Linear Dependency Modeling for Feature Fusion

Andy J Ma and Pong C Yuen
Department of Computer Science, Hong Kong Baptist University, Hong Kong

Overview

Introduction

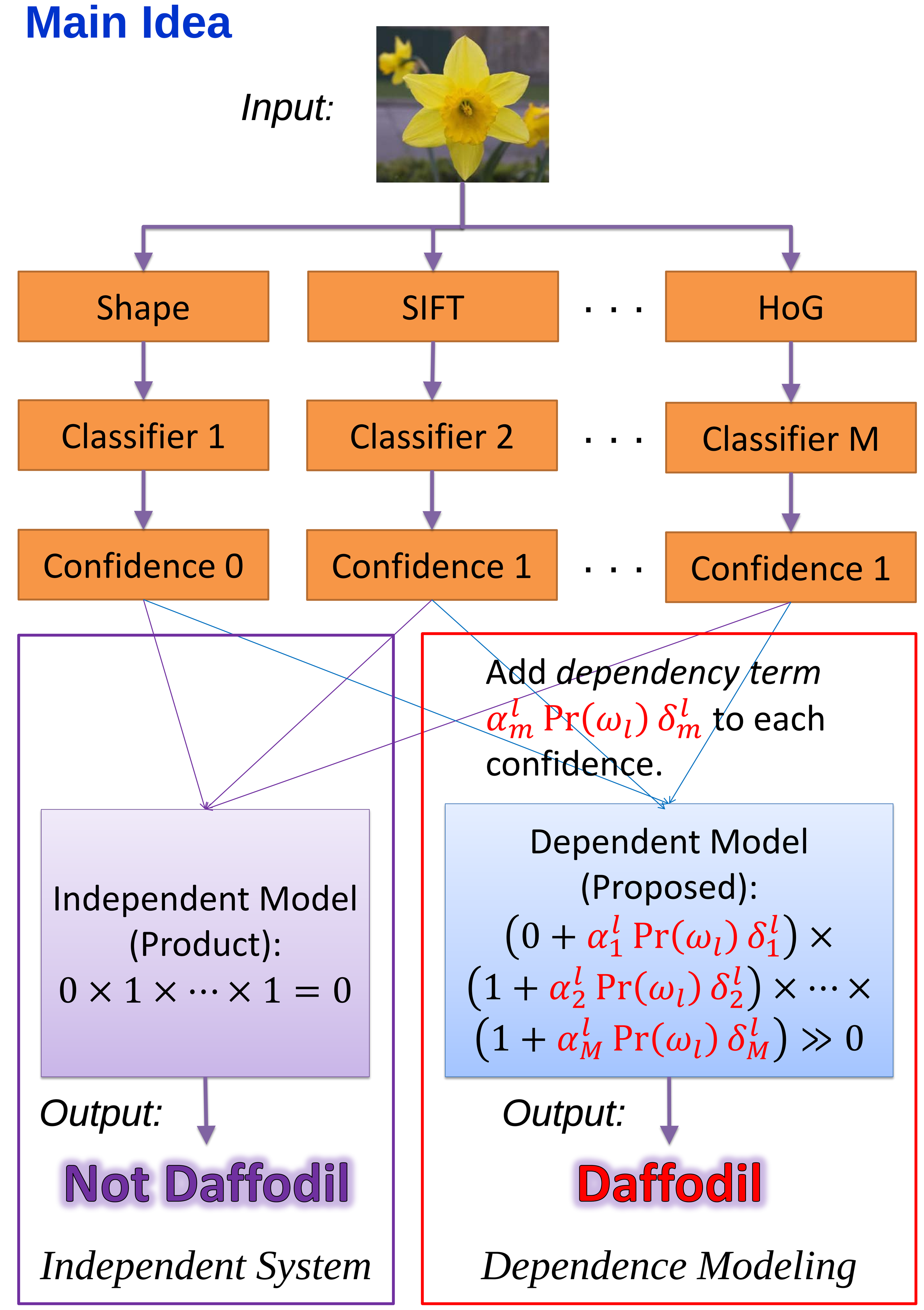
- Multiple features provide complementary information.
- Features can be combined in two levels
  - Classifier level: Train one classifier for each feature, then combine the classifiers.
  - Feature level: Combine the features directly to draw the conclusion.
- Problems in existing methods
  - Features are assumed to be *conditionally independent* in [1], while this may not be the case.
  - Feature level fusion, e.g. MKL [2], *do not* model feature dependency directly.
- *Explicitly* model *feature dependency* to improve the recognition performance.

Contributions

- Solve the problem of *independent* assumption in classifier combination.
- Prove that *linear combination* can model feature dependency under some mild assumptions.
- Develop a *novel* framework for dependency modeling.
- Propose two methods, LCDM and LFDM, for *classifier level* and *feature level fusion*.

