Lecture 2. Note Title 7/3/2011

Basic Properties of Images:

An image is an array of intensity values.
$\{I_{ij} : i = 1 \text{ to } n,\ j = 1 \text{ to } m\}$

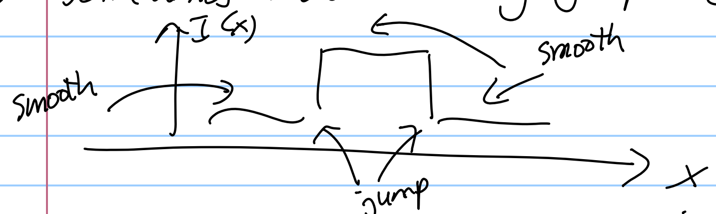
What is this image? →

140	131	132	140
137	20	15	141
143	14	17	144
151	145	132	143

Probably you say:
"It is an image of a dark box surrounded by a bright background."
Why?

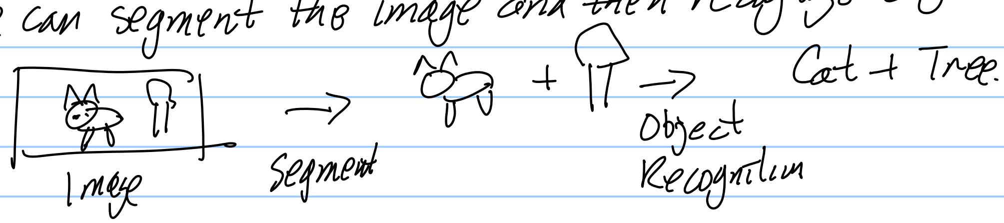
A simple model of images (~1980's) says that an image is piecewise smooth, or weakly smooth.

Neighboring pixels have similar intensity values (e.g. 20 to 15). But sometimes there is a big jump (e.g. 137 to 20)



A one-dimensional image.

This simple model says that images consist of regions. The intensity is roughly constant within each region. The intensity jumps between regions.
This can be used to segment images into regions.
This makes tasks like object detection easier.
– i.e. we can segment the image and then recognize objects.



Note: This model is too simple. Images are more complicated – e.g. they include texture regions.
In particular, we usually cannot segment objects without knowing what they are.

Are Images piecewise smooth?

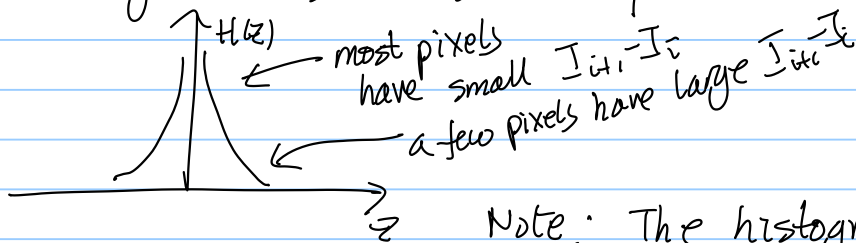
In one-dimension, calculate $\frac{dI}{dx}$ x-derivative. (or $I_{i+1}-I_i$ on lattice)

Calculate the histogram

$$H(z) = \frac{1}{N}\sum_{i=1}^{N} \underset{\leftarrow \text{ identity}}{\mathbb{I}}(I_{i+1}-I_i = z)$$

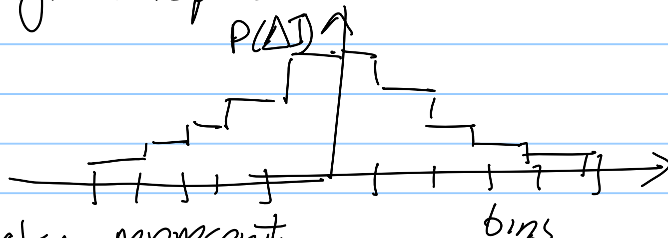
$\mathbb{I}(a=b) = 1$, if $a=b$
$= 0$, otherwise

The histogram has the standard form.



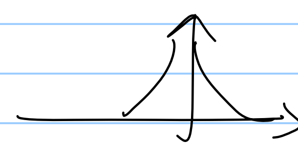
Note: The histogram is an image statistic — ie. a function on the image. It is the observed marginal distribution of the image derivative $P(\Delta I)$ $\Delta I \sim I_{i+1}-I_i$

The histogram represents this distribution by bins



Note: we could also represent $P(\Delta I)$ by a parameterized probability distribution — but we need to know the form of the distribution (it is not a Gaussian).

The statistics of $\frac{dI}{dx}$



suggest that images are piecewise smooth — at least as a very simple approximation.

We obtain similar statistics for two-dimension images — for $\frac{\partial I}{\partial x}$ and $\frac{\partial I}{\partial y}$. (Also for depth)

In fact, we get similar histograms for other derivatives. For $\frac{d^2 I}{dx^2}$, $\frac{d^3 I}{dx^3}$, .. See Mumford & Lee, Green.

(Note: higher order derivatives are non-local.)

