# Random Oracles and OAEP

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# So far...

- Symmetric encryption
  - Two people want to communicate
  - Share a secret key
  - Want their communication to be private and authenticated

# So far...

IND-CPA Symmetric Encryption Scheme

+

Strongly Unforgable MAC

↓

IND-CCA Authenticated Encryption Scheme

# Today

- Symmetric encryption
  - Two people want to communicate
  - Share a secret key
  - Want their communication to be private and authenticated

# Today

- Asymmetric encryption
  - Two people want to communicate
  - Don’t share a secret key
  - Want their communication to be private and authenticated (?)

# Asymmetric Encryption

- Also called *public key encryption*
- Instead of one key that both people share, now there are two per person
  - Public key which does not need to be kept secret ($k$)
  - Private key which only the owner should know ($k^{-1}$)

Public Key Public Key

Attack at dawn

Private key

Private key

- Public Key
- Public Key

Encrypt

- Attack at dawn

- Private key
- Private key

- Public Key
- Public Key

- Attack at dawn

- Private key
- Private key

- Public Key
- Public Key

- Attack at dawn

Decrypt

- Private key

- Private key

- Public Key
- Public Key

Message could have come from anyone

- Attack at dawn

Decrypt

- Private key

- Private key

# A New Atomic Primitive

- Family of one-way trapdoor permutations
- Family of permutations $(f, f^{-1})$
- One-way means that given f and y, it's hard to come up with the x where f(x) = y
- The inverse, $f^{-1}$, is the trapdoor
- Examples: RSA, Rabin, etc...

RSA is a one-way trapdoor permutation, *not an encryption scheme*

# OAEP

- Just like we built secure symmetric encryption out of PRPs (CTR), we want to build secure asymmetric encryption schemes out of OWTPs (OAEP)
- Optimal Asymmetric Encryption Protocol

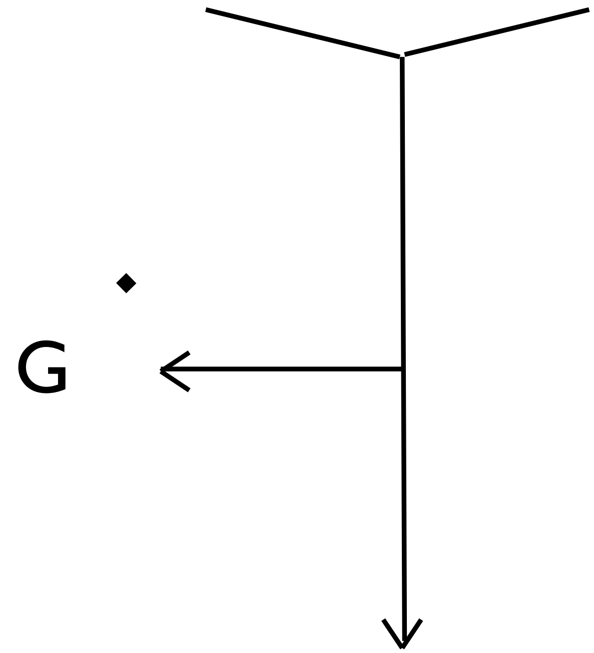
# Message

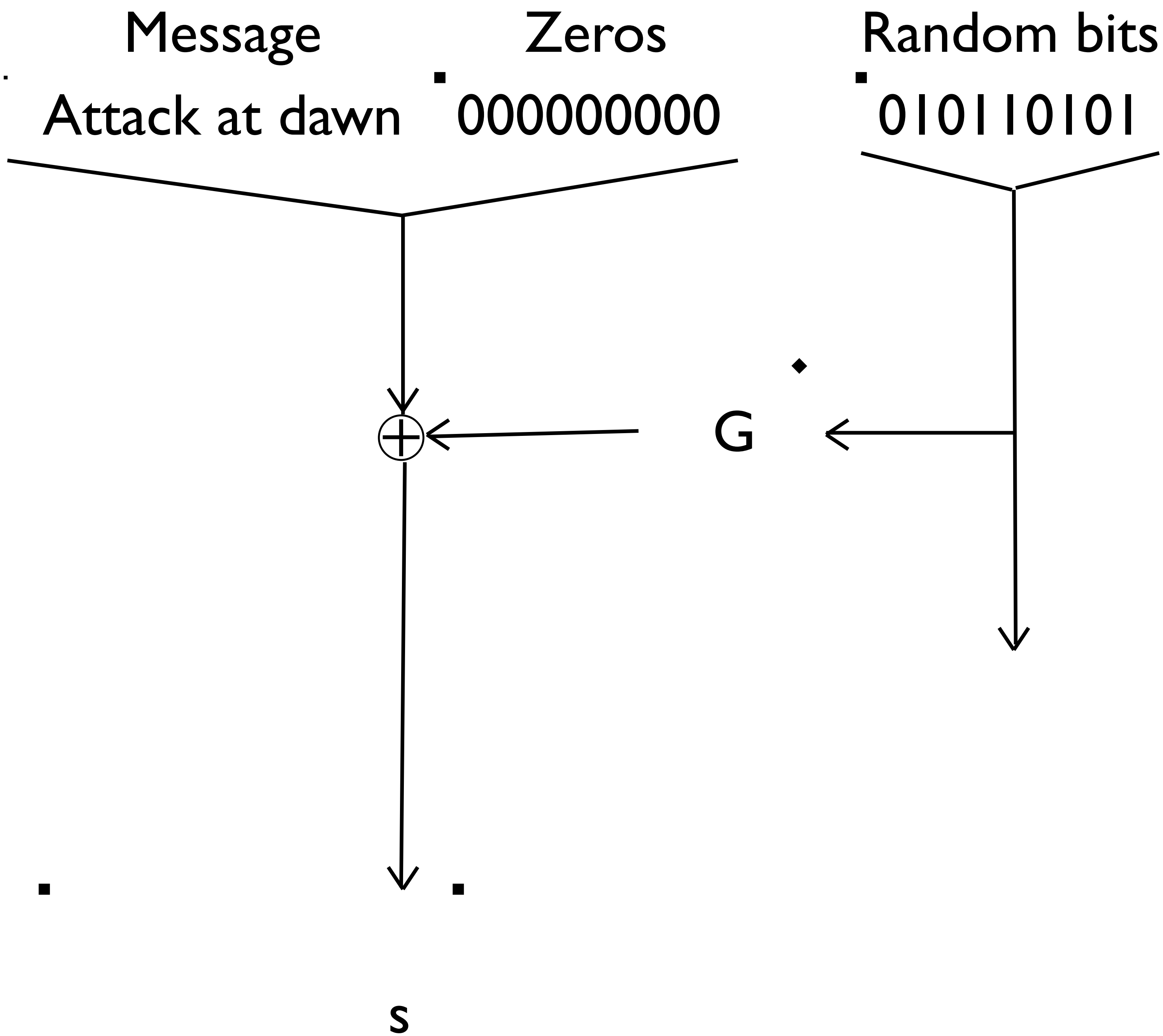
Attack at dawn

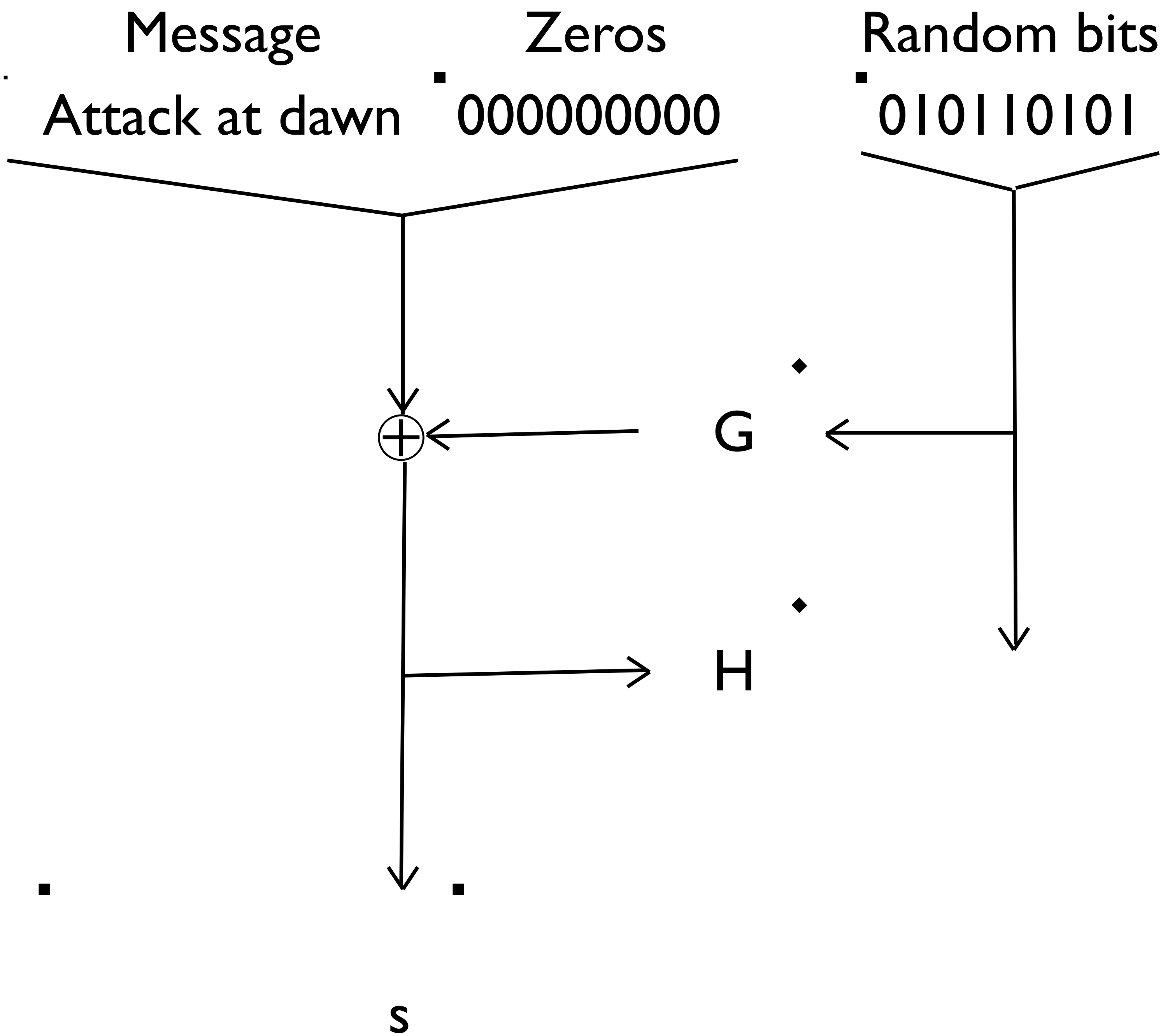
| Message | Zeros |
| --- | --- |
| Attack at dawn | 000000000 |

| Message | Zeros | Random bits |
| --- | --- | --- |
| Attack at dawn | 000000000 | 010110101 |