Linear Dependency Modeling

Main Idea



Linear Classifier Dependency Model (LCDM)

- Given scores $\Pr(\omega_l|\vec{x}_m)$,
  $\Pr(\omega_l|\vec{x}_1, \dots, \vec{x}_M)$
  $$\propto \sum_{m=1}^{M} \beta_m^l [\Pr(\omega_l|\vec{x}_m) - \Pr(\omega_l)] + \Pr(\omega_l)$$
  with $0 \le \beta_m^l \le 2$ and $\sum_{m=1}^{M} \beta_m^l = M$.

Linear Feature Dependency Model (LFDM)

- Given feature vectors $\vec{x}_1, \dots, \vec{x}_M$,
  $\Pr(\omega_l|x_{11}, \dots, x_{MN_M})$
  $$\propto \sum_{m=1}^{M} \sum_{n=1}^{N_m} \gamma_{mn}^l [\Pr(\omega_l|x_{mn}) - \Pr(\omega_l)] + \Pr(\omega_l)$$
  with $0 \le \gamma_{mn}^l \le 2$ and $\sum_{m=1}^{M} \sum_{n=1}^{N_m} \gamma_{mn}^l = \sum_{m=1}^{M} N_m$.

Learn Optimal Dependency Model

- Maximize the margin between genuine and imposter posterior probabilities.
- Learn the optimal β in LCDM and γ in LFDM by solving Linear Programming problems.

Sensitivity to Density Estimation Error

- The error factors in LCDM and LFDM are
  $$E_c = \frac{\sum_{m=1}^{M} \beta_m^l e_m^l}{\sum_{m=1}^{M} \beta_m^l \Pr(\omega_l|\vec{x}_m)} \text{ and } E_f = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N_m} \gamma_{mn}^l \epsilon_{mn}^l}{\sum_{m=1}^{M} \sum_{n=1}^{N_m} \gamma_{mn}^l \Pr(\omega_l|x_{mn})}.$$
- The upper bound of E_f is smaller than that of E_c.

Experiments

Mean error rate (%) and standard deviation on synthetic data.

Test	Sum [1]	LPBoost [3]	LP-B [4]	IN [5]	DN [5]	LCDM
IndNormal	4.66±0.71	3.19±0.48	2.49±0.48	**2.33±0.40**	2.44±0.42	2.34±0.46
DepNormal	13.56±1.16	4.71±0.85	6.83±1.30	7.48±0.98	**4.36±0.89**	6.12±0.93
IndNonNor	25.33±1.50	9.9±1.61	0.11±0.08	15.2±1.65	8.59±1.38	**7.00±0.07**
DepNonNor	36.67±0.89	31.05±1.30	30.63±1.9	34.54±0.92	30.16±1.35	**27.86±1.52**

- LCDM outperforms others in non-normal cases.

Mean error rate (%) and standard deviation on Flower Database.

Best Feature	Sum [1]	LPBoost [3]	LP-B [4]	IN [5]	DN [5]	LCDM
70.4±1.4	85.4±3.1	82.7±0.8	85.5±2.4	85.5±1.7	84.2±1.9	**86.3±2.4**

Best accuracy (%) on Weizmann and KTH databases.

Classifier level	Wei	KTH
Sum [1]	84.44	84.72
LPBoost [3]	83.33	83.33
LP-B [4]	84.44	**85.19**
IN [5]	**85.56**	84.26
DN [5]	84.44	83.80
LCDM	**85.56**	**85.19**

Feature level	Wei	KTH
Sum-F [1]	57.78	78.70
MKL [2]	**81.11**	**82.42**
LPBoost-F [3]	68.89	75.93
LP-B-F [4]	70.00	76.56
IN-F [5]	68.89	77.31
LFDM	**86.67**	**88.43**

- Both Flower and Action databases convince the proposed methods, LCDM and LFDM.
- LFDM outperform the others in action databases.

Reference

1. J. Kittler, M. Hatef, R. P. W. Duin, and J. Matas. On combining classifiers. TPAMI, 20(3):226–239, 1998.
2. A. Rakotomamonjy, F. Bach, S. Canu, and Y. Grandvalet. SimpleMKL. JMLR, 9:2491–2521, 2008.
3. A. Demiriz, K. P. Bennett, and J. Shawe-Taylor. Linear programming boosting via column generation. JMLR, 46(1-3):225-254, 2002.
4. P. Gehler and S. Nowozin. On feature combination for multiclass object classification. ICCV, pages 221–228, 2009.
5. O. R. Terrades, E. Valveny, and S. Tabbone. Optimal classifier fusion in a non-bayesian probabilistic framework. TPAMI, 31(9):1630–1644, 2009.