Lecture 3   Note Title   4/15/2007

Snakes & Region Competition.

Snakes: Witkin, Terzopoulos, Kass.

Contour: $\underline{\Gamma}(s) = (x(s), y(s))$ $s \sim$ arc length

usually, $\underline{\Gamma}(s)$ is a closed boundary of a region R (e.g. $\Gamma = \partial R$).

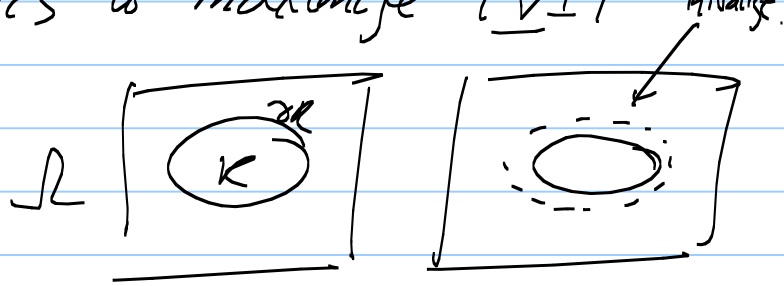
$$E[\Gamma(s)] = \int_{\Gamma(s)} \Big\{ \underbrace{\tfrac{1}{2}\left(\alpha |\underline{\Gamma}_s|^2 + \beta |\underline{\Gamma}_{ss}|^2\right.}_{\text{prior term on smoothness.}} \left. - \lambda |\underline{\nabla} I|^2 \right) \Big\} ds$$

$$\underline{\Gamma}_s = \frac{\partial \underline{\Gamma}}{\partial s}$$

Minimize $E[\Gamma(s)]$ by steepest descent

– requires good initialization.

Energy $E[\Gamma(s)]$ encourages a smooth & short boundary, and tries to maximize $|\underline{\nabla} I|^2$ along the boundary.



Steepest Descent:

$$\frac{d\underline{\Gamma}(s)}{dt} = -\frac{\partial F}{\partial \underline{\Gamma}} = -\alpha \underline{\Gamma}_{ss} + \beta \underline{\Gamma}_{sss} + \lambda \underline{\nabla} \cdot |\underline{\nabla} I|^2.$$

Bayesian Justification:

$I(x,y) \to |\underline{\nabla} I|(x,y)$

P_{on} & P_{off} first lecture.

$$P(\{|\underline{\nabla} I|\} | \Gamma) = \prod_{x \in \Gamma} P_{on}(|\nabla I|(x)) \prod_{x \in \Omega/\Gamma} P_{off}(|\underline{\nabla} I|(x))$$

(i.e. gradient on contour is generated by P_{on} and off contour by P_{off}.)

prior on contour:

$$P(\underline{\Gamma}(s)) = \frac{1}{Z} e^{-\int_{\Gamma(s)} \left\{ \frac{\alpha}{2} |\underline{\Gamma}_s|^2 + \frac{\beta}{2} |\underline{\Gamma}_{ss}|^2 \right\} ds}$$

$$P(\Gamma | \{\underline{\nabla} I\}) = \frac{P(\{|\underline{\nabla} I|\} | \Gamma) \, P(\Gamma)}{P(|\nabla I|)}.$$

MAP estimation.

select $\Gamma^* = \underset{\Gamma}{\mathrm{ARG\,MIN}} \left\{ -\log P(\Gamma | \{|\underline{\nabla} I|\}) \right\}$

<u>Minimize</u> $-\sum_{x \in \Gamma} \log P_{on}(|\nabla I|(x))$

$$-\sum_{x \in \Omega/\Gamma} \log P_{off}(|\nabla I|(x)) + \sum_{\underline{x} \in \Gamma(s)} \log P(\Gamma(s))$$

<u>Re-express</u> as $-\sum_{x \in \Gamma} \log \frac{P_{on}(|\underline{\nabla} I|(x))}{P_{off}(|\underline{\nabla} I|(x))} + \sum_{\underline{x} \in \Gamma(s)} \log P(\Gamma(s))$

local evidence for edge

page 3) Compare the Bayesian expression to the Snake energy function.

$$\log \frac{P_{on}(|\nabla I|(x))}{P_{off}(|\nabla I|(x))} \sim \lambda |\nabla I(x)|^2$$

priors are equivalent.