Message | Zeros | Random bits
Attack at dawn | 000000000 | 010110101




Message
Zeros
Random bits
Attack at dawn
000000000
010110101
G
s


Message
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Random bits
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000000000
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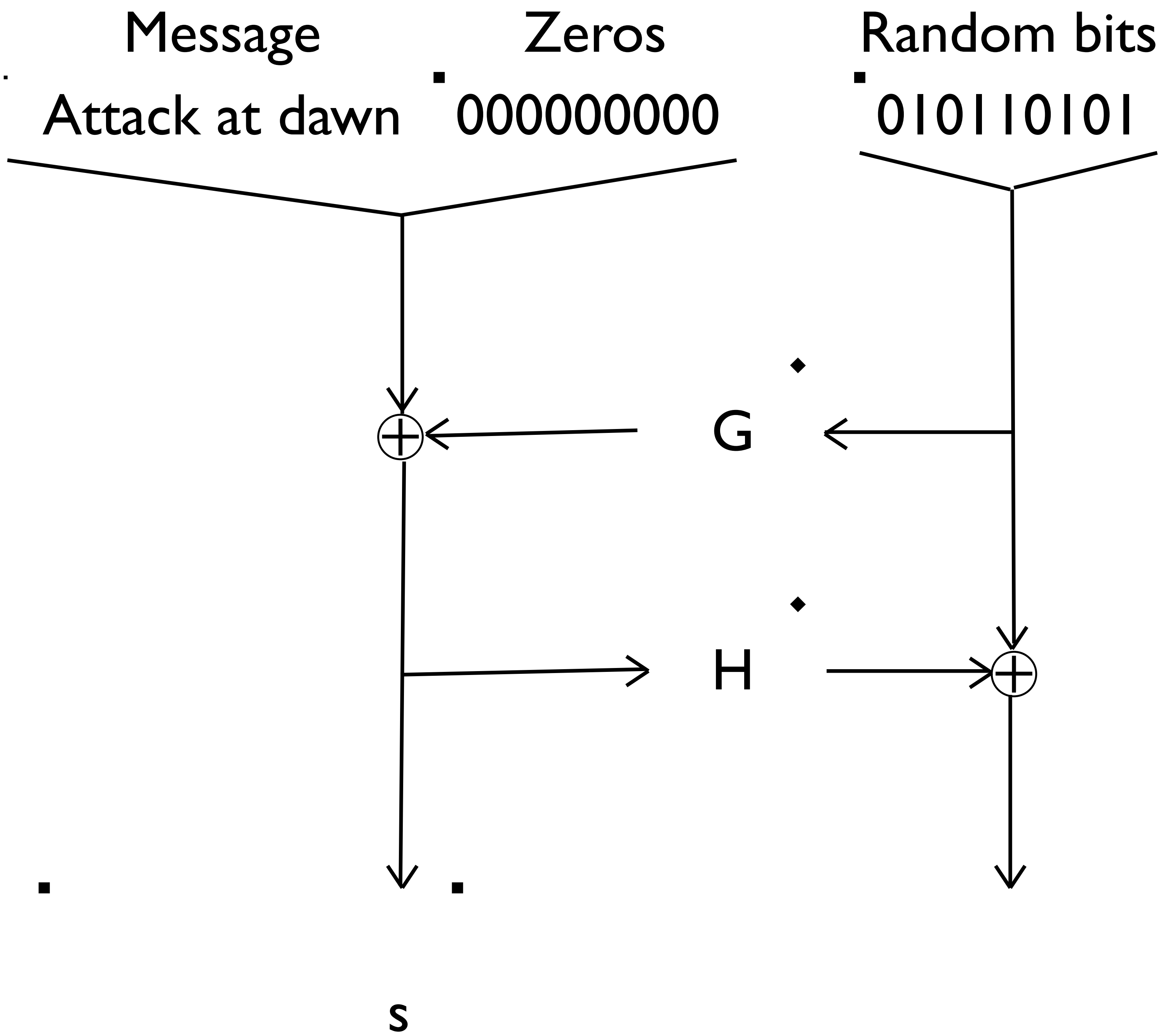
s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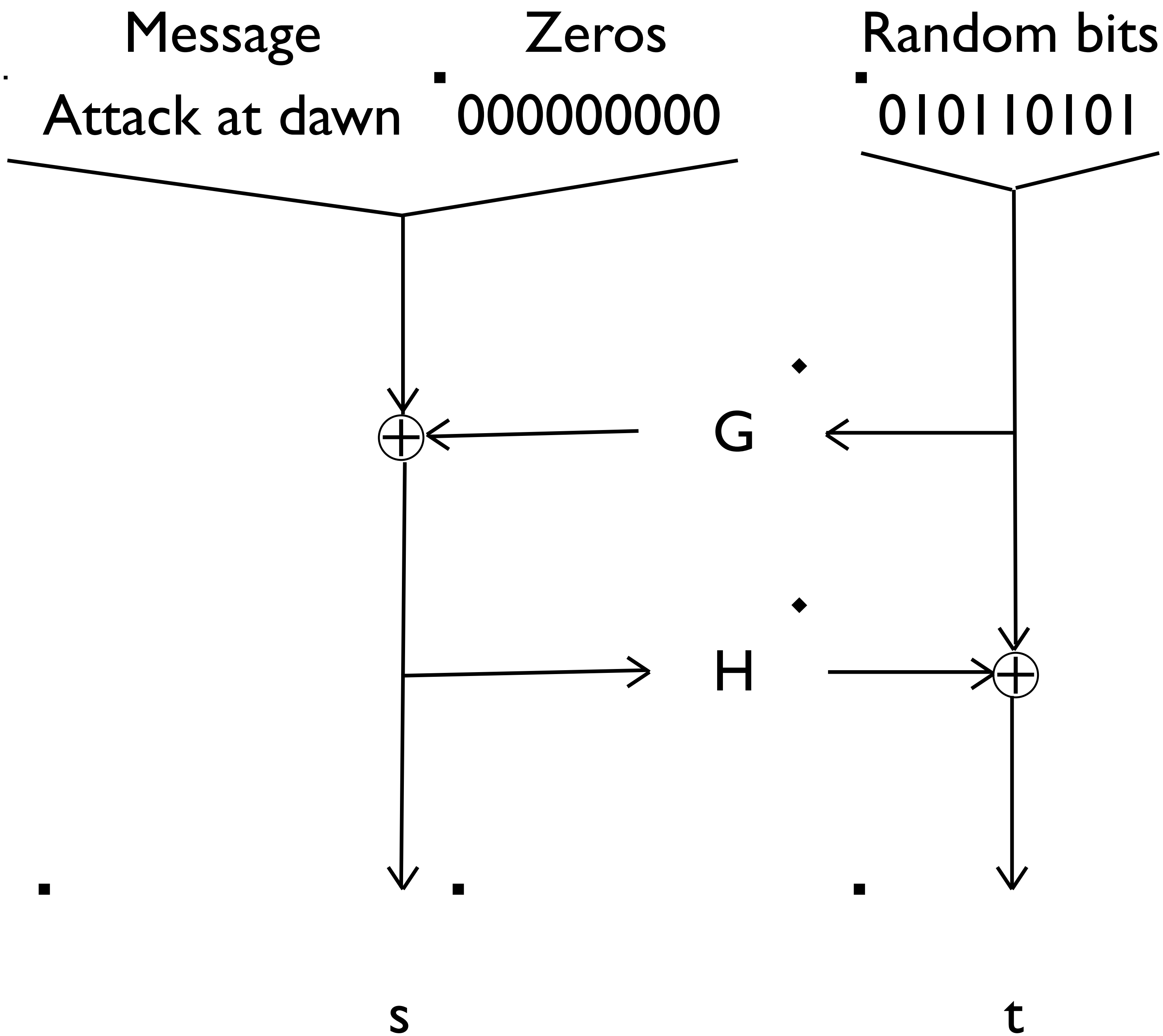
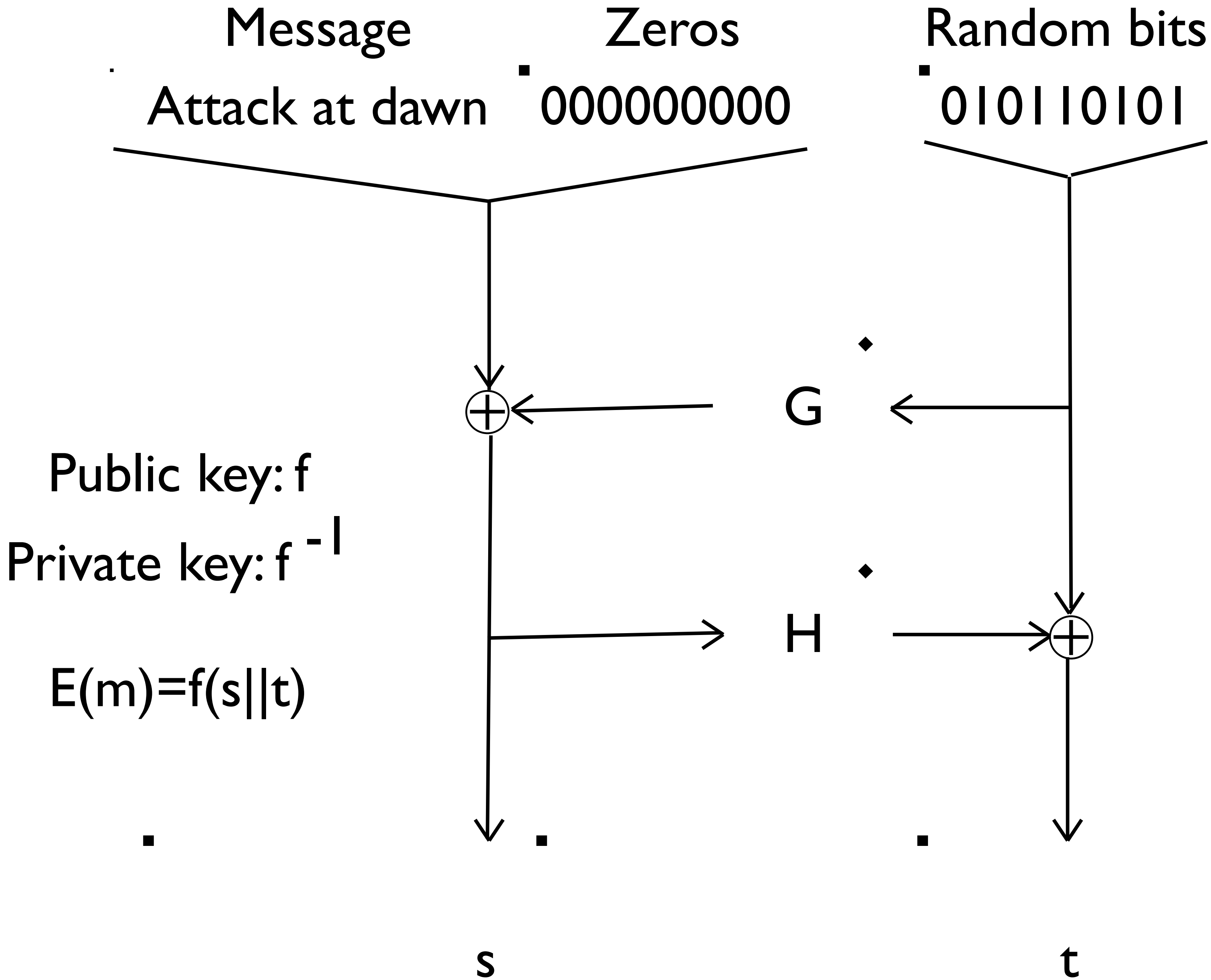


