

# Stacking Sigma

A Framework to Compose  $\Sigma$ -Protocols for Disjunctions

Aarushi Goel

Matthew Green

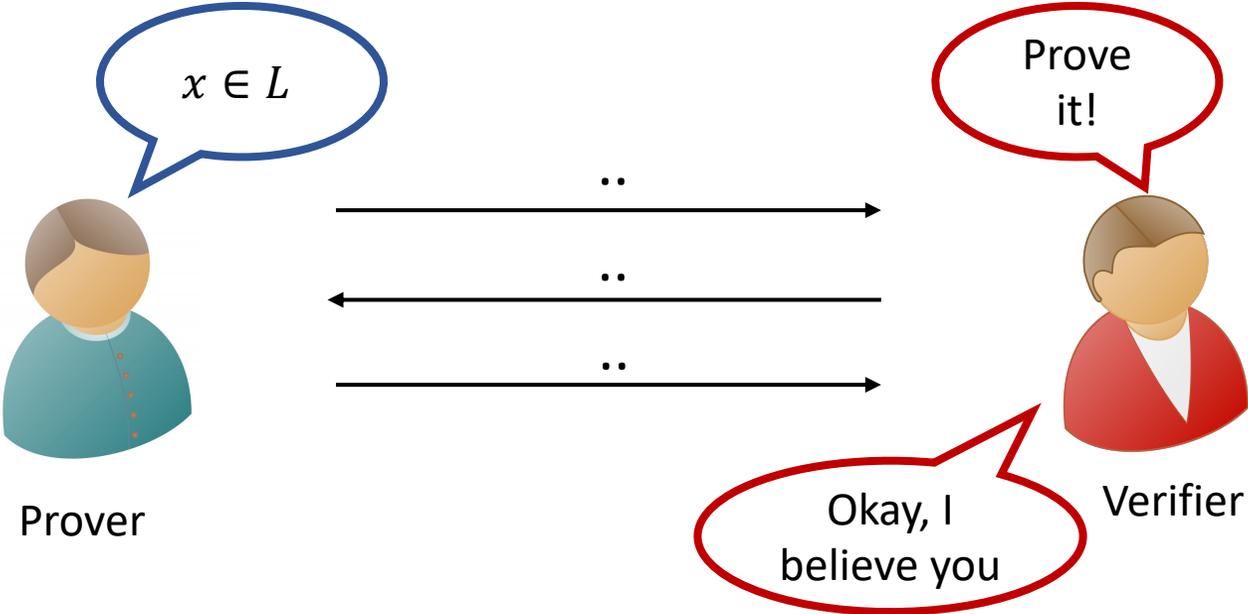
Mathias Hall-Andersen

Gabriel Kaptchuk

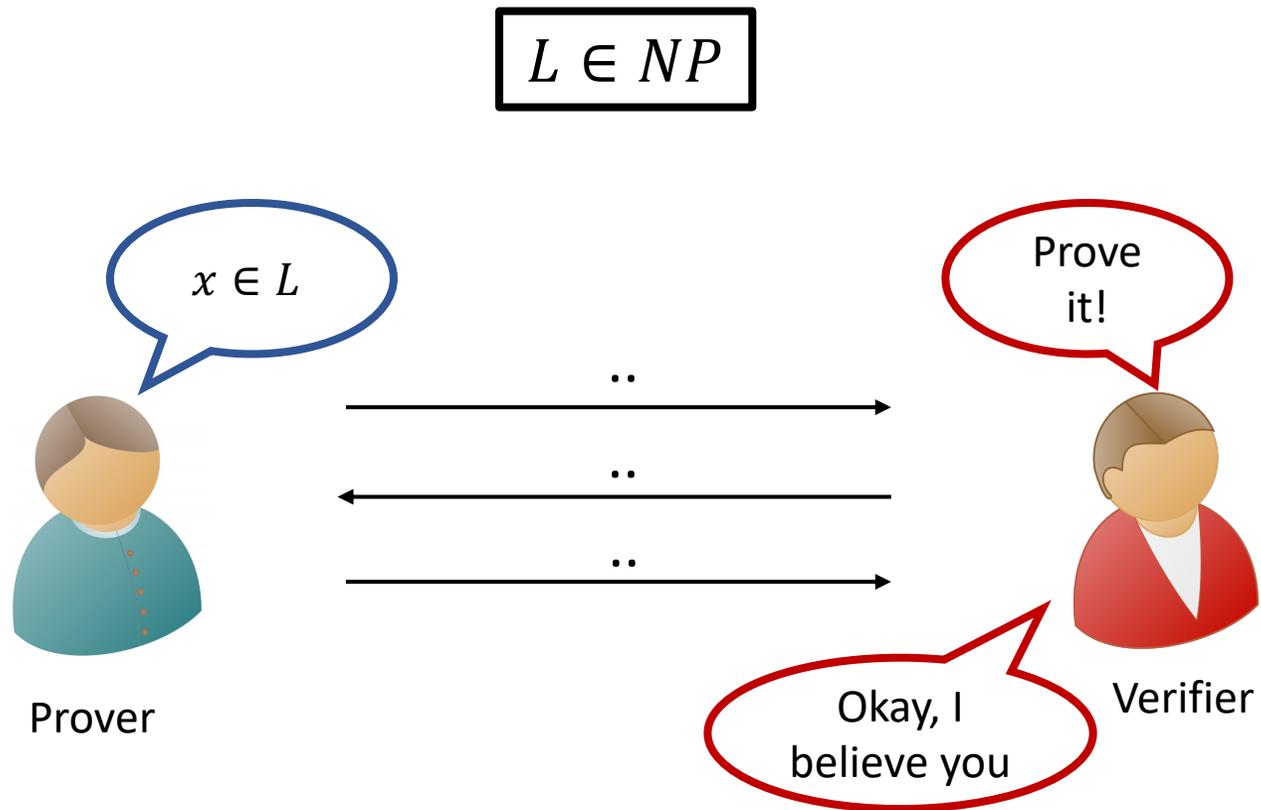


# Zero Knowledge Proofs

$L \in NP$



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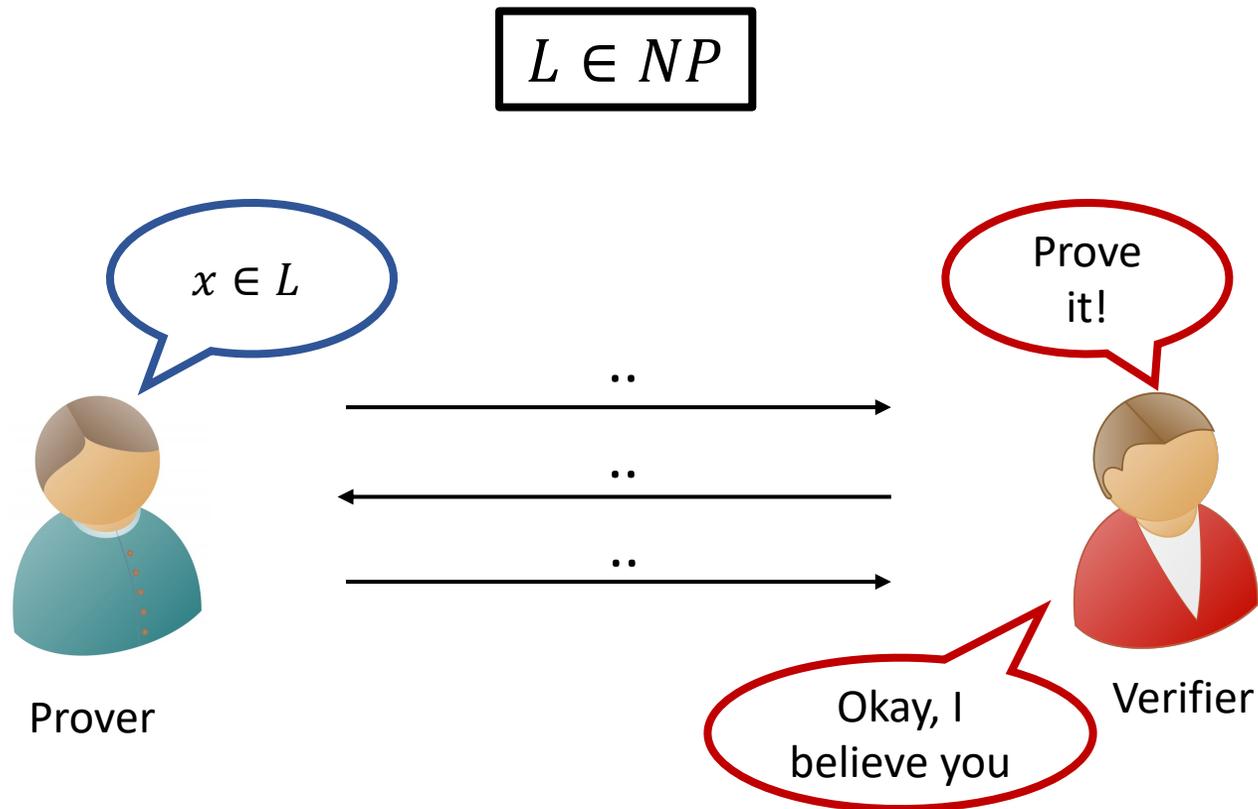


Soundness

Cheating prover should not be able to convince the verifier if

$x \notin L$

# Zero Knowledge Proofs



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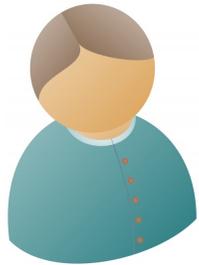
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Zero knowledge

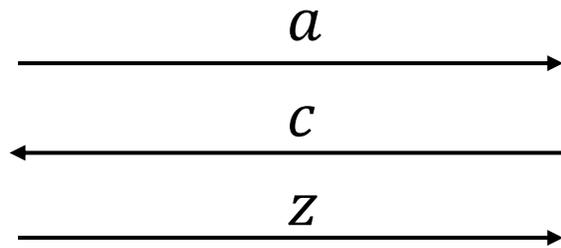
Verifier should not learn anything other than the validity of the statement

# Sigma Protocols

$$L \in NP$$



Prover



Verifier

Public coin proofs

Honest verifier zero-knowledge

Can be made non-interactive in the  
random oracle model

# Disjunctive Statements: Interesting Class of Languages

$x_1 \in L_1$  or  $x_2 \in L_2$  or ..... or  $x_n \in L_n$

Where each  $L_i \in NP$

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## Applications:

Set-Membership Proofs – Ring signatures, confidential transactions

Proving existence of bugs in codebase

Proving correct execution of a processor

.....

