Probabilistic Models of the Visual Cortex

Fall 2020 Homework 2

Prof. Alan Yuille

Due on Oct 20 before the class. Late homework will not be accepted unless permission is obtained well in advance in documented extenuating circumstances. Submit pdf file on the Gradescope (entry code: posted in Blackboard). Format file name as *firstname*-*lastname*-hw2.pdf. Do not include the iPython notebook code in the pdf submission as it is not required. Please start early on the coding part and make sure the software (i.e. the jupyter notebook) works for you. If you have any questions about the homework, please post on the Piazza Q&A - hw2.

Question 1. Fourier Basis and Sparse Coding (15 points)

1. Recall that Fourier theory represents an image by a linear combination of basis functions, namely sinusoids, where the coefficients of the basis functions can be obtained by the Fourier inverse transform. Write down the formulas for the Fourier transform and inverse Fourier transform. (4 points) To generalize, if we have a new set of orthonormal basis, and we would like to use this set of basis to represent the image, how do we calculate the coefficient of the image in the new basis? (2 points)

2. Use three to four sentences to explain what is sparse coding? (3 points) The sparsity penalty encourages many of the coefficients of basis functions to be zero. How does the degree of sparsity depend on the parameter λ which penalizes the sum of the magnitude of the coefficients. (2 point) Motivate this by studying the one-dimensional case with the function $f(x;a) = (x-a)^2 + \lambda|x|$ and the rule $\hat{x} = \arg\min f(x;a)$. Show that, for some values of a, the answer is sparse (i.e. $\hat{x} = 0$). (4 points)

Question 2. Hebbian Learning and Binocular Stereo (15 points)

1. Is Hebbian learning a kind of supervised learning? Why or why not? (2 points) In the Hebbian learning equation, there is a term to decrease the value of all weight by the amount proportional to their strength, what is the purpose of this term, and if there is no such term in the equation, what will be the problem? (3 points)

2. Observe the stereokinetic effect and the Ames Windows illusion, write two to three sentences to explain each . (4 points)
   Stereokinetic effect
   `http://www.michaelbach.de/ot/mot-ske/index.html`
   Ames Windows illusion:
   `http://www.michaelbach.de/ot/sze-AmesBallerina/index.html`

3. Using the stereo disparity model, is it possible for a single cell to detect disarity by itself when $\rho_l - \rho_r = \omega D$? (2 points) Briefly describe how a neural network estiamte disparity. (4 points)

Question 3. Decision Theory (28 points)

1. In Bayes Decision Theory, what are priors, likelihood functions, and loss functions? (2points each) What are the formulas for Bayes Risk and the Bayes Decision Rule? (2 points each) What are maximum likelihood (ML) estimation or maximum a posteriori (MAP) estimation? (2 points each) When does the Bayes Decision Rule reduce to them? (2 points each)

2. What are false positives and false negatives? (1 point each) Give a formula for these when the likelihood functions are Gaussians of one variable with the same variance σ^2, but different means μ_T, μ_D. Express the false positives and false negatives in terms of the error function (integrals of Gaussians). (2 points)

Question 4. Experimental Section: Sparse Coding (20 points)

This consists the project as described in this IPython notebook:

`http://nbviewer.jupyter.org/github/ccvl/VisualCortexCourse/blob/master/HW2_part1/HW2_part1_Intro.ipynb`

Project: Learn a Sparse Code for Natural Images. Use an unsupervised sparse coding technique to learn receptive fields from naturally occurring image statistics. *You are allowed to complete Question 4 in a teamwork (up to two people per team). Since the code is unstable on Windows, it is best to use OS X or Linux. Additionally, if you are still experiencing difficulties, the visualization is found in the last page.*

Updated information (10/15/2020): for those of you who experience difficulties in setting up the environments, we provide the visualization in Figure 1. If you choose to use these images in your homework write-up (without running the code), you must describe your observations in all three parts of Q4, along with a basic description of your working

understanding of the code.