Empirically, the form of $\log \frac{P_{on}(|\nabla I|)}{P_{off}(|\nabla I|)}$ is not like $|\underline{\nabla} I(x)|^2$.

more like $$\frac{|\underline{\nabla} I(x)|^2}{1 + |\underline{\nabla} I(x)|^2}$$

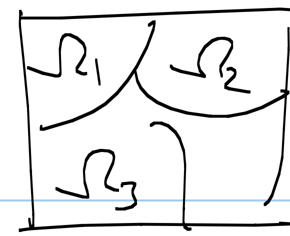
Balloons modify snakes by an additional energy term $E_B[\Gamma(s)] = -\nu \iint_R dx\,dy$.

Attempts to maximize the area.

This is another prior $$P(\underline{\Gamma}) = \frac{1}{Z} e^{\nu \iint_R dx\,dy} e^{-\frac{1}{2}\int (\alpha |\underline{\Gamma}_s|^2 + \beta |\underline{\Gamma}_{ss}|^2)\, ds}$$

Note: priors should be learnt from the data and not imposed by the modeler.

(page 4) <u>Region Competition</u>

Ω 

$$E[\{\partial \Omega_i\},\{\alpha_i\},M] = \sum_{i=1}^{M} \frac{\mu}{2}\int_{\partial \Omega_i} ds + \lambda M$$

$$- \sum_{i=1}^{M} \log P(\{I_{(x,y)} : (x,y)\in \Omega_i\} \mid \alpha_i).$$

M-regions, $\{\Omega_i\}$, $\bigcup_{i=1}^{M} \Omega_i = \Omega$

s.t. $\Omega_i \cap \Omega_j = \phi$ for $i \neq j$.

Boundaries $\{\partial \Omega_i\}$

Model Parameters, $\{\alpha_i\}$

$$P(\{I\} \mid \{\Omega\},\{\alpha\})\, P(\{\Omega\} \mid M)\, P(M)$$

$P(M) = \frac{1}{Z_1} e^{-\lambda M}$ prior on no. of regions.

$$P(\{\Omega\} \mid M) = \frac{1}{Z_2} e^{-\sum_{i=1}^{M} \frac{\mu}{2}\int_{\partial \Omega_i} ds}$$

$$P(\{I\} \mid M) = \prod_{i=1}^{N} P(\{I_{(x,y)} : (x,y)\in \Omega_i\} \mid \alpha_i)$$

<u>Can</u> use different models for different regions

→ but have to select the models

(needs a difficult algorithm).

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We can re-derive a variant of the Mumford & Shah model, by choosing Gaussian models.

Replace $P(\{I(x,y)\} \mid \alpha_i)$

$$P(\{I(x,y)\} \mid \{F(x,y)\}) = \prod_{x,y} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}\{I(x,y) - F_i(x,y)\}^2}$$

$$P(\alpha) = \frac{1}{Z} e^{-\int_{R_i} |\nabla F_i(x,y)|^2 \, dx\, dy}$$

Note: $\{\alpha_i\}$ is replaced by a function $F_i(x,y)$ with prior $P(\alpha)$.

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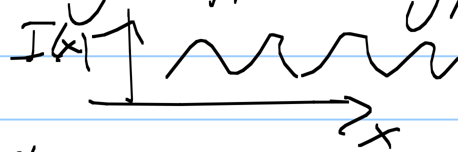
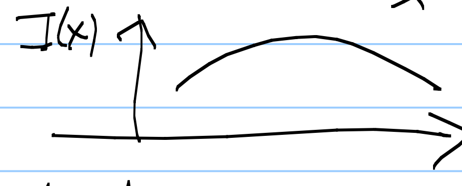
But in typical natural images, regions can be of many different types.

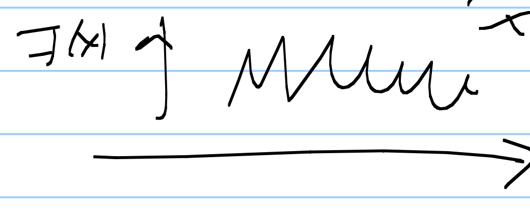
→ e.g. (i) texture



(ii) shading



(iii) junk



The only requirement is that the regions are homogeneous — in the sense of being generated by a single distribution.