# Zero-Knowledge Proofs for Disjunctive Statements

Result	General Compiler	Languages	Proof Size	Prover Time	Non-interactive
<b>Classical</b> [CDS94, AOS02]	For $\Sigma$ -Protocols	All	Linear in all the branches	Linear in all the branches	Random Oracle Model

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Our Work	For $\Sigma$ -Protocols	All	Linear in one branch	Linear in all the branches	Random Oracle Model

# Stacking $\Sigma$ -Protocols for Disjunctions

$x_1 \in L_1$  or  $x_2 \in L_2$  or ..... or  $x_n \in L_n$



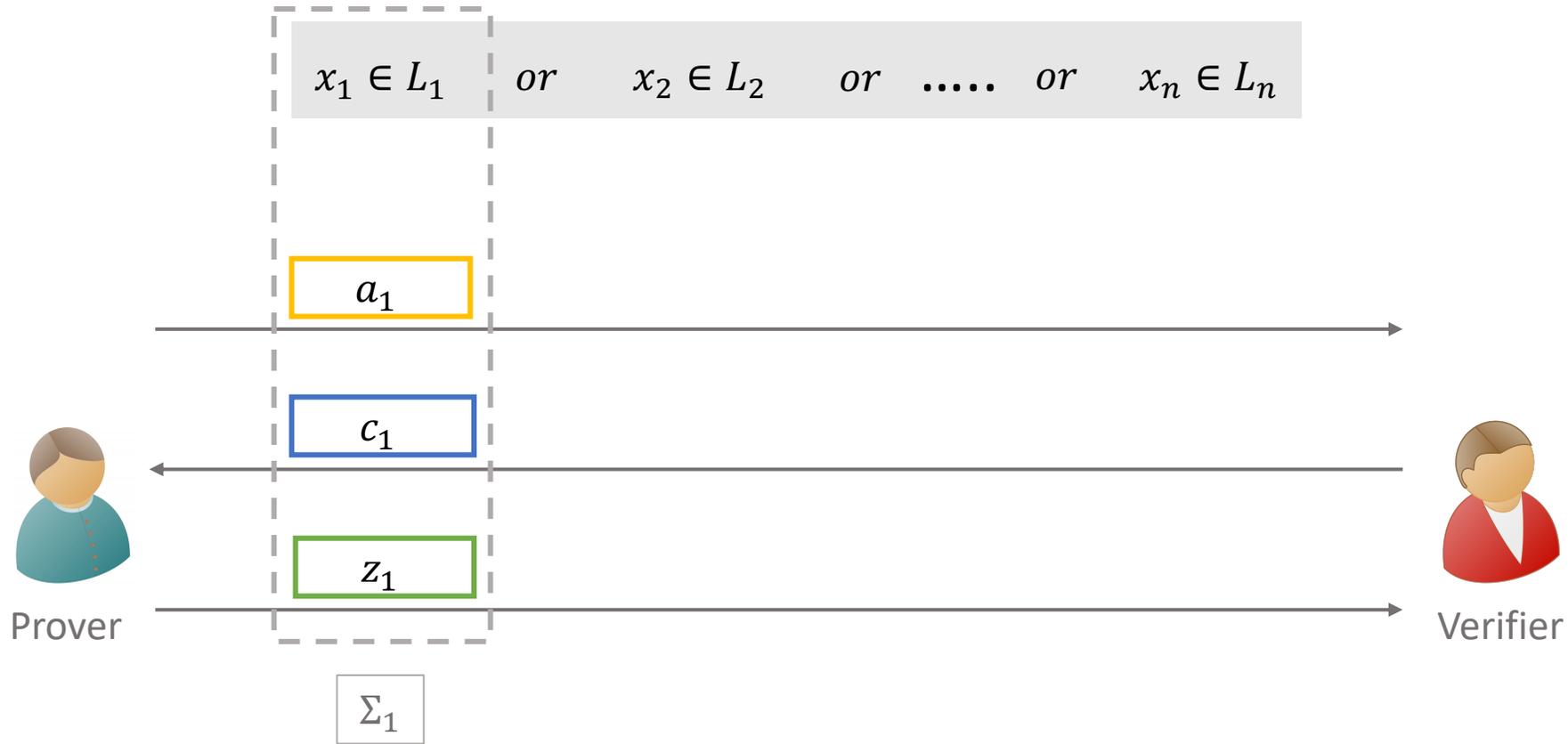
Prover



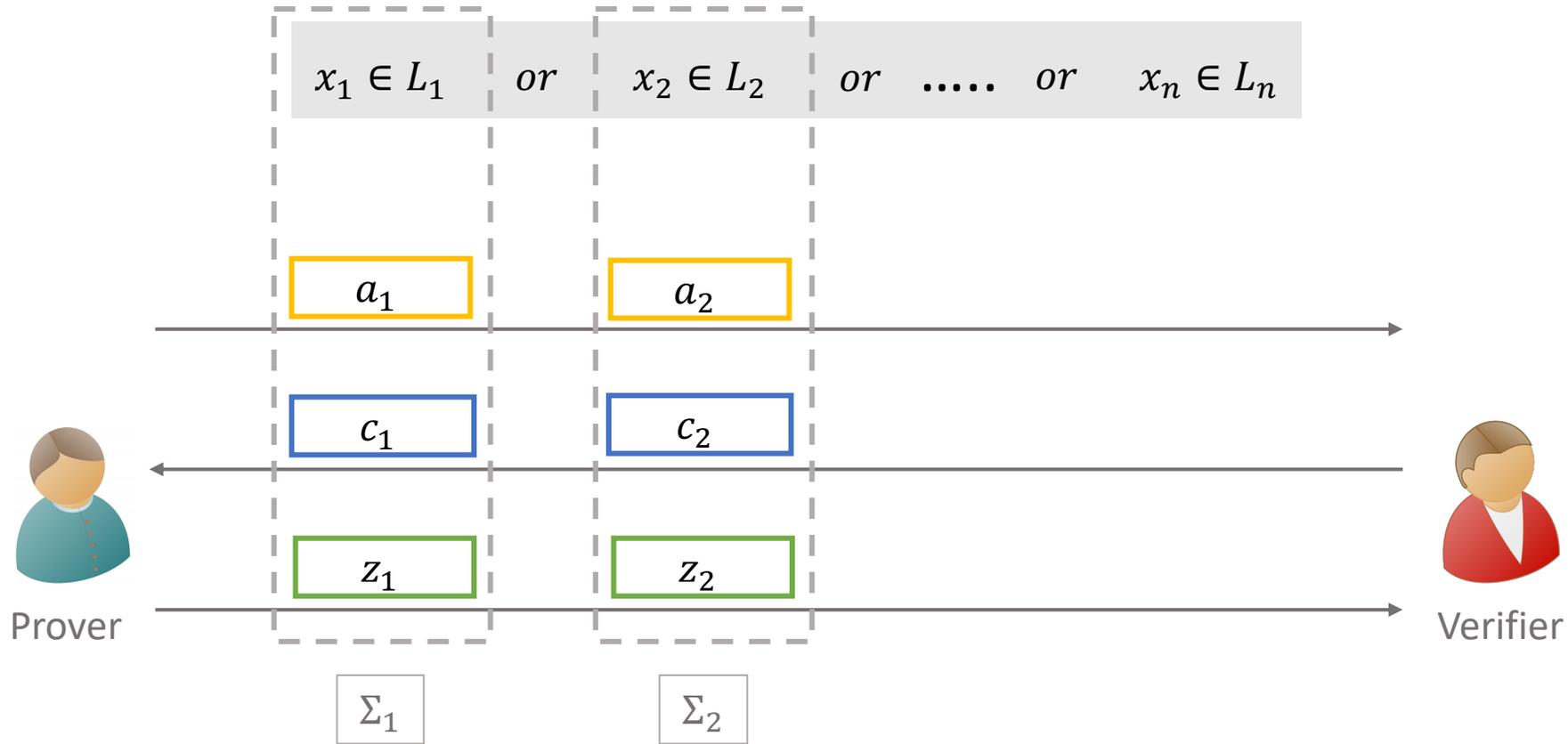
Verifier



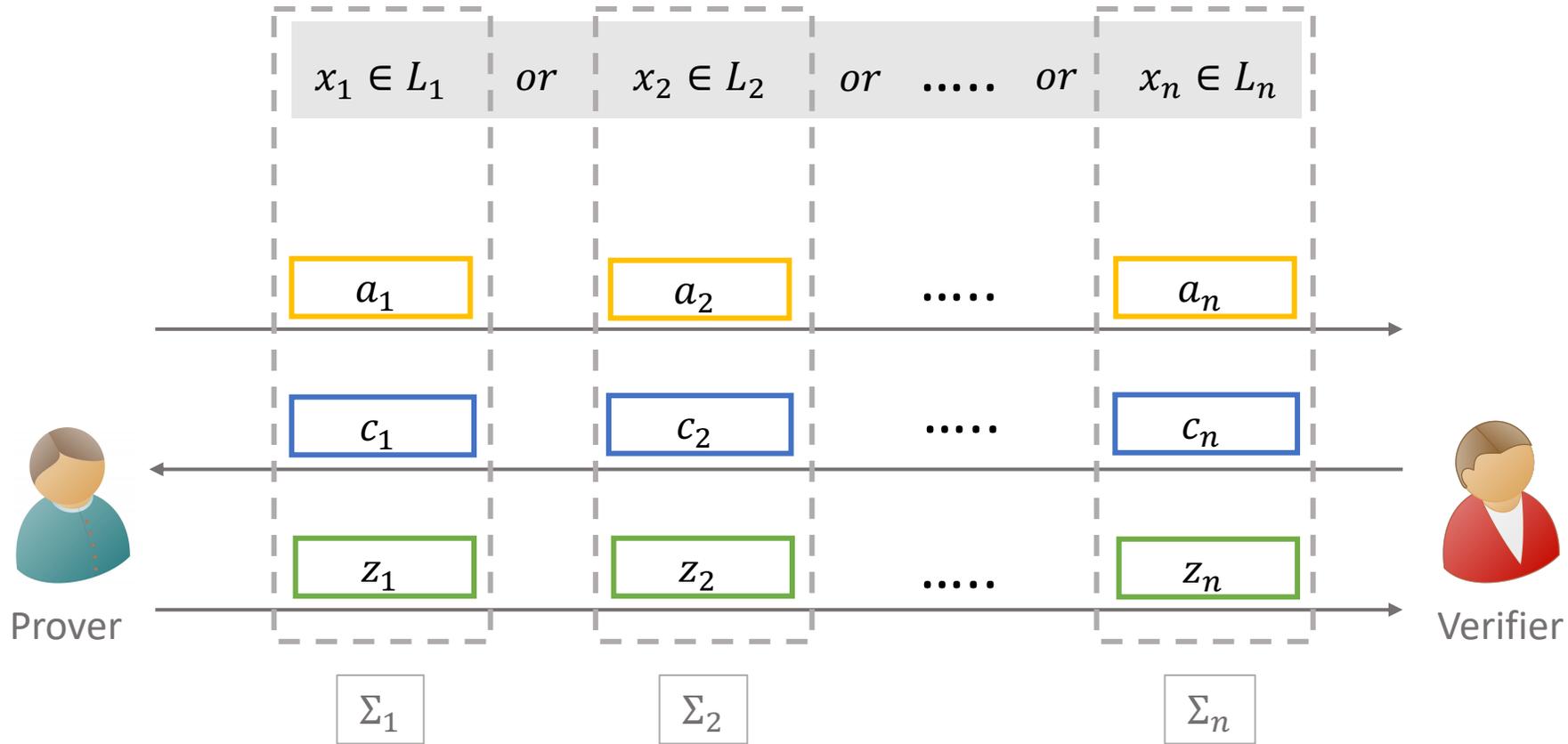
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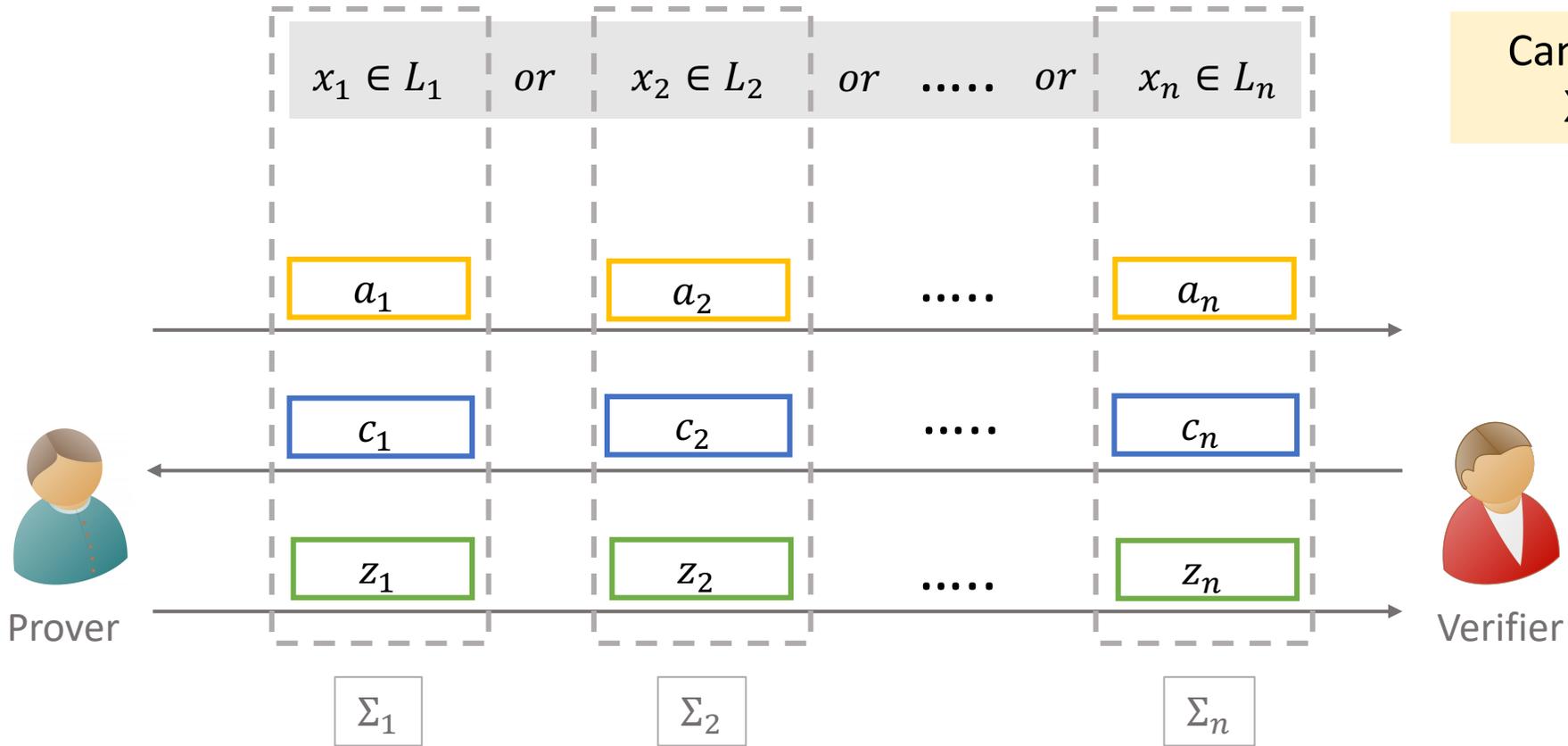
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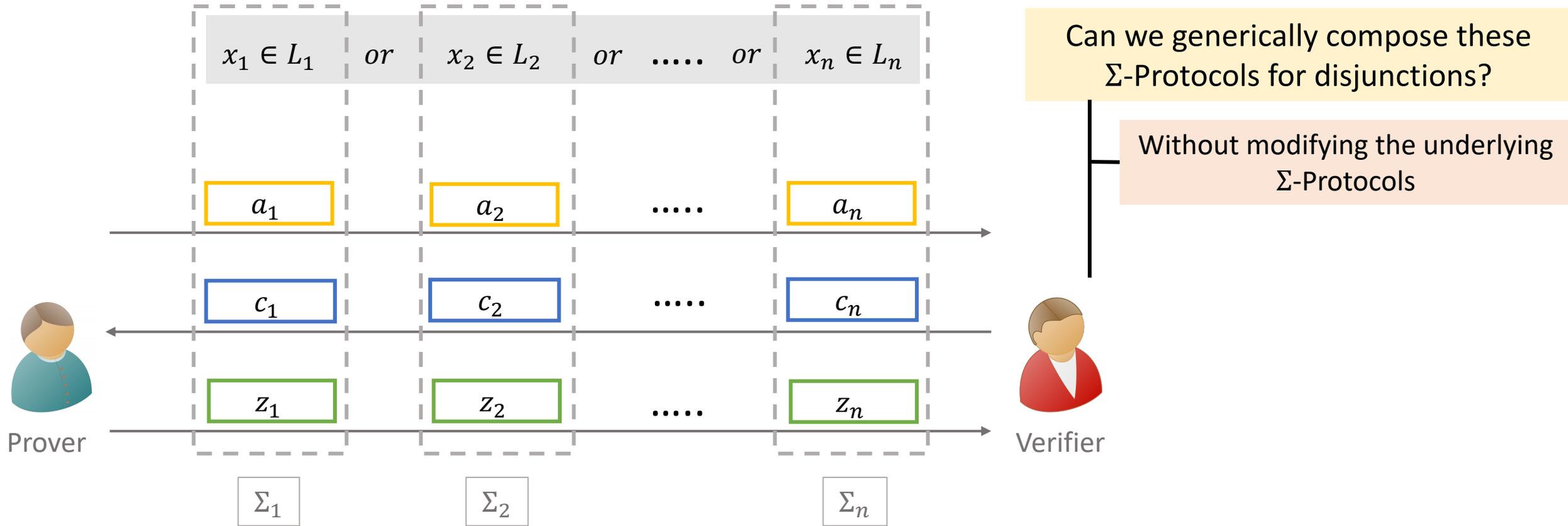