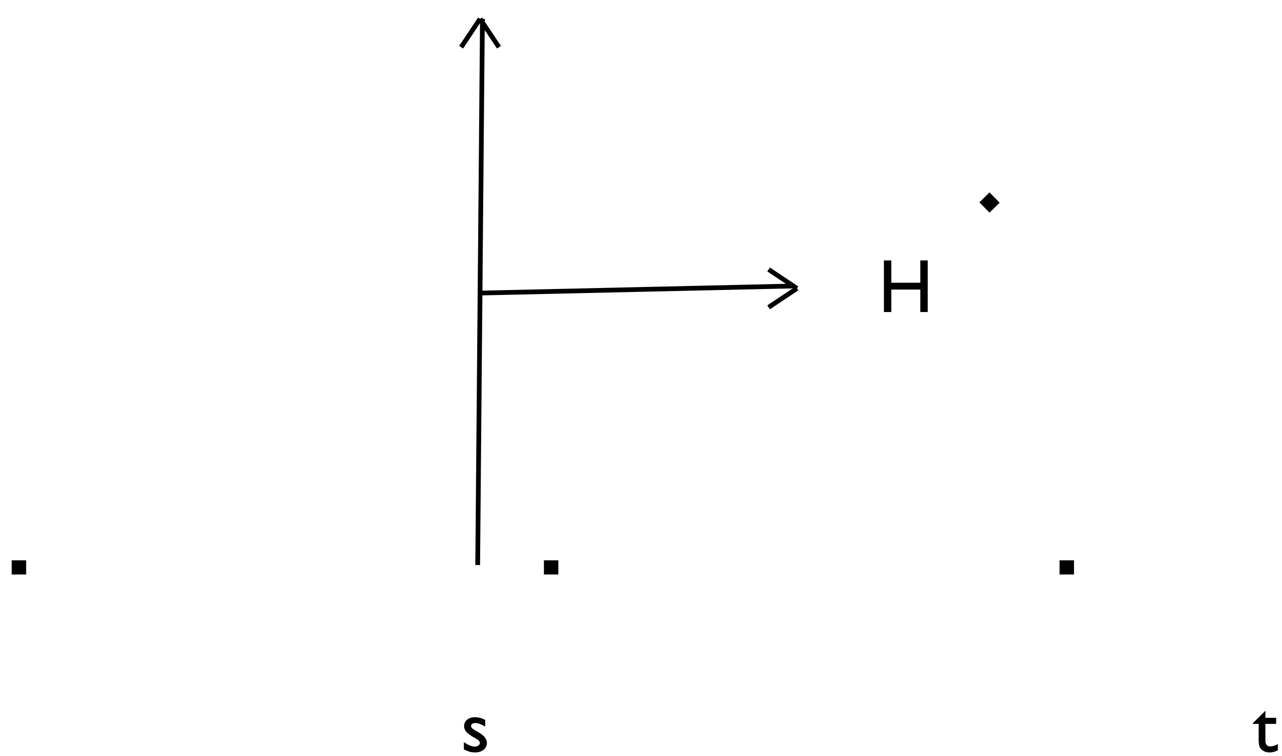
Message
Zeros
Random bits
Attack at dawn
000000000
010110101
G
H
s
t

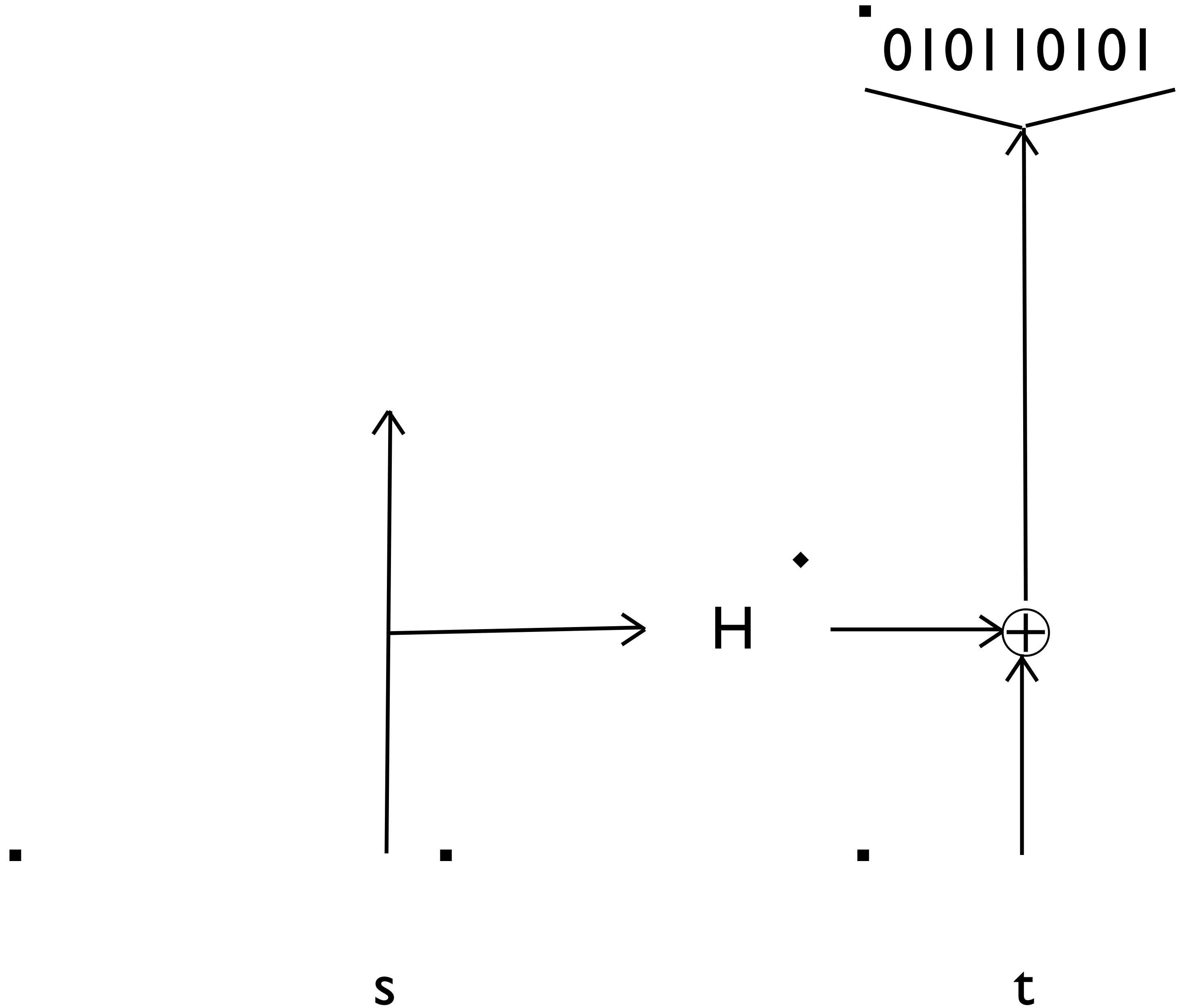

Message
Attack at dawn
Zeros
000000000
Random bits
010110101
G
H
Public key: f
Private key: f -1
E(m)=f(s||t)
s
t

