Overview

Introduction
- Multiple features provide complementary information.
- Features can be combined in two levels
  - Classifier level: Train one classifier for each feature, then combine the classifiers.
  - Feature level: Combine the features directly to draw the conclusion.
- Problems in existing methods
  - Features are assumed to be conditionally independent in [1], while this may not be the case.
  - Feature level fusion, e.g. MKL [2], do not model feature dependency directly.
- Explicitly model feature dependency to improve the recognition performance.

Contributions
- Solve the problem of independent assumption in classifier combination.
- Prove that linear combination can model feature dependency under some mild assumptions.
- Develop a novel framework for dependency modeling.
- Propose two methods, LCDM and LFDM, for classifier level and feature level fusion.

Main Idea

Input:

Shapes
SIFT
...  
HoG
Classifier 1
Classifier 2
...  
Classifier M
Confidence 0
Confidence 1
...  
Confidence 1

Output:

Not Daffodil
Independent System
Daffodil
Dependence Modeling

Linear Classifier Dependency Model (LCDM)
- Given scores $\text{Pr}(\omega_l | \mathbf{x}_m)$,
  $\text{Pr}(\omega_l | \mathbf{x}_1, ..., \mathbf{x}_M)
  \propto \sum_{m=1}^{M} \beta_m [\text{Pr}(\omega_l | \mathbf{x}_m) - \text{Pr}(\omega_l)] + \text{Pr}(\omega_l)$
with $0 \leq \beta_m \leq 2$ and $\sum_{m=1}^{M} \beta_m = M$.

Linear Feature Dependency Model (LFDM)
- Given feature vectors $\mathbf{x}_1, ..., \mathbf{x}_M$,
  $\text{Pr}(\omega_l | \mathbf{x}_{mn})
  \propto \sum_{m=1}^{M} \sum_{n=1}^{N_m} y_m^n \text{Pr}(\omega_l | \mathbf{x}_{mn}) - \text{Pr}(\omega_l) + \text{Pr}(\omega_l)$
with $0 \leq y_m^n \leq 2$ and $\sum_{m=1}^{M} \sum_{n=1}^{N_m} y_m^n = \sum_{m=1}^{M} N_m$.

Learn Optimal Dependency Model
- Maximize the margin between genuine and imposter posterior probabilities.
- Learn the optimal $\beta$ in LCDM and $\gamma$ in LFDM by solving Linear Programming problems.

Sensitivity to Density Estimation Error
- The error factors in LCDM and LFDM are
  $E_c = \frac{\sum_{m=1}^{M} \beta_m \text{Pr}(\omega_l | \mathbf{x}_m)}{\sum_{m=1}^{M} \beta_m \text{Pr}(\omega_l | \mathbf{x}_m)}$ and $E_f = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N_m} y_m^n \text{Pr}(\omega_l | \mathbf{x}_{mn})}{\sum_{m=1}^{M} \sum_{n=1}^{N_m} y_m^n \text{Pr}(\omega_l | \mathbf{x}_{mn})}$.
- The upper bound of $E_f$ is smaller than that of $E_c$.

Experiments

Mean error rate (%) and standard deviation on synthetic data.

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<tbody>
<tr>
<td>IndNormal</td>
<td>4.66±0.71</td>
<td>3.19±0.48</td>
<td>2.49±0.48</td>
<td>2.33±0.40</td>
<td>2.44±0.42</td>
<td>2.34±0.46</td>
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<tr>
<td>DepNormal</td>
<td>13.56±1.16</td>
<td>4.7±1.85</td>
<td>6.8±1.30</td>
<td>7.48±0.98</td>
<td>4.36±0.89</td>
<td>6.1±0.93</td>
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<tr>
<td>IndNonNor</td>
<td>25.33±1.50</td>
<td>9.9±1.61</td>
<td>1.1±0.08</td>
<td>15.2±1.65</td>
<td>8.59±1.38</td>
<td>7.0±0.07</td>
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<tr>
<td>DepNonNor</td>
<td>36.67±0.89</td>
<td>31.05±1.30</td>
<td>30.63±1.9</td>
<td>34.54±0.92</td>
<td>30.16±1.35</td>
<td>27.86±1.52</td>
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- LCDM outperforms others in non-normal cases.

Mean error rate (%) and standard deviation on Flower Database.

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<tbody>
<tr>
<td>70.4±1.4</td>
<td>85.4±3.1</td>
<td>82.7±0.8</td>
<td>85.5±2.4</td>
<td>85.5±1.7</td>
<td>84.2±1.9</td>
<td>86.3±2.4</td>
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</tbody>
</table>
- Both Flower and Action databases convince the proposed methods, LCDM and LFDM.
- LFDM outperform the others in action databases.

Linear Dependency Modeling for Feature Fusion
Andy J Ma and Pong C Yuen
Department of Computer Science, Hong Kong Baptist University, Hong Kong

Best accuracy (%) on Weizmann and KTH databases.

<table>
<thead>
<tr>
<th>Classifier level</th>
<th>Wei</th>
<th>KTH</th>
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<tbody>
<tr>
<td>Sum [1]</td>
<td>84.44</td>
<td>84.72</td>
</tr>
<tr>
<td>LPBoost [3]</td>
<td>83.33</td>
<td>83.33</td>
</tr>
<tr>
<td>LP-B [4]</td>
<td>84.44</td>
<td>85.19</td>
</tr>
<tr>
<td>IN [5]</td>
<td>85.56</td>
<td>84.26</td>
</tr>
<tr>
<td>DN [5]</td>
<td>84.44</td>
<td>83.80</td>
</tr>
<tr>
<td>LCDM</td>
<td>85.86</td>
<td>85.19</td>
</tr>
</tbody>
</table>
- Feature level Wei and KTH
| Feature level Sum-F [1] | 67.78 | 78.70|
| LP-B-F [4]       | 81.11 | 82.42|
| IN-F [5]         | 68.89 | 77.31|
| LFDM             | 86.67 | 88.43|

Reference