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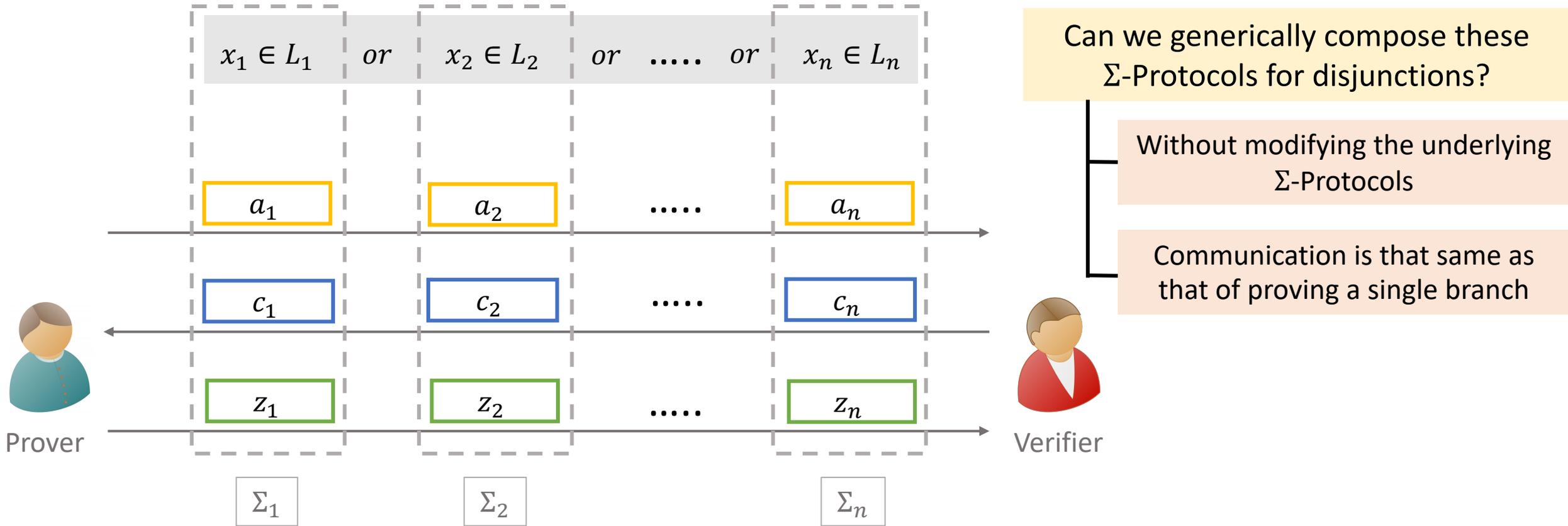


Can we generically compose these  $\Sigma$ -Protocols for disjunctions?

# Stacking $\Sigma$ -Protocols for Disjunctions



# Stacking $\Sigma$ -Protocols for Disjunctions



# Applications of such Stacking Compilers

Reduces manual effort of modifying existing techniques

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Newly developed  $\Sigma$ -protocols can also be used to produce stacked proofs immediately

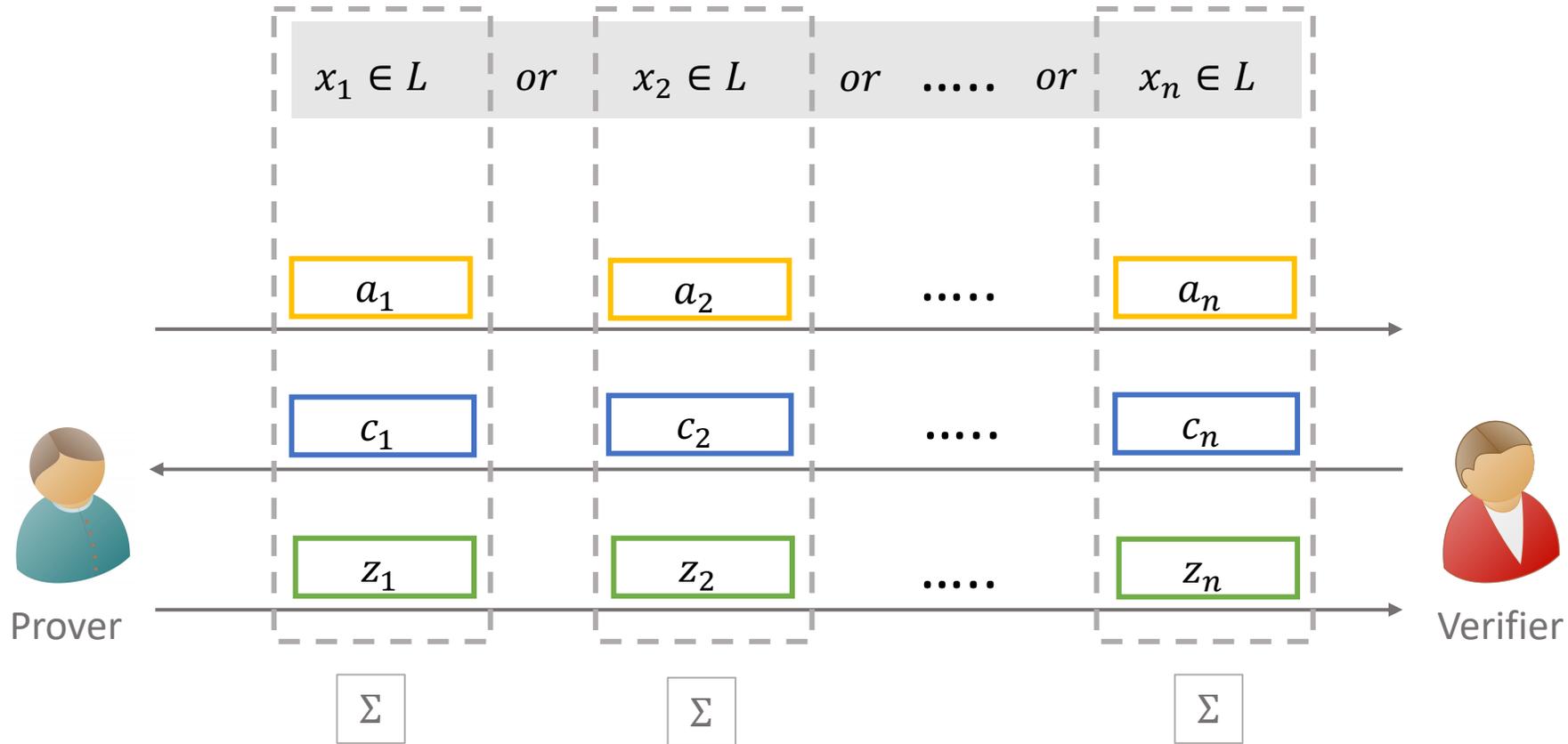
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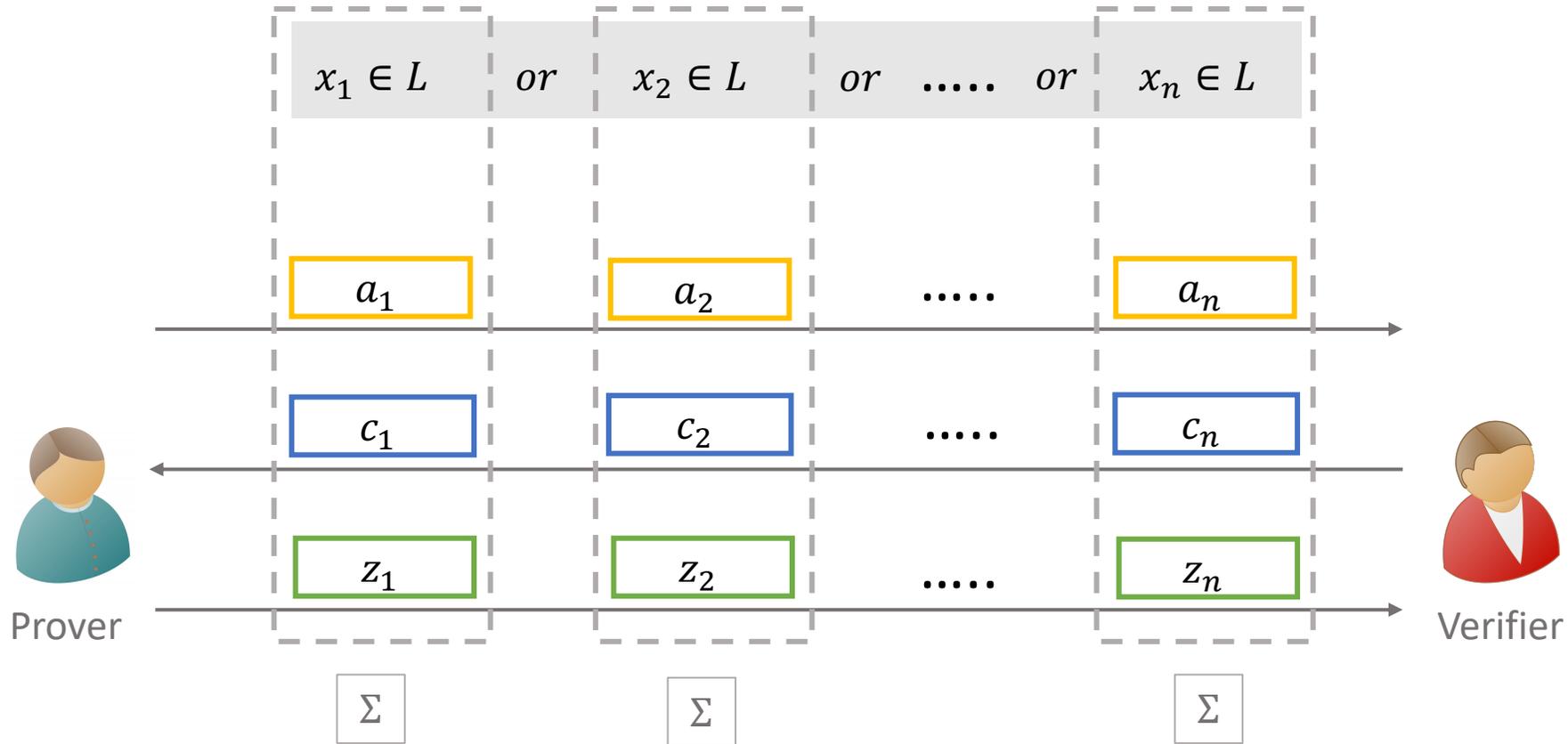
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Empowering protocol designers to choose appropriate  $\Sigma$ -protocols based on their application