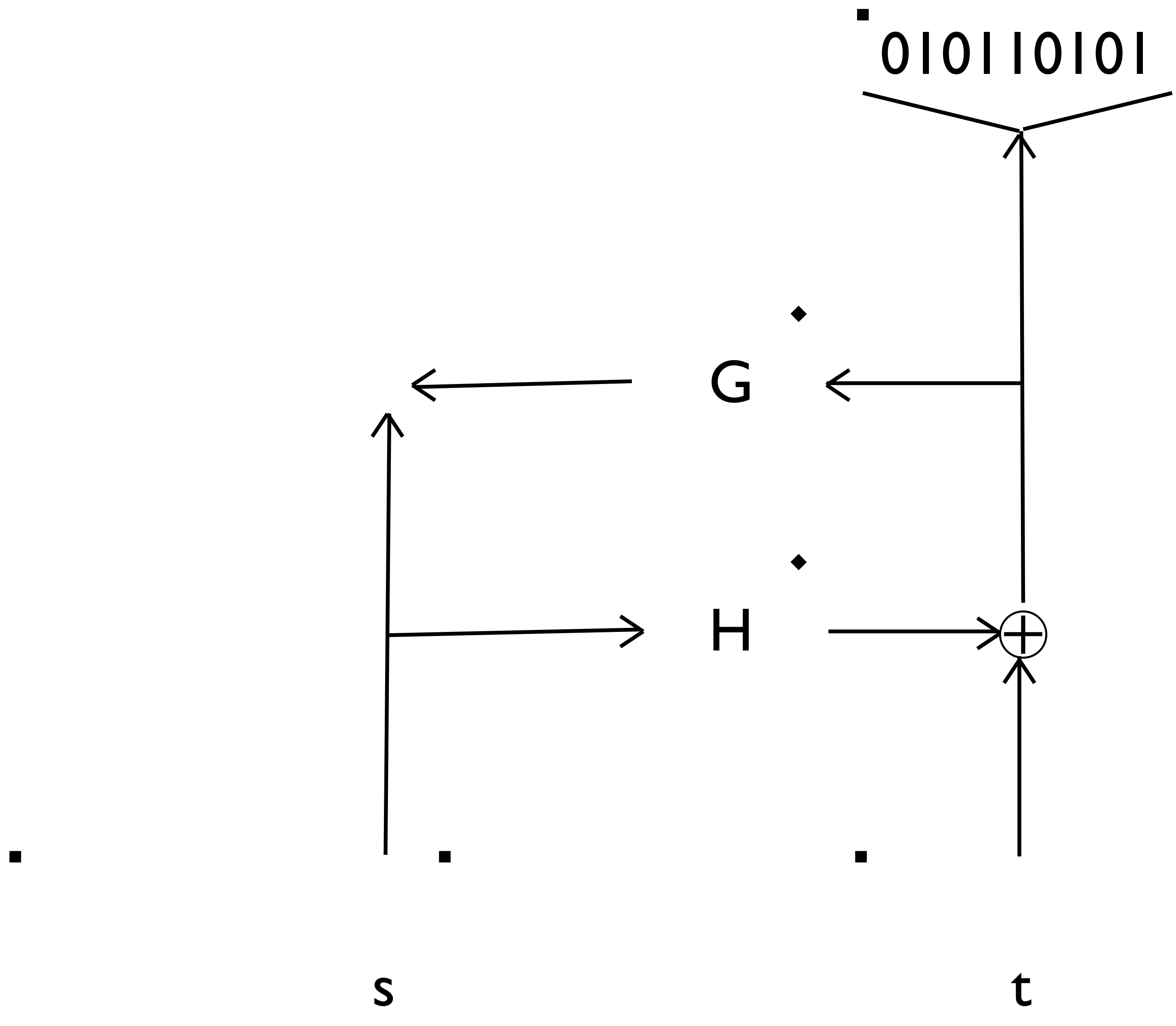
$$s||t = f^{-1}(c)$$

▪ ▪ ▪

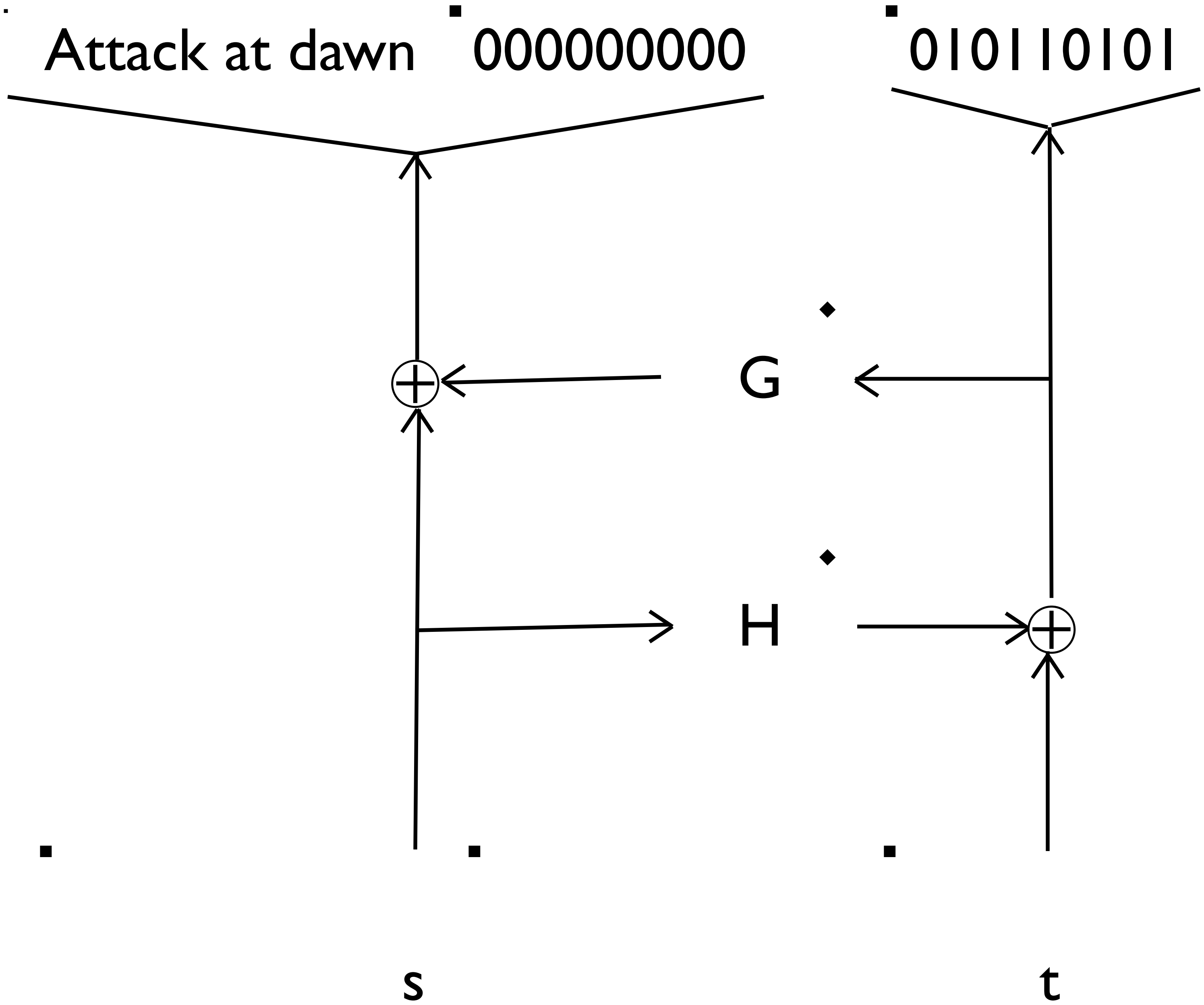
s t

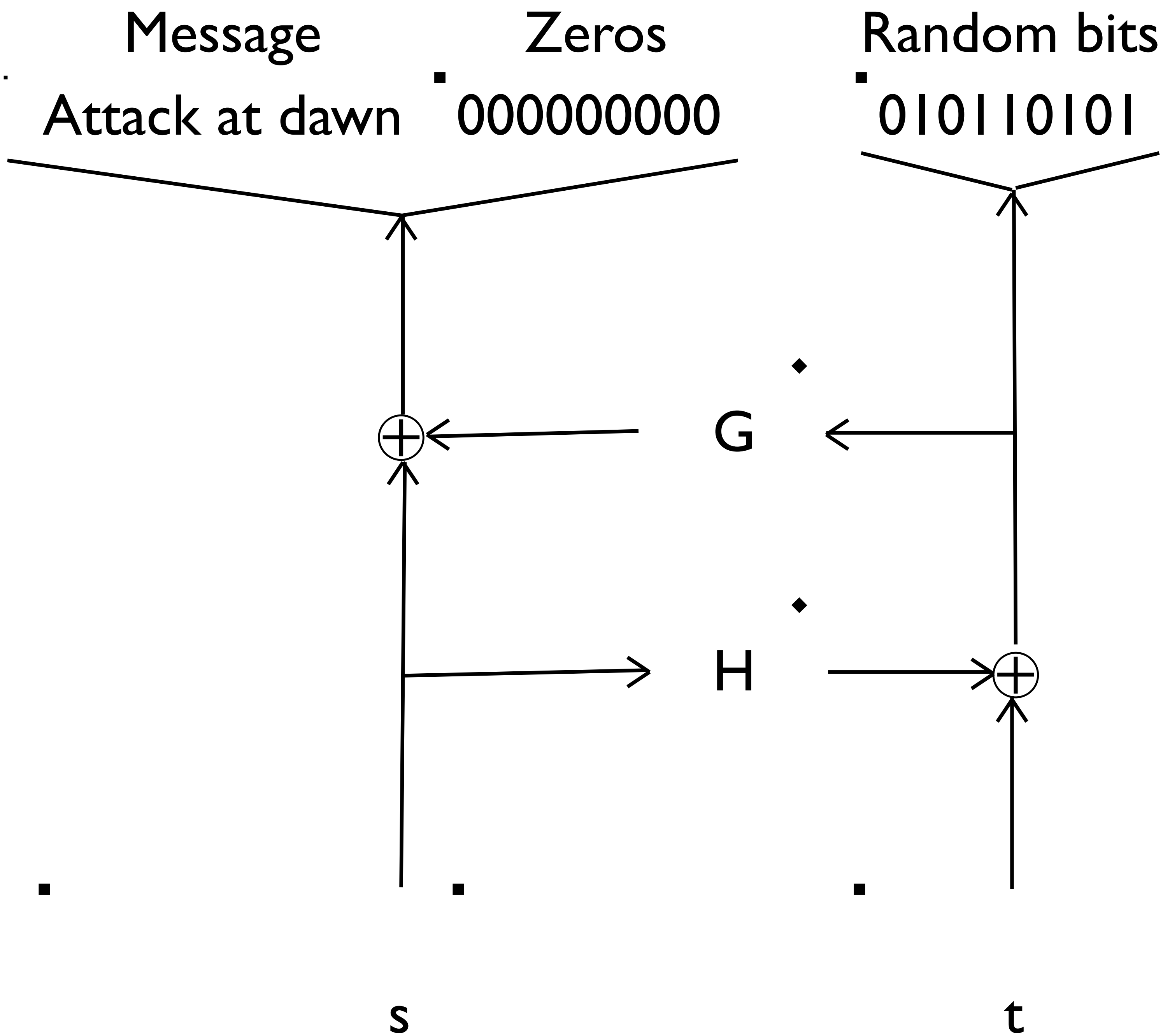

H
s
t

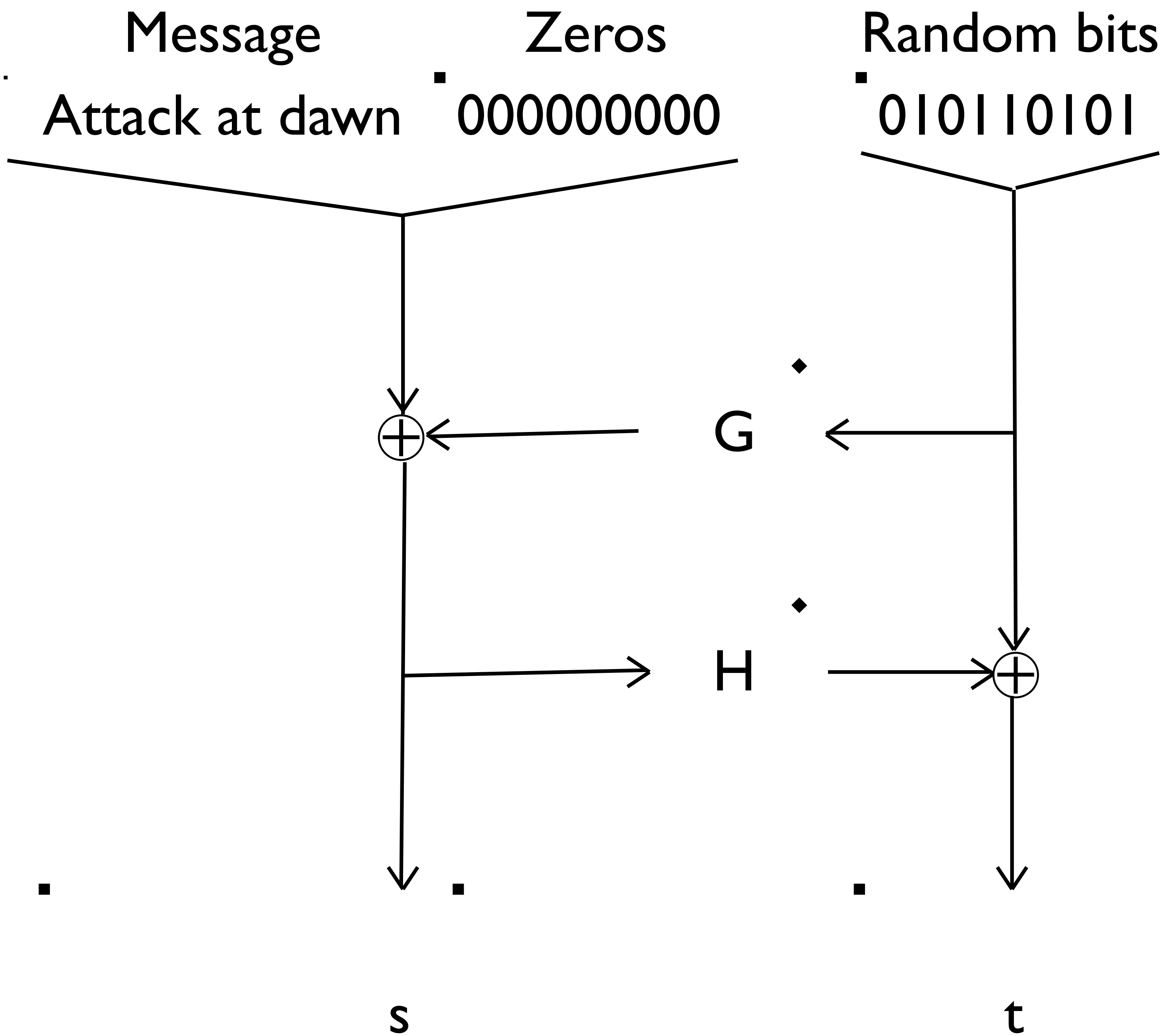

010110101
H
s
t



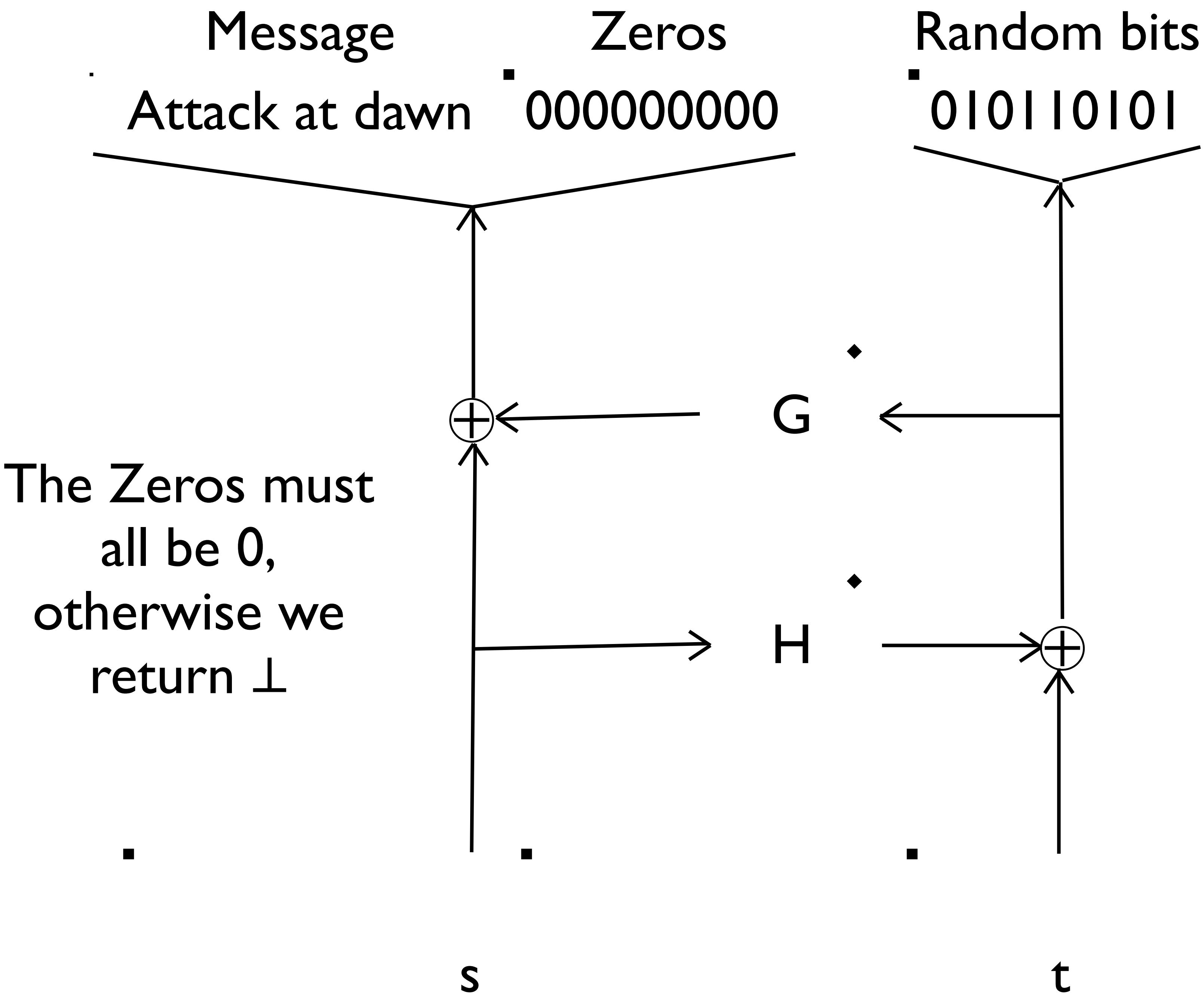
010110101
G
H
s
t


Attack at dawn
000000000
010110101
G
H
s
t


Message
Zeros
Random bits
Attack at dawn
000000000
010110101
G
H
s
t



Message
Zeros
Random bits
Attack at dawn
000000000
010110101
G
H
s
t


Message
Attack at dawn
Zeros
000000000
Random bits
010110101
G
H
The Zeros must all be 0, otherwise we return ⊥
s
t

# What are G and H?

- Publicly computable (no keys)
- Randomish
- Onewayish
- Collision resistantish
- None of these properties are *sufficient*

# Real Cryptographic Hash Functions

- Unkeyed SHA-1 is (hopefully):
  - Collision resistant
  - One-way
  - “Random looking”
  - And more...

