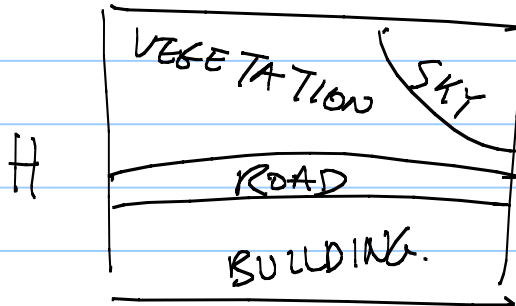


## Images



Dataset of images. The images are segmented by hand into different regions labelled by  $m \in M$   $M$ -set of labels  
 Eg.  $M = \{ \text{sky, vegetation, road, building, other} \}$ .

## Images.

$$I(i,j) \quad i = 1 \text{ to } W$$

$$j = 1 \text{ to } H$$

$$I(i,j) \in \{0, 1, 2, \dots, 255\}$$

Colour images:  $R(i,j), B(i,j), G(i,j)$

red  $\uparrow$  blue  $\uparrow$  green  $\uparrow$

(2)

Apply filters to the images.

Filter  $\phi_{\mu}(i, j)$ , linear filter.  $\mu \in \Delta$

Discrete  $\phi_{\mu} * I(i, j) = \sum_{k, l} \phi_{\mu}(i-k, j-l) I(k, l)$

Continuous  $\phi_{\mu} * I(x, y) = \int \phi_{\mu}(x-u, y-v) I(u, v) du dv$

Strategy: used the labelled images to learn conditional distributions.

$$P(\phi_{\mu}(i, j) \mid (i, j) \in \text{Region class } m)$$

Problem: (i) how to represent the distribution and learn it.

(ii) how to generalize the distribution to next images?

Representation of the distribution:

- (i) parametric distributions (e.g. Gaussian)
- (ii) non-parametric (e.g. histogram).

(3)

### Trade-offs.

Probability models involve only a small number of parameters (e.g. mean and variance of Gaussian).

This makes it easier to learn them and requires less training data. Typically you need  $k \times m$  examples to learn a probability model with  $m$  degrees of freedom, and  $k \sim 5, 10$ .

Learning: Generalization versus Memorization

Basic Assumptions of Learning:

Training examples.  $D = \{x_i : i=1 \text{ to } N\}$   
assumed to come from an unknown  $n$  distribution  $P(x)$ .

How can we learn the distribution from  $D$ , so that the model applies to data from  $P(x)$  than we haven't seen yet.

(4)

Memorization: the model learnt from data  $D$  is un-representative of  $P(x)$ .

Generalization: the model learnt from  $D$  is similar to  $P(x)$ .

Learning requires generalization — otherwise it is only memorization.

A sufficient amount of data is required for memorization —  $k \times M$  (M no. of parameters)

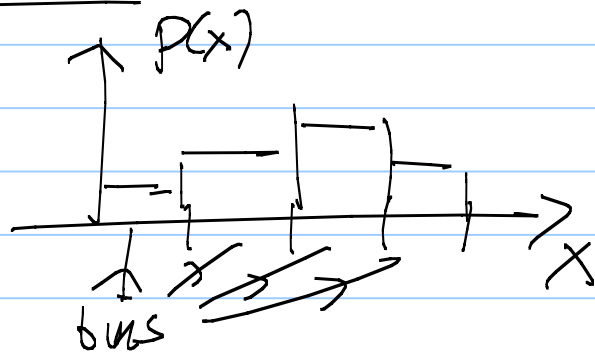
### Curse of Dimensionality

the number of parameters increases with the dimensionality of the data. E.g. Gaussian model requires  $O(N^2)$  parameters, where  $N$  is the dimension of the space.

Amount of data required for generalization grows by  $O(N^2)$  as the dimensionality of the data increases.

(5)

## Histogram Representation.



$$P(x) = \frac{n_i}{n}, \quad i) \quad x \in [x_{i-1}, x_i]$$

$\{x_i: i=0, \dots, m\}$  bin boundaries

$$\sum_{i=1}^m n_i = n.$$

$\{n_i \geq 0, i=1, \dots, m\}$ .

Note: the bin boundaries can be evenly spaced, or adaptively spaced (better, but more complicated).

The histogram makes few assumptions about the distribution.

Bad in many dimensions — the number of bins grows like  $\{m^D\}$  (exponentially)

(6)

## Back to Images and Segmentation.

Apply filters to images

Filters: Typical image filter.

$$|\underline{\nabla} I| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \quad \text{Image derivatives.}$$

$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

$$\underline{N} = \int G(x) \underline{\nabla} I \underline{\nabla} I^T dx$$

← Gaussian

$$\underline{\nabla} I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \text{ vector.}$$

On image lattice, approximate the derivative by finite differences. Ex.  $\frac{\partial I}{\partial x} \rightarrow (I_{i+1,j} - I_{i,j})$   
or higher order discretizations

We can smooth images by a Gaussian, and then apply these operators. Gives a scale space representation.

Page (7)

Choose a representation for the distribution.

Learn  $P(\phi_\mu(i,j) | (i,j) \in \text{Region } m)$

Classify  $m^*(i,j) = \text{ARG-MAX}_m P(\phi_\mu(i,j) | m)$

To obtain errors, calculate the confusion table  $P(m^* | m)$ . If  $m^* = m$ , then correct classification - otherwise an error.

Results → see Kontski & Vuille paper

This approach is effective at classifying pixels for classes {ROAD, SKY, VEGETATION}

Combine cues by joint distribution  $P(\phi_\mu(i,j), \phi_\nu(i,j) | (i,j) \in \text{Region } m)$

Moral: simple statistics can work.