# Stacking $\Sigma$ -Protocols for Disjunctions



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# Stacking $\Sigma$ -Protocols for Disjunctions

$x_1 \in L$  or  $x_2 \in L$  or ..... or  $x_n \in L$

$a_1$   $a_2$  .....  $a_n$

$c$

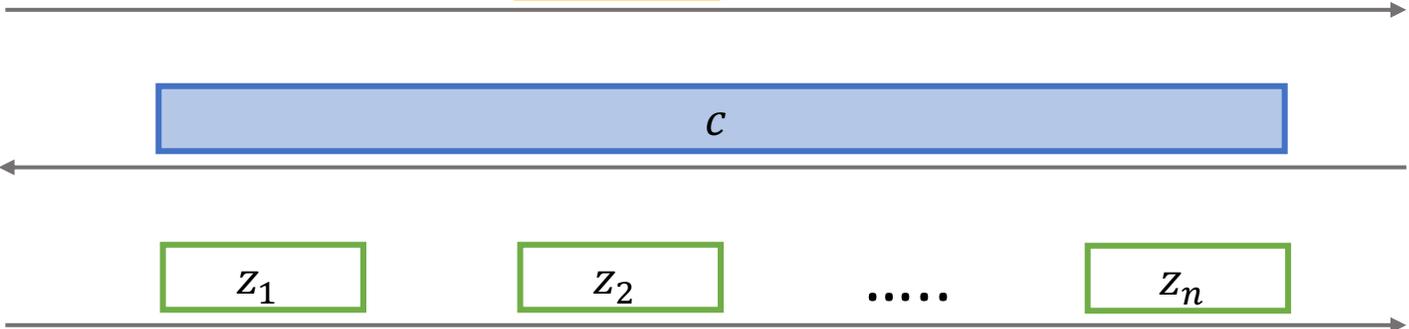
$z_1$   $z_2$  .....  $z_n$



Prover

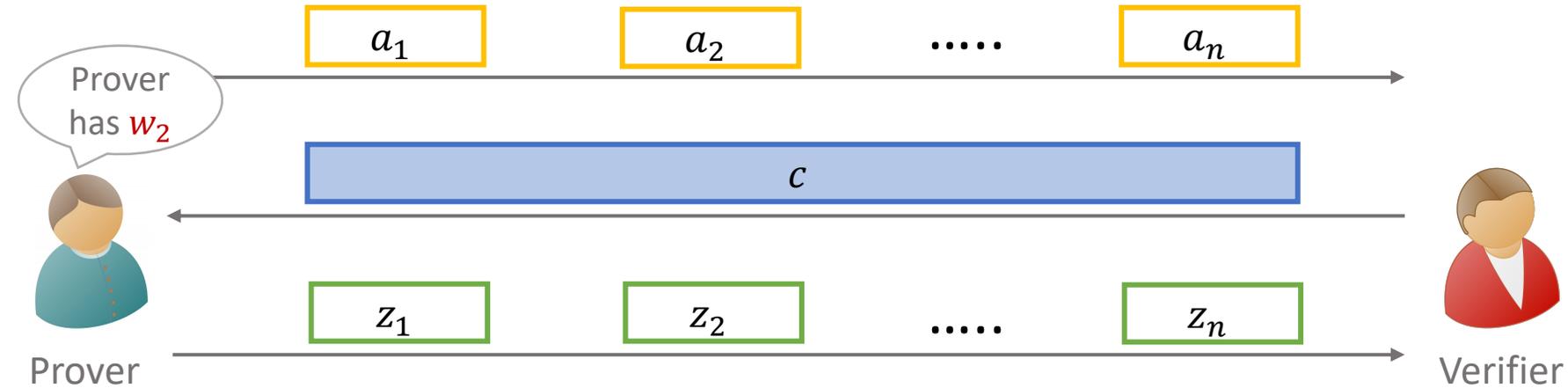


Verifier



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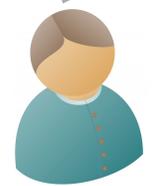
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$a_1$        $a_2$       .....       $a_n$

