{I}^{\mu}(\underline{x}) = a(\underline{x})\, \underline{n}(\underline{x}) \cdot \underline{s}^{\mu} \qquad \mu = 1 \ldots m$$

This is a bilinear equation.

Set $\underline{b}(\underline{x}) = a(\underline{x})\, \underline{n}(\underline{x})$ $\qquad |\underline{b}(\underline{x})| = a(\underline{x})$

$$\hat{\underline{b}}(\underline{x}) = \underline{n}(\underline{x}).$$

Least squares cost function

$$E[\underline{b}, \underline{s}] = \sum_{\mu, \underline{x}} \left\{ I(x, \mu) - \sum_{i=1}^{3} b_i(x)\, s_i(\mu) \right\}^2.$$

Solve by Singular Value Decomposition (SVD).

$I(x, \mu) \to N \times |\Omega|$ matrix $\underline{\underline{J}}$

N – no. of images

$|\Omega|$ – size of the image.

SVD $\qquad \underline{\underline{J}} = \underline{\underline{U}}\, \underline{\underline{D}}\, \underline{\underline{V}}^T$

$$\underline{\underline{U}}\, \underline{\underline{U}}^T = \underline{\underline{I}}, \quad \underline{\underline{V}}\, \underline{\underline{V}}^T = \underline{\underline{I}} \qquad \underline{\underline{D}} = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & \\ 0 & & \ddots \end{pmatrix}$$

$\underline{\underline{U}}$ & $\underline{\underline{V}}$ are eigenvectors of

$\underline{\underline{J}}^T \underline{\underline{J}}$ and $\underline{\underline{J}}\, \underline{\underline{J}}^T$ respectively

(4) If the Lambertian model is correct, then only the first three diagonal elements of $\underline{\underline{D}}$ are non-zero.

Let $\{\underline{f}(\mu)\}$ be the first three columns of $\underline{\underline{U}}$

$\{\underline{e}(\underline{x})\}$ be first three columns of $\underline{\underline{V}}$.

Then the solution for $\underline{b}$ & $\underline{s}$ are of form.

$$\underline{b}^*(\underline{x}) = \underline{\underline{P}}_3\, \underline{e}(\underline{x}), \quad \underline{s}(\mu) = \underline{\underline{Q}}_3\, \underline{f}(\mu)$$

where $\underline{\underline{P}}_3^T \underline{\underline{Q}}_3 = \underline{\underline{D}}_3$ ← the top three components of $\underline{\underline{D}}$

There is an ambiguity.

$$\underline{\underline{P}}_3 \to \underline{\underline{A}}^T \underline{\underline{P}}_3 \quad \& \quad \underline{\underline{Q}}_3 \to \underline{\underline{A}}^{-1} \underline{\underline{Q}}$$

for any matrix $\underline{\underline{A}}$. Except that the surface consistency must be enforced.

$$\frac{\partial}{\partial x}\left(\frac{b_2(x)}{b_3(x)}\right) = \frac{\partial}{\partial y}\left(\frac{b_1(x)}{b_3(x)}\right)$$

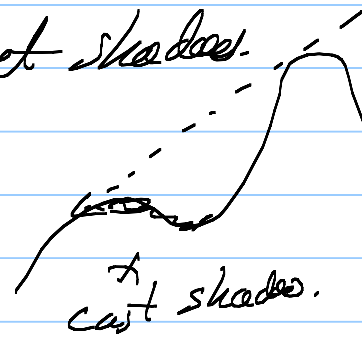
$$z = f(x,y)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

(5) Generalized Bas Relief (GBR) ambiguity.

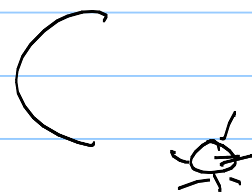
$$f(x) \to \underbrace{\lambda}_{\text{bas relief}} f(x) + \underbrace{\mu x + \nu y}_{\text{additive plane.}}$$

Can also be shown that the GBR exists when there are attached and cast shadows.



Recover the classic concave convex ambiguity as a special case when $\lambda = -1$.

or



But GBR usually requires changes in albedo.

GBR is inherent to the Lambertian lighting model (including cast & attached shadows)

(6) KGBR Ambiguity.

Object Geometry $\underline{r}(u,v)$

surface normal $\underline{n}(u,v)$

albedo $a(u,v)$

(u,v) parameter on the surface.

Light source $\underline{s}$, viewpoint $\underline{v}$.

Lambertian gives

$$I(u,v) = \max\{0, a(u,v)\,\underline{n}(u,v)\cdot\underline{s}\}$$

For geometry, there are known ambiguities

$\underline{r}(u,v) \to \underline{K}\,\underline{r}(u,v)$ $\quad \underline{K}$ affine transformation.

(Koenderink & van Doorn
Faugeras)

Views of objects are invariant up to affine transformations in the image plane.

Affine, corresponds to an approximation to perspective projection.

(7) Extend affine transformations to deal with lighting. KGBR.

$$\underline{r}(u,v) \to \underline{\underline{K}}\,\underline{r}(u,v)$$

$$a(u,v) \to a(u,v)\,|\,\underline{\underline{K}}^{T,-1}\underline{n}(u,v)|$$

implies $a(u,v)\,\underline{n}(u,v) \to \underline{\underline{K}}^{T,-1} a(u,v)\,\underline{n}(u,v)$

Claim, this is equivalent to changing viewpoint by $\underline{v} \to \dfrac{\underline{\underline{K}}\,\underline{v}}{|\underline{\underline{K}}\,\underline{v}|}$

and lighting by $\underline{s} \to \underline{\underline{K}}\,\underline{s}$.

Cast shadows and attached shadows are also preserved (by geometric reasoning)

If two objects are related by a KGBR, then we can always finding corresponding viewpoints and lighting conditions for which they appear identical.

To reduce to the GBR, we require that the objects appear identical from the same viewpoint (but different lighting).