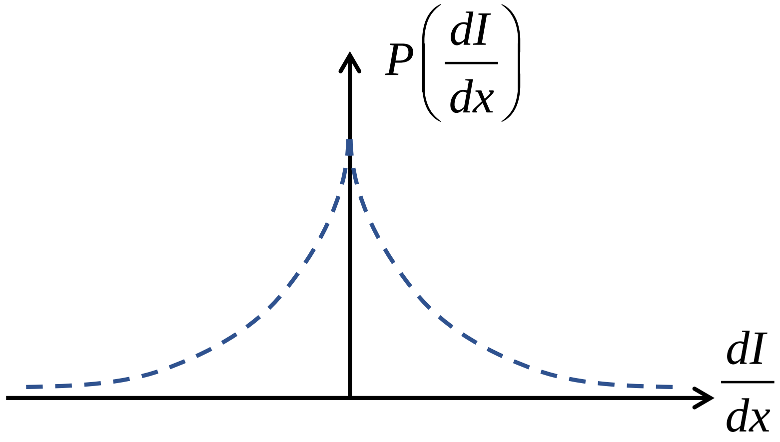
Statistics on Image Derivatives

Image Statistics

Estimate $P\left(\frac{dI}{dx}\right)$ or $P\left(\frac{dI}{dy}\right)$ from an image



This can be modeled by Laplace distribution

$$P(x) = 2\lambda e^{-\lambda|x|}$$

λ varies a little between images

But this finding is very consistent

Intuition: Images are locally smooth

Most derivatives in images are very small,
but at some places (e.g. edges), they take large values

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Image Statistics

This observation has motivated many models of image context, or is consistent with exiting models (see later in the course)

But further studying of image statistics shows a richer structures

The statistics of $\frac{d^2 I}{dx^2}, \frac{d^3 I}{dx^3}, \cdots, \frac{d^8 I}{dx^8}$ are also of the same Laplace form

Even the statistics of a general "derivative" operator take this form:

$$\sum_{x \in W} a(x) I(x), \quad \text{s.t.} \sum_{x \in W} a(x) = 1$$

➡ We are losing information by only considering the statistics of the first order derivative

Image Statistics

Note: Statistics of high order derivatives corresponds to increasingly non-local image regularities

Statistics of first order derivatives are nearest neighbors (on lattice)

Statistics of n^{th} order derivatives are of n^{th} nearest neighbors

Recall $$\frac{dI}{dx} \approx I(x+1) - I(x)$$

$$\frac{d^2 I}{dx^2} \approx \frac{1}{2}\{I(x+1) + I(x-1) - 2I(x)\}$$

Handout by Mark Green studies theses properties in detail