$c$

$z_1$        $z_2$       .....       $z_n$

Prover has  $w_2$



Prover

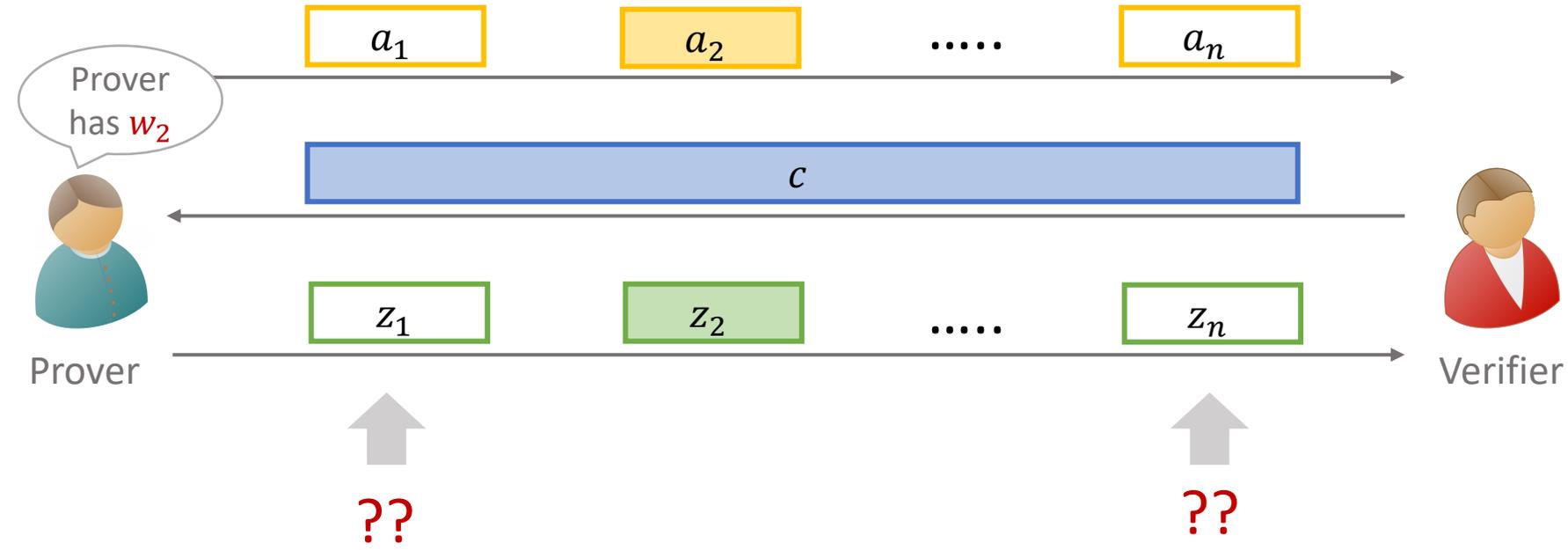


Verifier

↑  
Prover can compute these messages

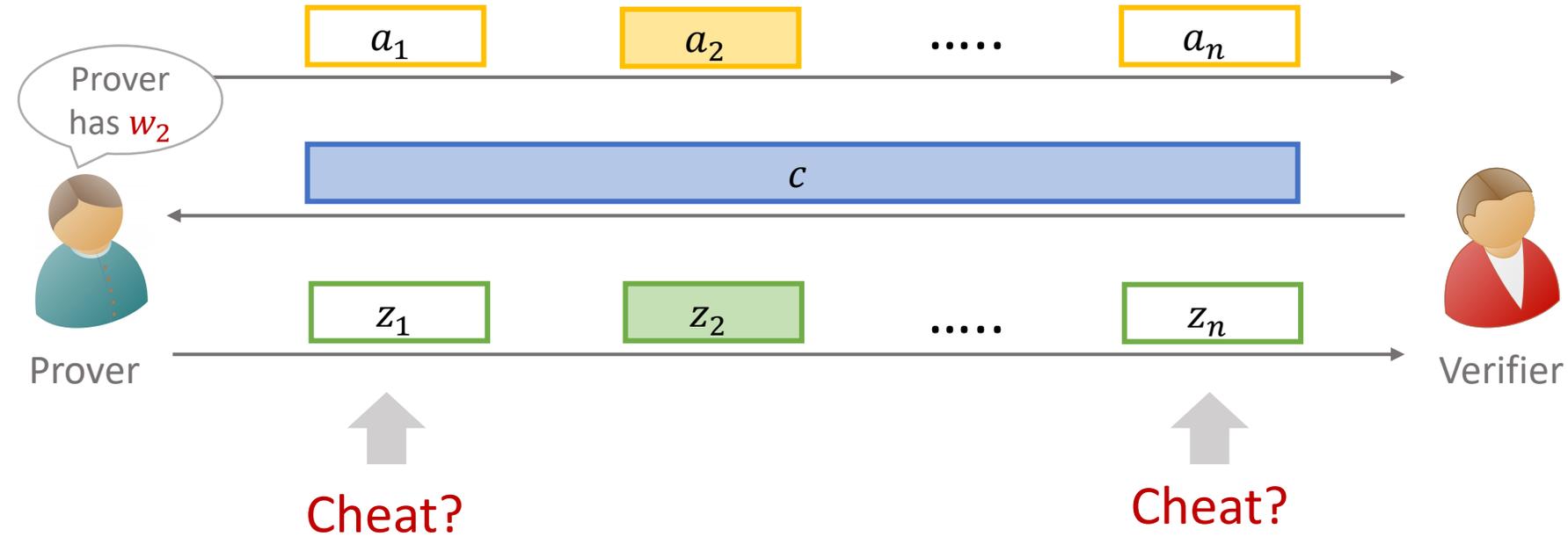
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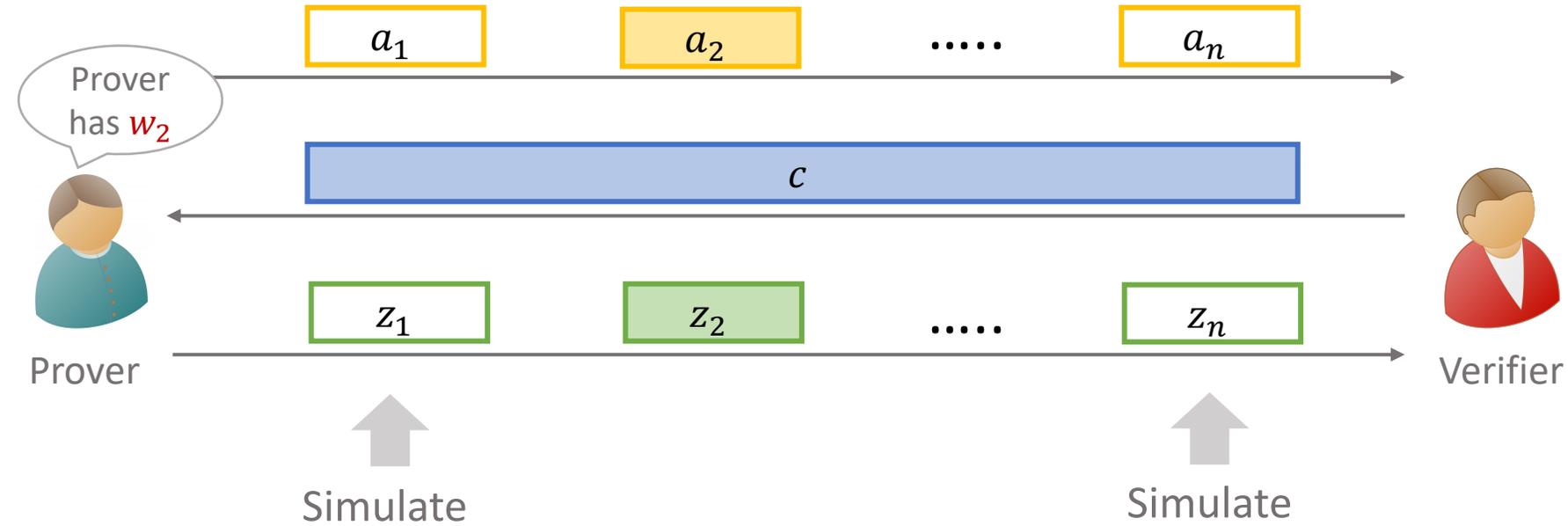
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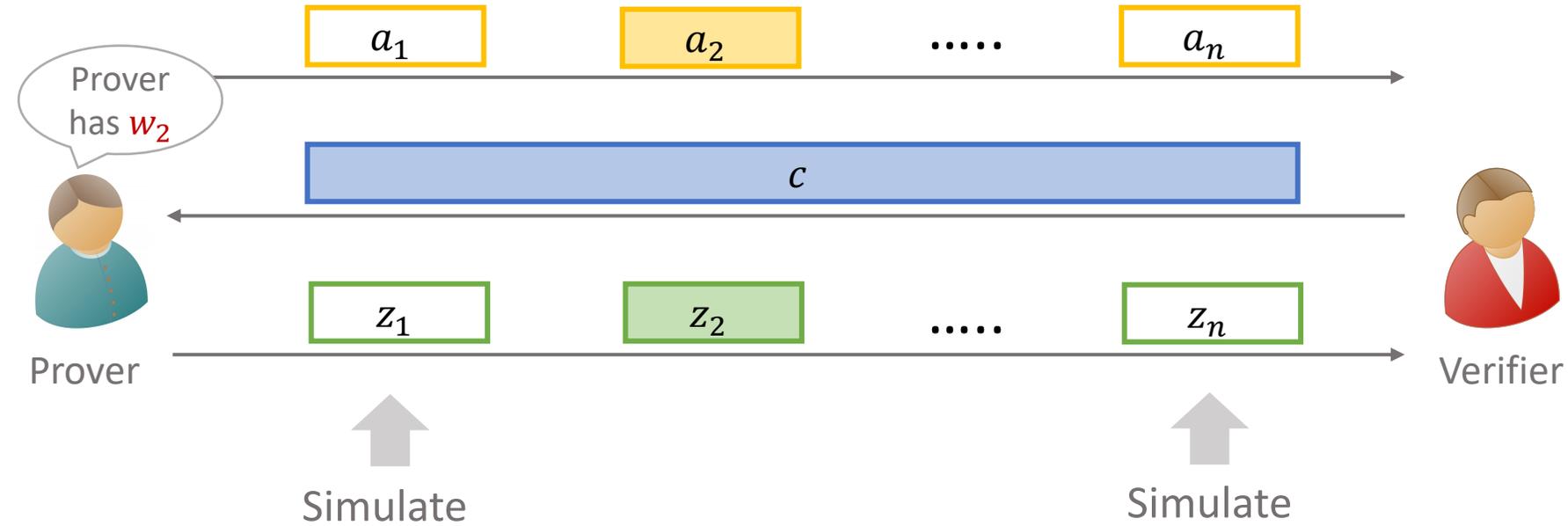
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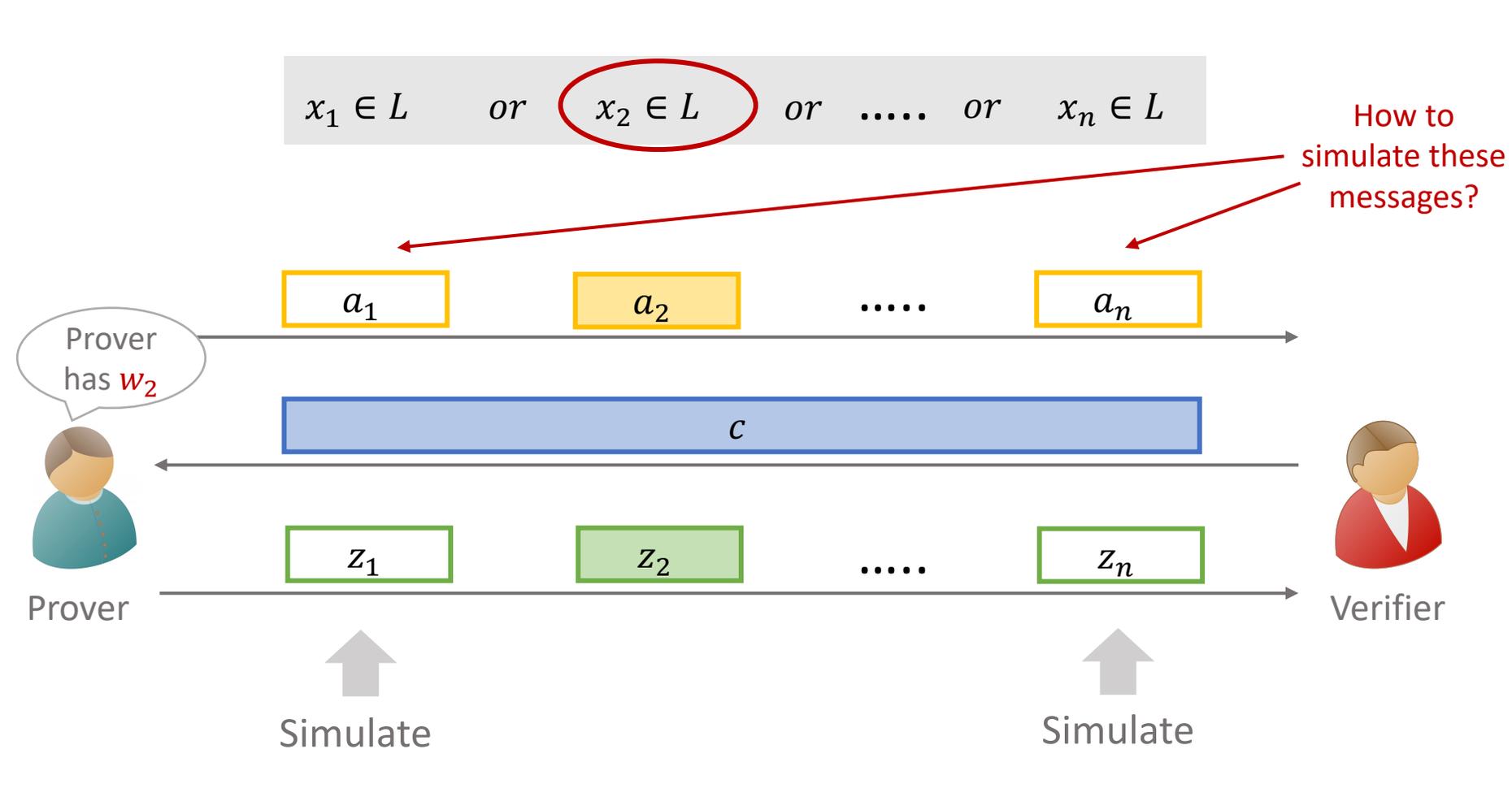