See (Zhu & Yuille, Tu & Zhu)

E.g. Shading model $I(x,y) = A + Bx + Cy + Dx^2 + Exy + Fy^2 + n(x,y)$

zero mean Gaussian additive noise.

(Page 7) Algorithm to minimize $E[\Gamma, \{\alpha_i\}]$ by steepest descent:

Two stages:

(1) Fix Γ, solve for the $\{\alpha^i\}$ within each region

$$\hat{\alpha}_i = \arg\min_{\alpha_i} \left\{ -\iint_{R_i} \log P(\alpha_i \mid I(x,y))\, dx\, dy \right\} \quad \forall i$$

E.G. i) Gaussian distribution:

$$\hat{\mu}_i = \frac{1}{|R_i|} \sum_{(x,y)\in R_i} I(x,y)$$

$$\sigma_i^{*2} = \frac{1}{|R_i|} \sum_{(x,y)\in R_i} I^2(x,y) - \{\mu_i^*\}^2$$

(2.) Fix $\{\alpha_i\}$, solve for Γ by steepest descent

$$\frac{d\underline{\Gamma}}{dt} = -\frac{\delta E[\underline{\Gamma}, \alpha_i]}{\delta \Gamma} \qquad \underline{\Gamma} \text{ on the boundary}$$

$$\frac{d\Gamma}{dt} = -\frac{\mu}{2} \underline{K}_v \underline{n}_v + \log \frac{P(I(v) \mid \alpha_k)}{P(I(v) \mid \alpha_{k+1})} \underline{n}_v$$

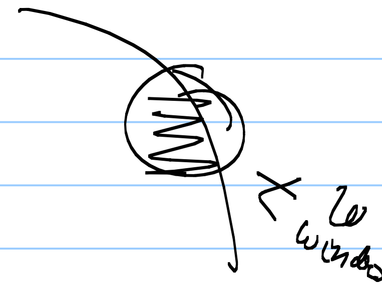
curvature

intuitions — the regions k & $k+1$ compete for ownership of the pixels on the boundary.

(page 8) If the models include texture statistics, (e.g. $\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}$), then need to use a window to measure the statistics.

$$\log \frac{P(I_v \mid \alpha_n)}{P(I_v \mid \alpha_{n+1})}$$ is not a good test for texture properties.

<ins>Compare</ins> $$\log \frac{P(I_v \in W \mid \alpha_n)}{P(I_v \in W \mid \alpha_{n+1})}$$



<ins>E.G.</ins> for Gaussian:

$$\frac{d\Gamma}{dt} = -\mu \kappa \underbrace{\underline{n}}_{\text{normal}} - \frac{1}{2}\left\{ \log \frac{\sigma_i^2}{\sigma_j^2} + \left\{ \frac{(\bar{I}-\mu_i)^2}{\sigma_i^2} - \frac{(\bar{I}-\mu_j)^2}{\sigma_j^2} \right\} + \left\{ \frac{s^2}{\sigma_i^2} - \frac{s^2}{\sigma_j^2} \right\} \right\} \underline{n}$$

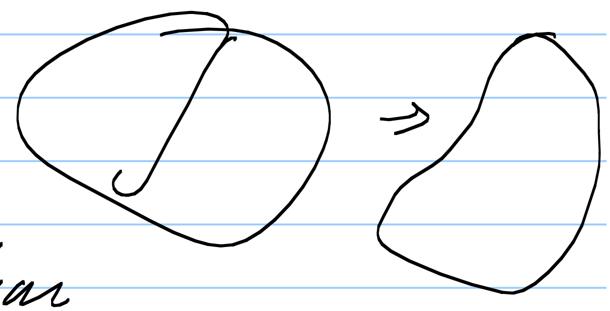
where $\bar{I}$ is the mean in the window W

s^2 is the variance in the window W.

(page 9) This steepest descent algorithm does not change the number of regions — ie it has fixed M.

To change M.

<u>merge</u> two regions R_i & R_j is the cost of describing them by a single model is lower than describing them by two and the boundary



<u>Strategy</u>:

- Initialize the algorithm with many regions (over segmentation)
- Run the 2-stage iterative process
- Merge regions if it reduces the energy.

(Also many of initial regions collapse to zero size, and can be eliminated).