# Need Some Way To Model These Functions

- Can't enumerate all the properties they're supposed to have, but have some intuition
- We will replace these functions with something that has all the properties that we want hash functions to have, but we'll overshoot
- No real function has the properties we claim

# Random Oracles

R

# Random Oracles

x R ◆

# Random Oracles

x R 010010110101...

Each bit of the output
is chosen uniformly
at random

# Random Oracles

y R ◆ 110100100111...

# Random Oracles

x R 01001011010101...

On the same input always returns the same output

# Random Oracles

x R 010010110101...

If you want a shorter output just ignore the rest

# Key Thing To Note

- There's no way to figure out anything about the output of R when given x short of asking R for the output
- So, if the adversary knows R(x) we know he must have asked R for it

# Random Oracles Can't Exist

- We will *approximate* them with cryptographic hash functions
- We will *prove* that a construction that uses random oracles is secure
- We then implement the construction using cryptographic hash functions and *hope* that the hash functions are a good approximation

# Why Does This Make Sense?

- We want to accomplish some real world goal
- Some construction is going to be used no matter what
- If we can't prove anything about any of the efficient constructions without random oracles, we might as well use one that we can prove secure under the R.O. assumption

# Proof of Security

- Similar game to before:
  - Adversary given access to encryption and decryption oracles
  - Also given access to the random oracles G and H
  - Given the encryption of either $m_0$ or $m_1$, has to decide which it is

# Break OAEP, you've broken the OWTP

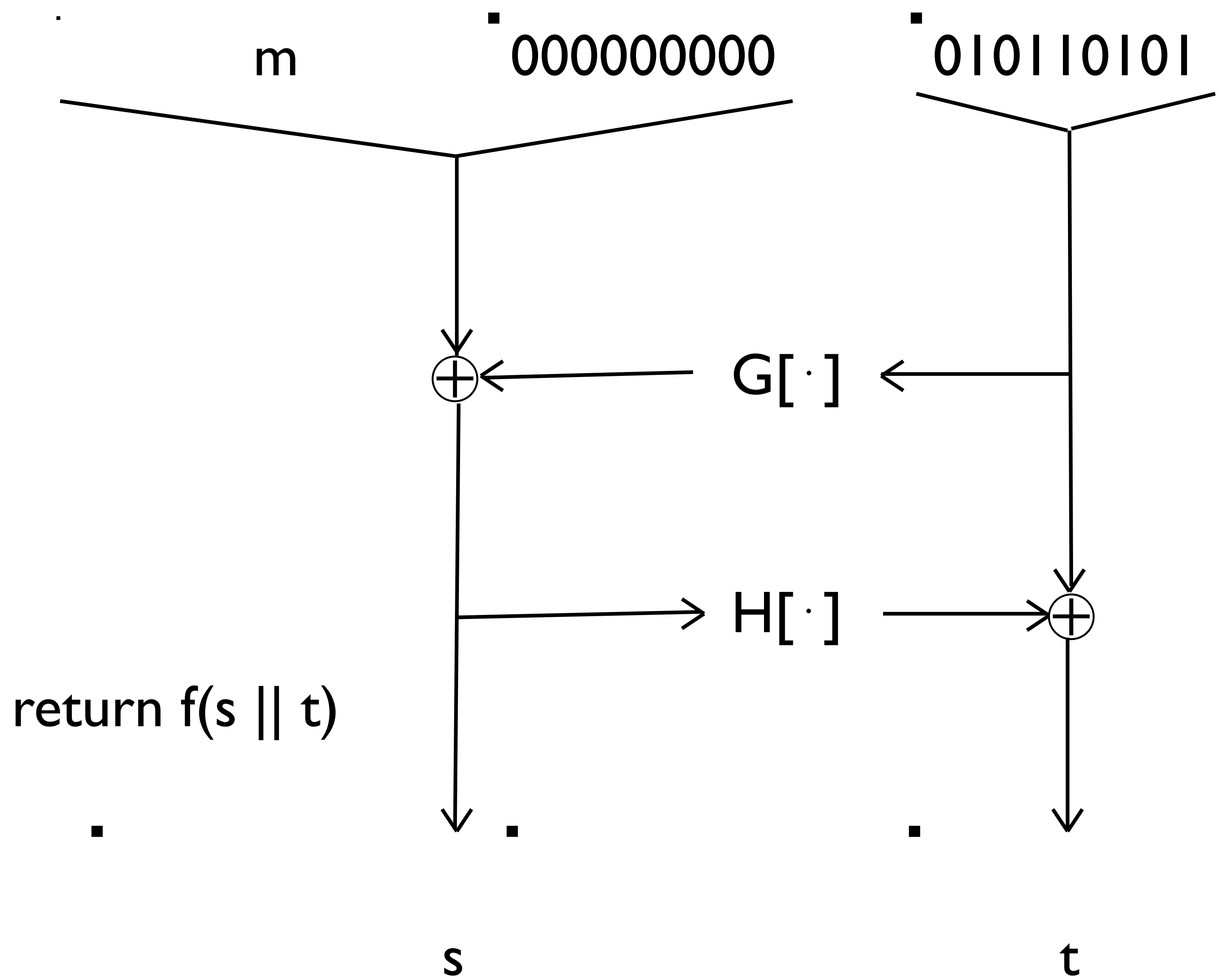
- Use the adversary that breaks OAEP to break the underlying one-way trapdoor permutation
- If the adversary can win at the $m_0$ or $m_1$ game, we can invert f (i.e. given a y, come up with x s.t. $f(x) = y$)

```
Adversary B(f, y)
// Wants to find x s.t. f(x) = y
Run A
      When A asks for G(x):
        See if G[x] exists, if so return it
        Generate G[x] at random, return it
      When A asks for H(x):
        See if H[x] exists, if so return it
        Generate H[x] at random, return it
...
```

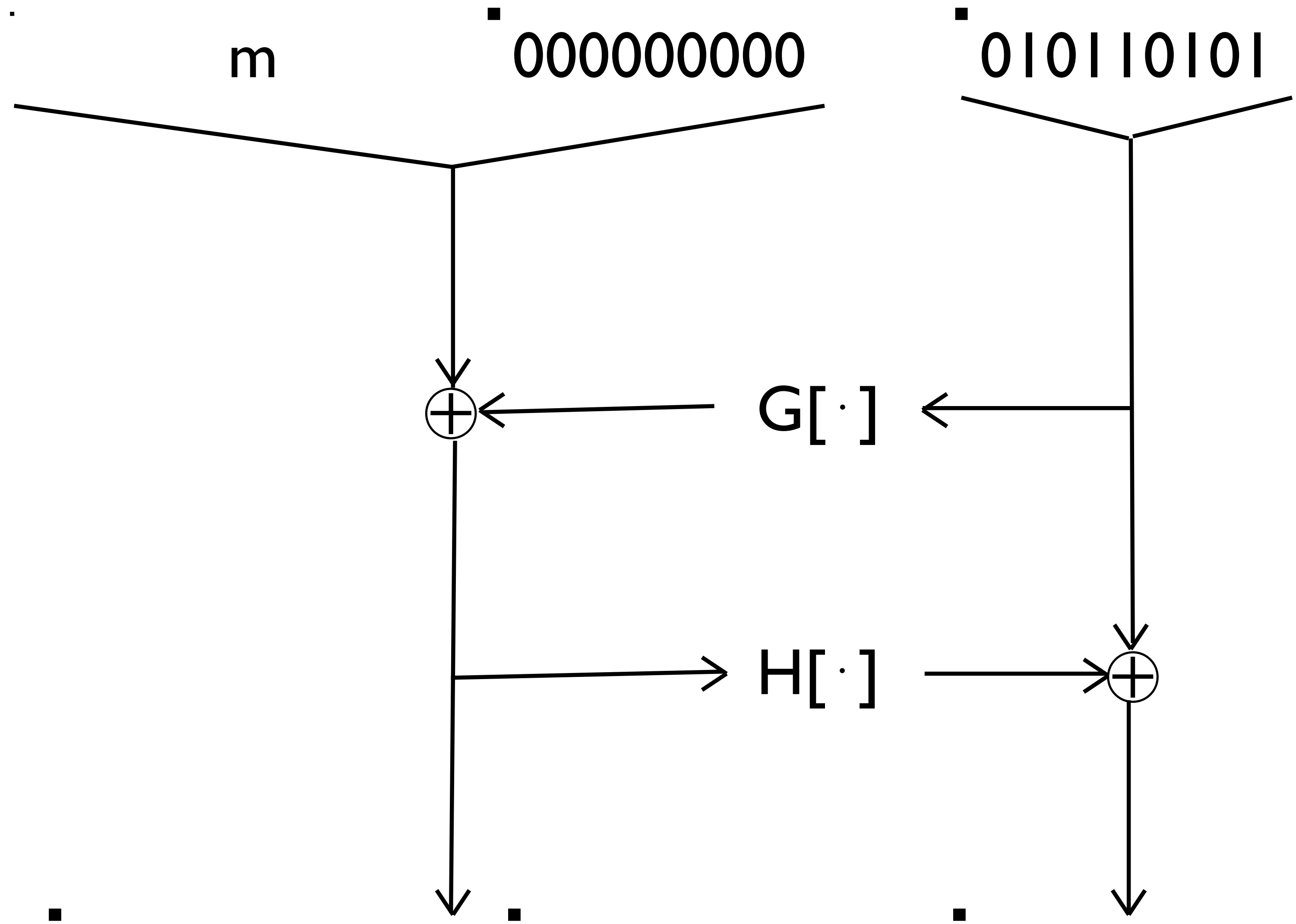
Adversary B(f, y)
// Wants to find x s.t. f(x) = y
Run A
    When A asks for G(x):
        See if G[x] exists, if so return it
        Generate G[x] at random, return it
    When A asks for H(x):
        See if H[x] exists, if so return it
        Generate H[x] at random, return it
...