Common **simulation** strategy in  $\Sigma$ -protocols:

Step 1: Sample  $c$

Step 2: Compute  $a, z$

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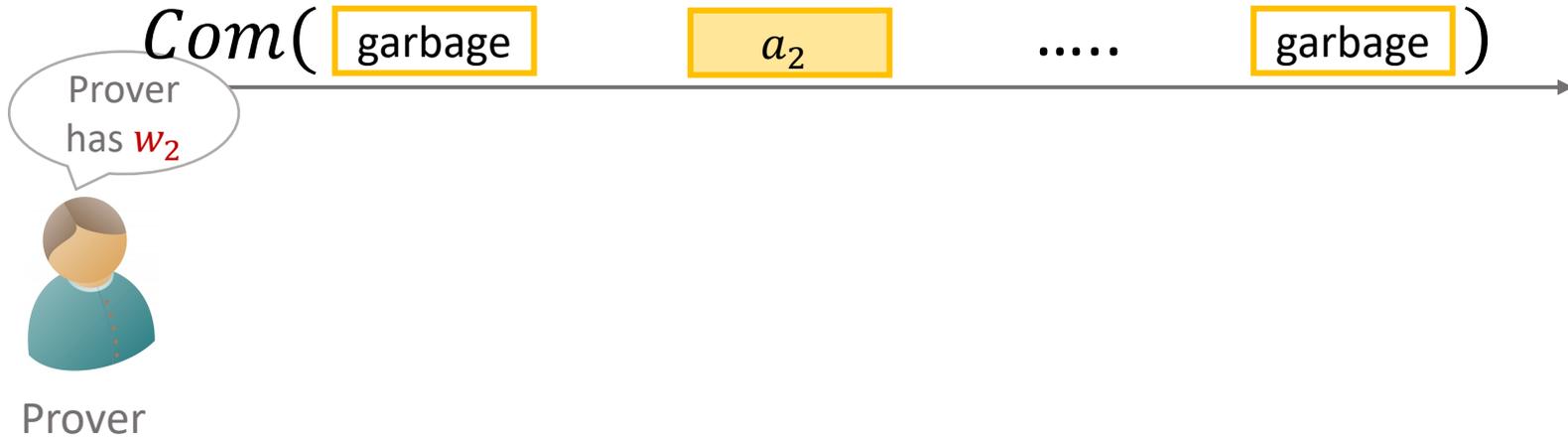


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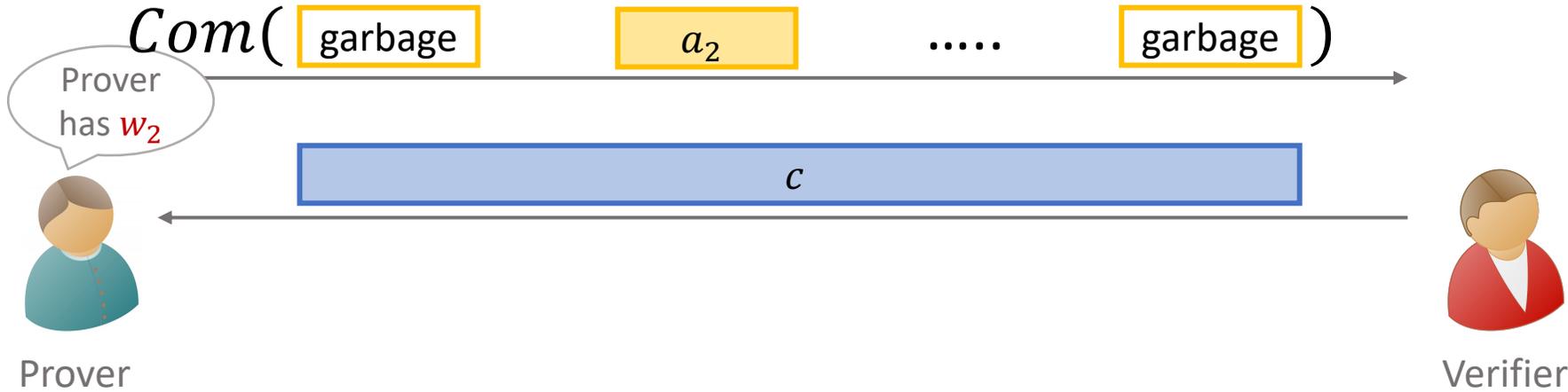


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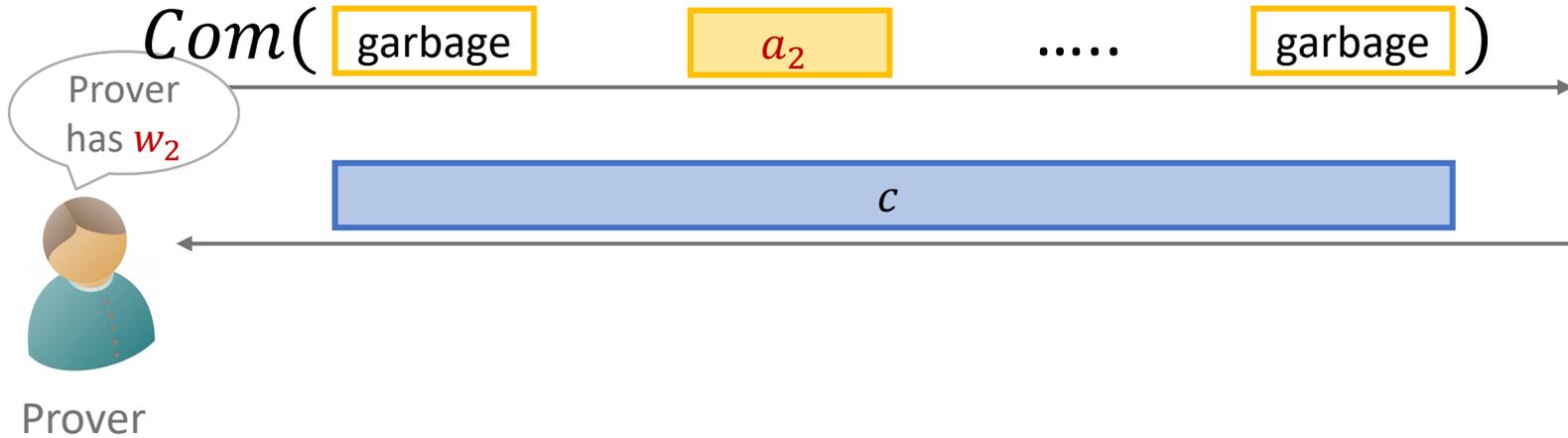


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Simulate  $(a_1, z_1), (a_3, z_3) \dots, (a_n, z_n)$