Linear Filter Theory

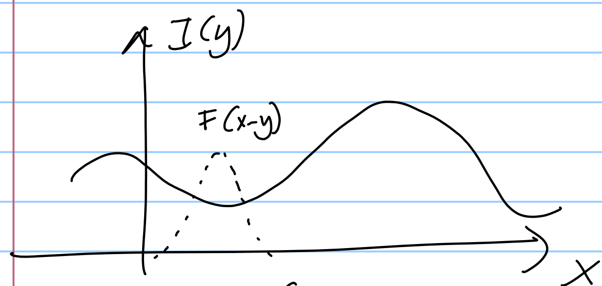
$\delta_{ij} = 1$, if $i = j$
$= 0$, otherwise

$$\underset{\text{Filter}}{F} * \overset{\text{image}}{I_i} = \sum_j F_{i-j} I_j$$

– E.G. $F_{i-j} = \delta_{i+1,j} - \delta_{i,j}$
gives the difference
$F * I_i = I_{i+1} - I_i$

A derivative filter obeys $\sum_j F_{i-j} = 1$, for any i.

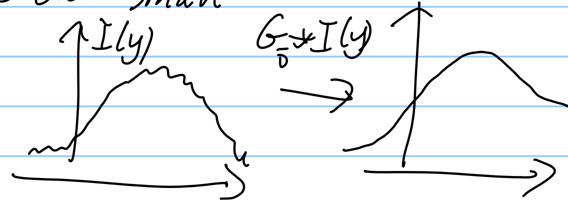
$F_{i-j} = \delta_{i+1,j} - 2\delta_{i,j} + \delta_{i-1,j}$
gives $I_{i+1} - 2I_i + I_{i-1}$
2nd order derivative.



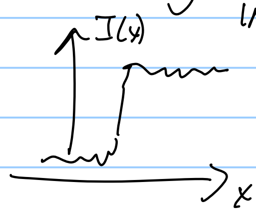
$$F * I(x) = \int F(x-y) I(y)\, dy$$

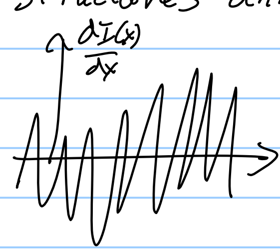
Example: Gaussian smoothing: $G_\sigma(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-x^2/2\sigma^2}$

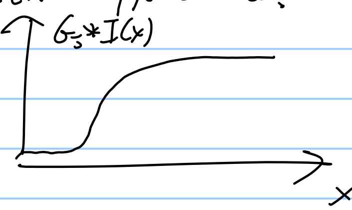
This blurs the image and smooths out small structures in the image:

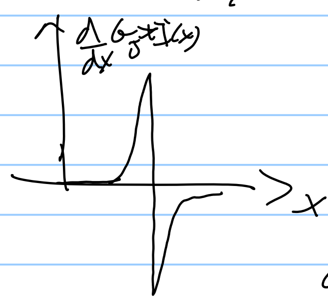


Often combine differentiation and smoothing. → e.g. smooth the image to eliminate small image structures and then differentiate.









Increasing σ in G_σ smooths the image more and removes more image structure.

$$\frac{d G_\sigma * I(x)}{dx} = \frac{d}{dx}\int G_\sigma(x-y) I(y)\, dy$$

Continuous Filters → Discrete Filters

e.g. $\frac{d}{dx} I(x) \rightarrow I_{i+1} - I_i$

Note: many ways to discretize derivatives. Some give better approximations.

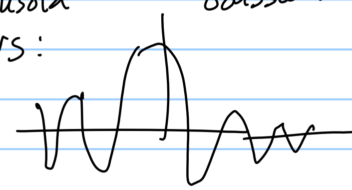
FilterBanks

Sets of Filters:

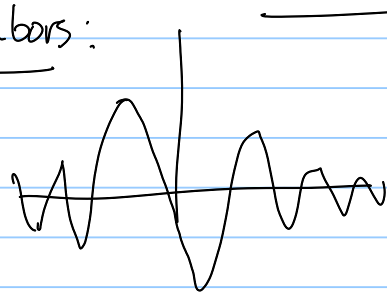
- Derivatives, Smoothing, Derivatives and Smoothing.
- Gabor filters.

$$G_{\omega}(\underline{x}) = \underbrace{e^{i\underline{\omega}\cdot\underline{x}}}_{\text{Sinusoid}} \underbrace{e^{-\frac{1}{2}\underline{x}^T \underline{\underline{\Sigma}}^{-1}\underline{x}}}_{\text{Gaussian}}.$$

Cosine Gabors:



Sine Gabors:



Gabor's detect local frequency structure in images.

Color Images: $\{\underbrace{R_{ij}}_{\text{red}}, \underbrace{B_{ij}}_{\text{blue}}, \underbrace{G_{ij}}_{\text{green}}\}$, ij ← pixel locations

Normalized Color:

$$\frac{R_{ij}}{R_{ij}+B_{ij}+G_{ij}}$$