Question 5. Experimental Section: Edge Detection (14 points)

Project: Statistical Edge Detection. Apply Bayes Decision Theory to edge detection. You will need to use IPython notebook for this project. Follow:

`http://nbviewer.jupyter.org/github/ccvl/VisualCortexCourse/blob/master/HW2_part2/HW2_part2_Intro.ipynb`

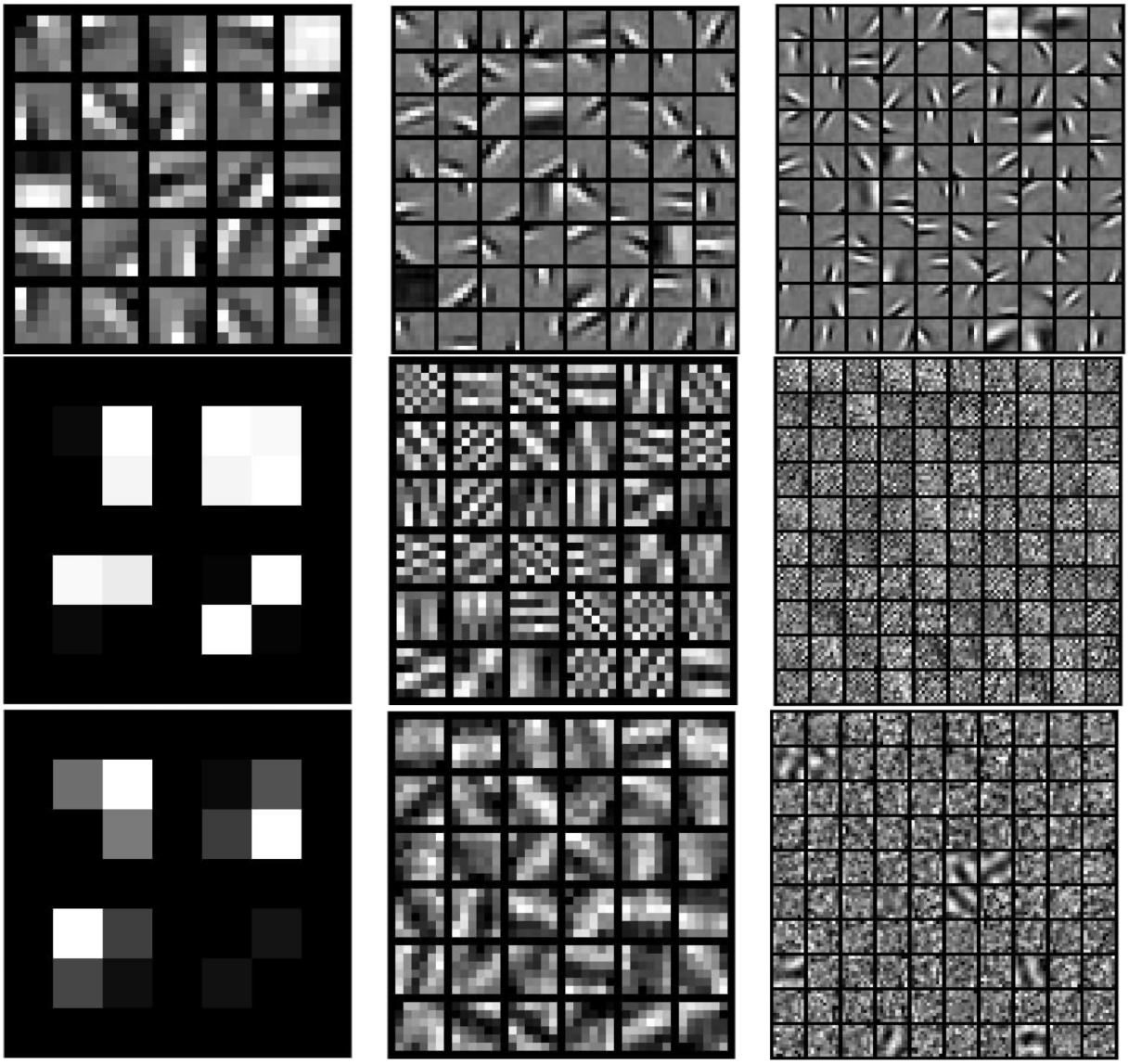

Figure 1: Visualization of the over-complete set in Question 4. First row: natural images with M = 25, 64, and 100. Second row: high frequency sinusoid images with M = 4, 36, and 100. Third row: low frequency sinusoid images with M = 4, 36, and 100.