Just a table

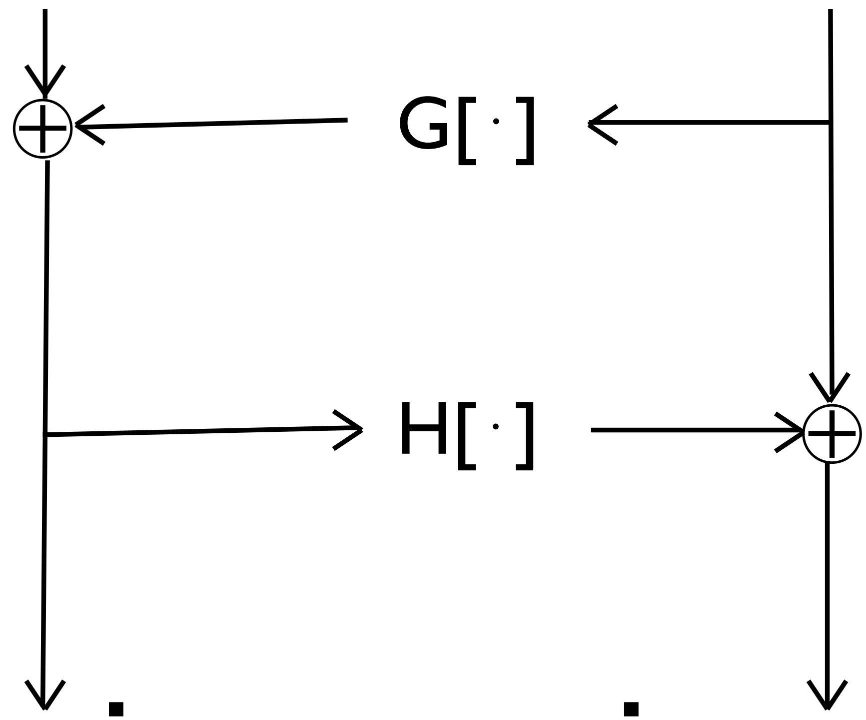
When A asks for E(m):



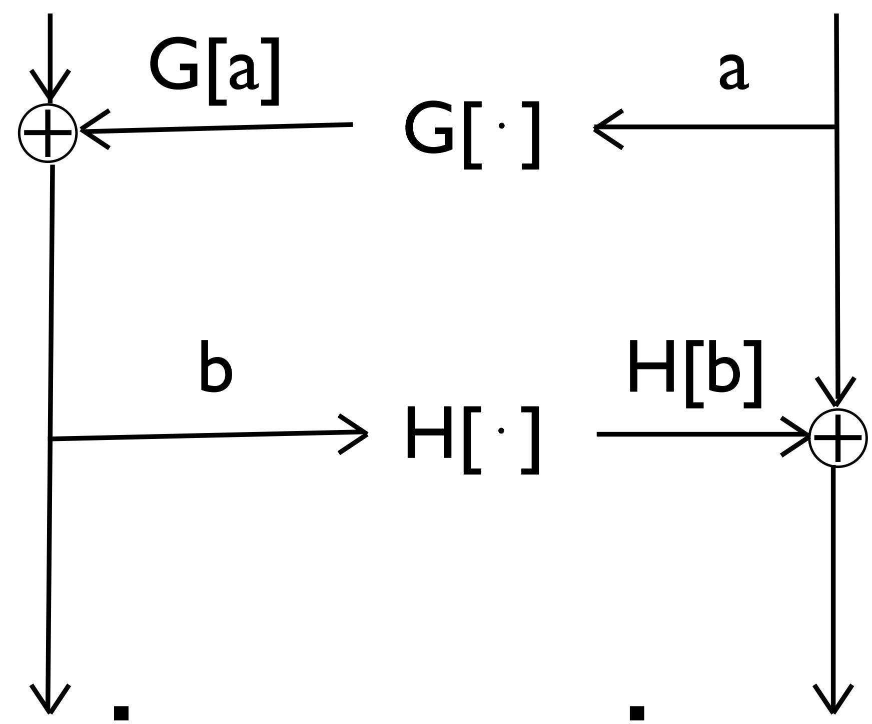
When A asks for D(c):



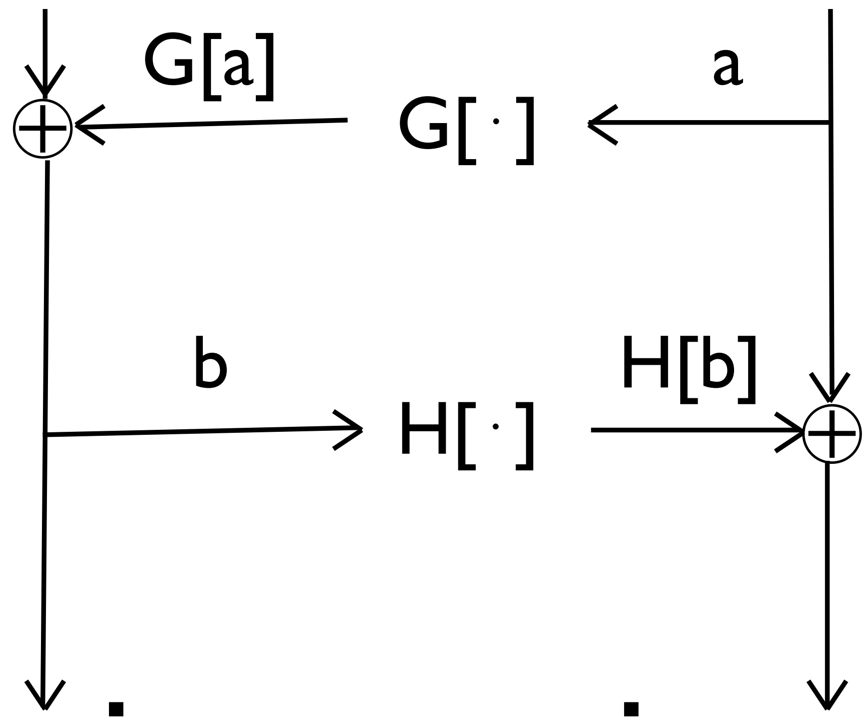
# When A asks for D(c):



When A asks for D(c):



# When A asks for D(c):

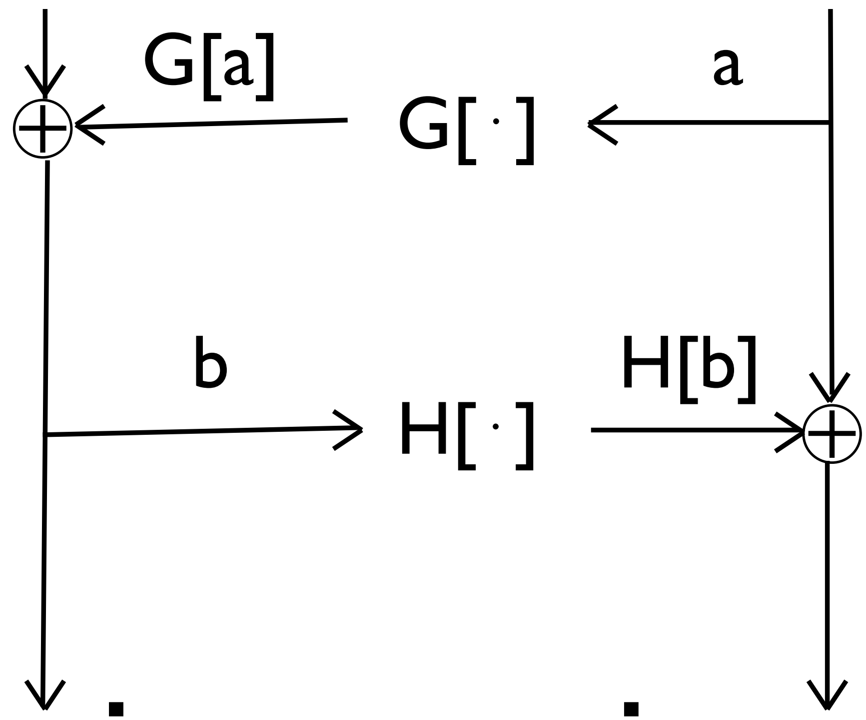



s = b

$t = a \oplus H[b]$

When A asks for D(c):

