(1) DP can be extended to perform inference on any graph which does not have closed loops

Key idea: decompose

The inference into steps which can be computed independently

(2) DP can be extended to some graphs with closed loops – junction trees

(3) DP can be modified to compute other quantities

\[
P_i(x_i) = \sum_{\{x_j : j \neq i\}} P(x) \quad \text{or} \quad P(x_i, x_j) = \sum_{\{x_k : k \neq i, k \neq j\}} P(x)
\]
Binocular Stereo

Epipolar Line Constraint

The geometry means that a point in the left eye can only match to points on one line in the right eye.

Special cue: cameras (eyes) are parallel.
Binocular Stereo

So, stereo matching is a **1-D problem** if the epipolar geometry is known (calibrated cameras)

Disparity: Point $i$ in left image matches point $i+d_i$ in right image \( \{d_i\} - \text{disparity} \)

Task: estimate disparity \( \{d_i\} \) determines depth if matching is known, depth is estimated by trigonometry
Energy Function / Probability Formulation

\[ E[\{d_i\}] = \sum_i \Phi(d_i, I_L, I_R) + \sum_i \psi(d_i, d_{i+1}) \]

- data cues
- weak smoothness constraint

\[ e.g. \vert F(I_L)_i - F(I_R)_{i+d_i} \vert \quad e.g. \, K(d_i - d_{i+1}) \]

\( F(I_L)_i \): image feature, computed on left image at position \( i \),
(e.g. derivative filters, smoothing filters)

\( F(I_R)_{i+d_i} \): image feature, computed on right image at position \( i+d_i \)

**Note:** for some images the data cues are enough

E.G. \( I_L = \begin{bmatrix} + & \bullet & \star \end{bmatrix} \quad I_R = \begin{bmatrix} + & \bullet & \star \end{bmatrix} \)

Then match (+ to +), (• to •), (★ to ★): But this almost never happens.
Energy Function / Probability Formulation

Need weak smoothness of disparity to resolve matching ambiguities

Task: solve for $D = \{d_i\}$ by minimizing

$$E[\{d_i\}] = \sum_i \Phi(d_i, I_L, I_R) + \sum_i \psi(d_i, d_{i+1})$$

Use Dynamic Programming: restrict the disparity $d_i$ to take a finite set of $k$-values.
Energy Function / Probability Formulation

**Note:** This is a very simple model of stereo

There are situations where some features are visible in one eye (camera) only

⇒ These *half-occluded* points cannot be matched

Half-occlusion gives information.

- It occurs when the surface orientation changes – i.e. where smoothness of disparity is violated

- Can exploit this to get better stereo.

  (e.g. Geiger, Ladendorf, Yuille 1995, Belhaven & Mulford 1996)

Better stereo algorithms enforce weak smoothness across the epipolar lines – not just along -
2D Problem

\[ E \left[ \{d_{i,j}\} \right] = \sum_{i,j} \Phi(d_{i,j}, I_L, I_R) + A \sum_{i,j} \psi(d_{i,j}, d_{i+1,j}) + B \sum_{i,j} \psi(d_{i,j}, d_{i,j+1}) \]

along epipolar line

across epipolar line

weak smoothness

Use Belief Propagation → or Max-Flow to estimate \( \{\hat{d}_{i,j}\} = \arg \min E \left[ \{d_{i,j}\} \right] \)