$G[a] \oplus b$ $\qquad$ $a$

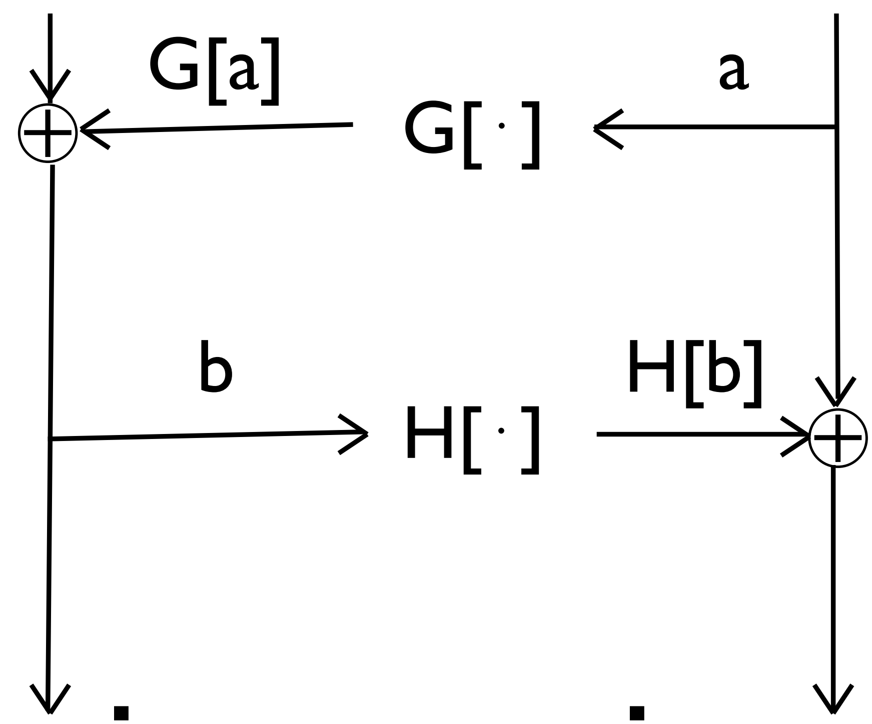



$s = b$ $\qquad$ $t = a \oplus H[b]$

When A asks for D(c):

$G[a] \oplus b$ $\qquad\qquad$ $a$

For index a of G[]
  For index b of H[]
    if $f(b \,||\, a \oplus H[b]) = c$
      if $G[a] \oplus b$ has Zeros
        return $G[a] \oplus b$
return $\perp$



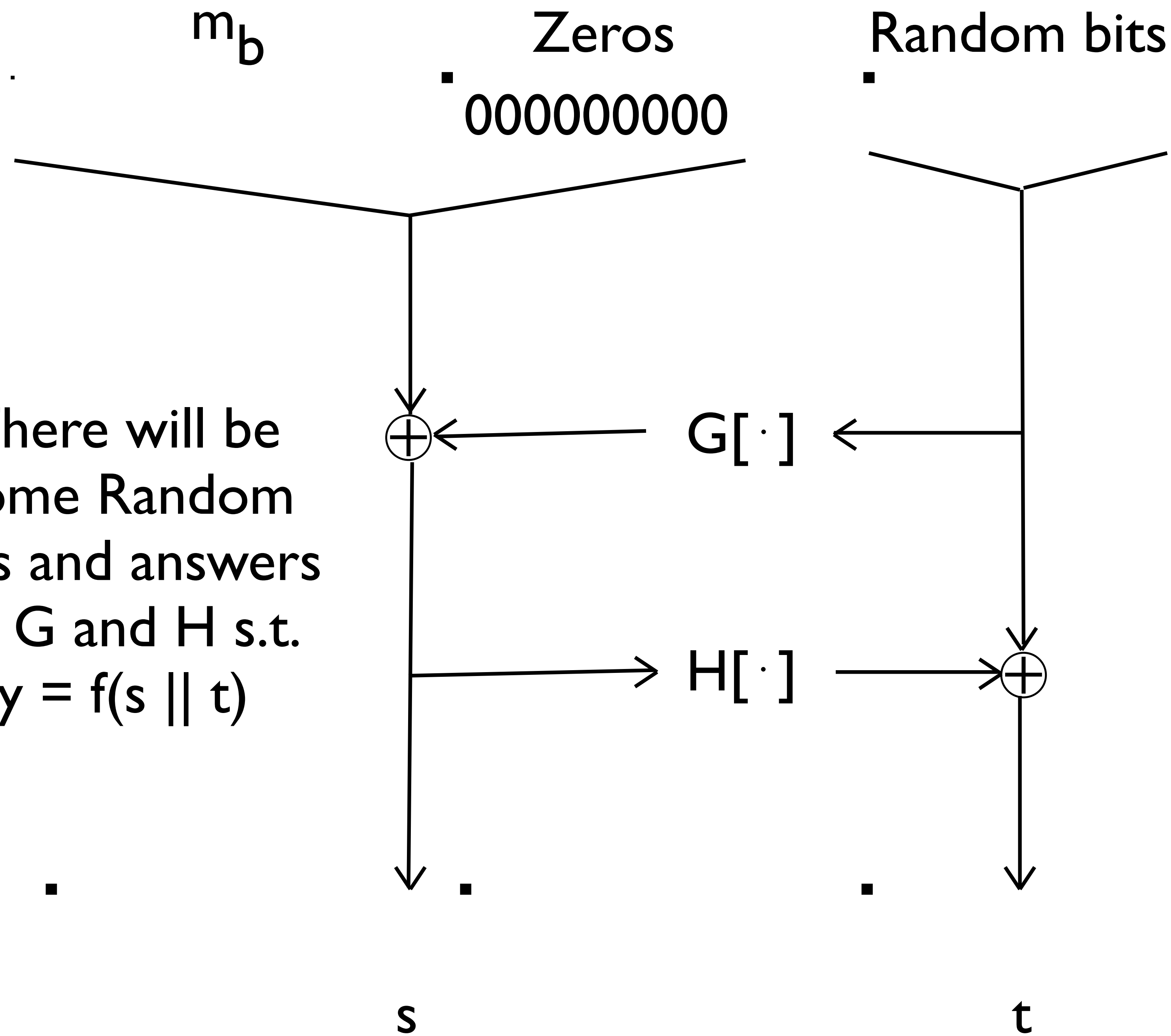

$s = b$ $\qquad\qquad$ $t = a \oplus H[b]$

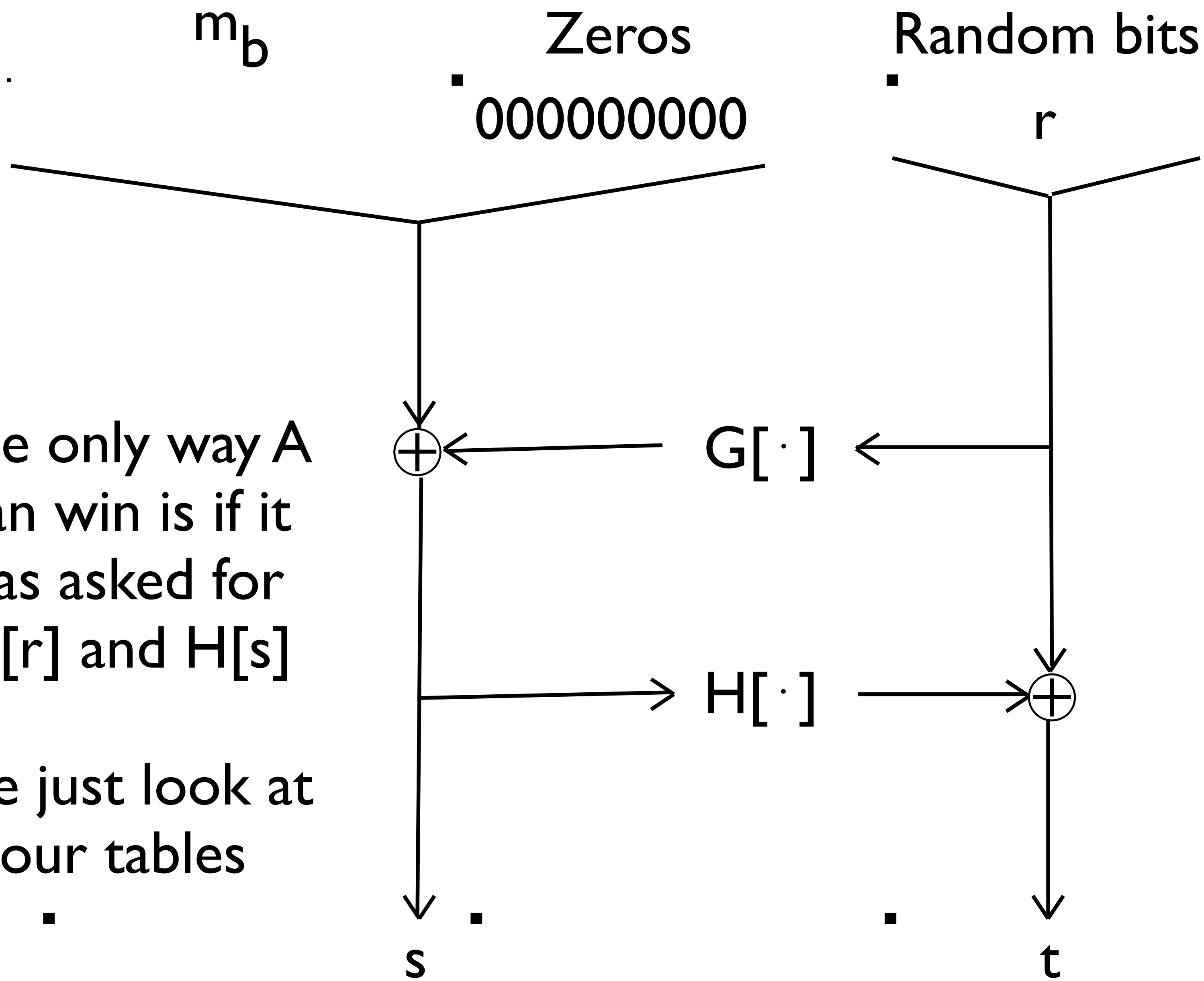
A gives us $m_0$ and $m_1$

No matter what, we say that the encryption is y (remember that y is the thing we're trying to invert)

What if y isn't the encryption of either $m_0$ or $m_1$?

There will be some Random Bits and answers to G and H s.t. $y = f(s \,||\, t)$






The only way A can win is if it has asked for G[r] and H[s]

We just look at our tables

$y = f(x) = f(s \,||\, t)$

# The Result

- If someone can mount a chosen ciphertext attack on OAEP, they can invert the underlying trapdoor permutation *in the random oracle world*

# Not So Fast...

- There's a subtle flaw in the proof
- It took 7 years for someone to find
- OAEP was already being used
- We'll look at what happened