$op =$  Equivocate  $com$  to  $[a_1, a_2, \dots, a_n]$

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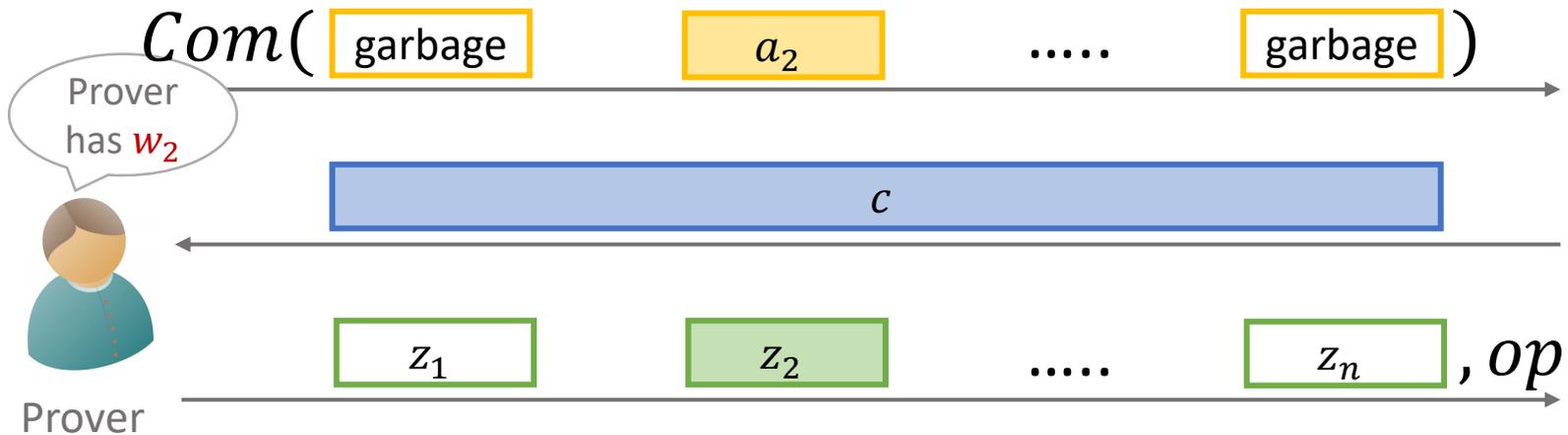
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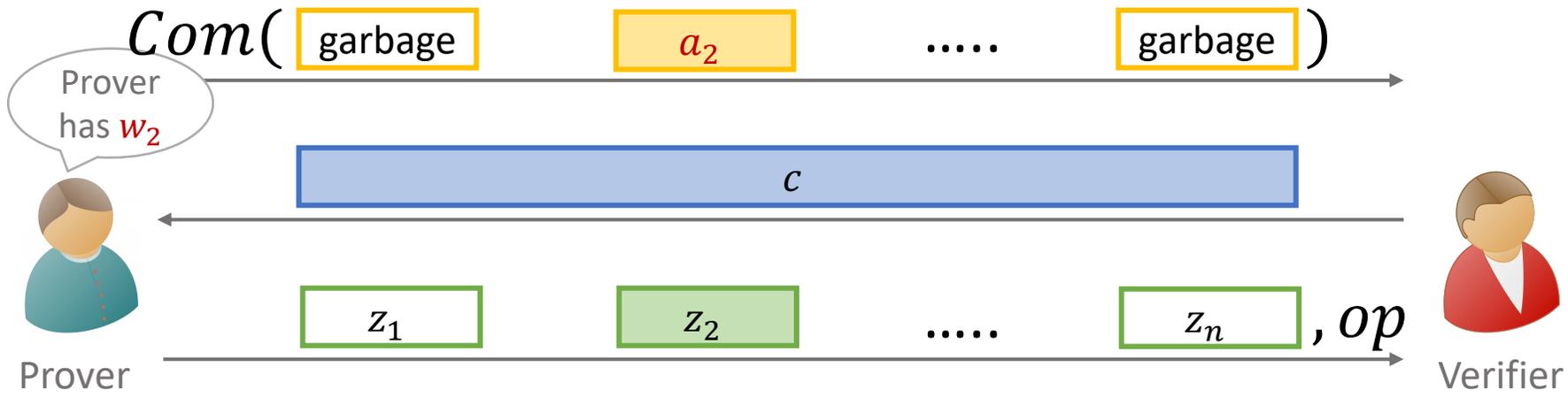


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Properties of these commitment schemes?

# Partially Binding Vector Commitments

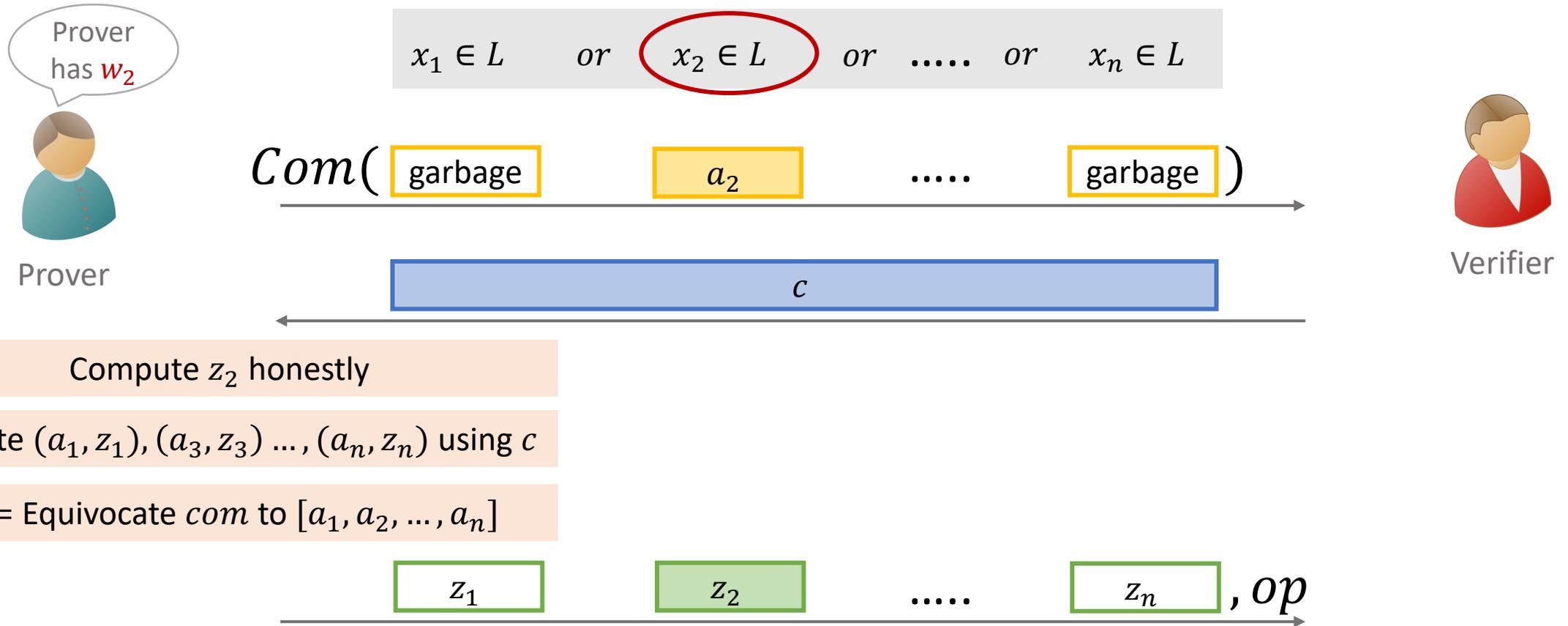
$t$ -out-of- $n$  positions are binding. Rest can be equivocated.

Binding positions are fixed at the time of commitment.

Binding positions remain hidden from the receiver.

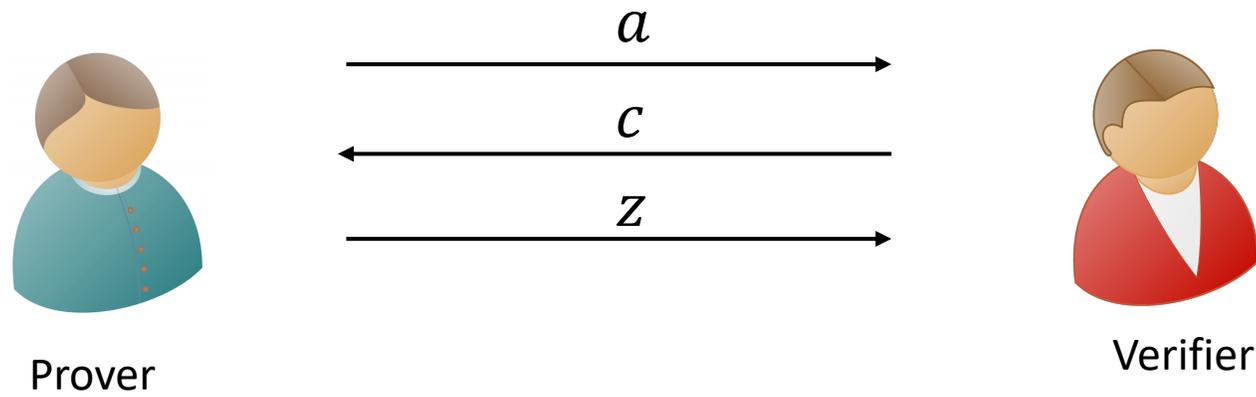
We propose a construction using Discrete Log

# Stacking $\Sigma$ -Protocol for Disjunctions

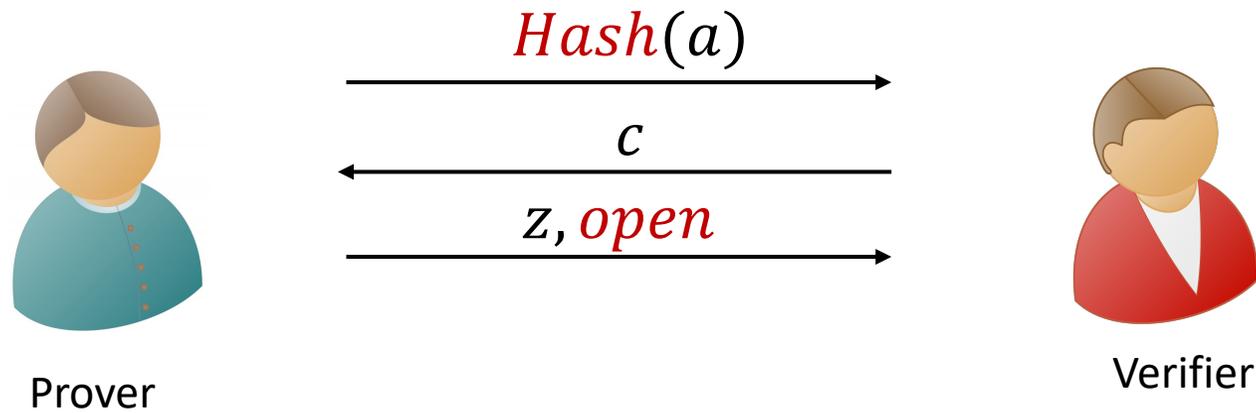


This is a valid  $\Sigma$ -protocol for disjunctions. But we haven't really saved any communication?

# Bulkiest Part of a $\Sigma$ -Protocol

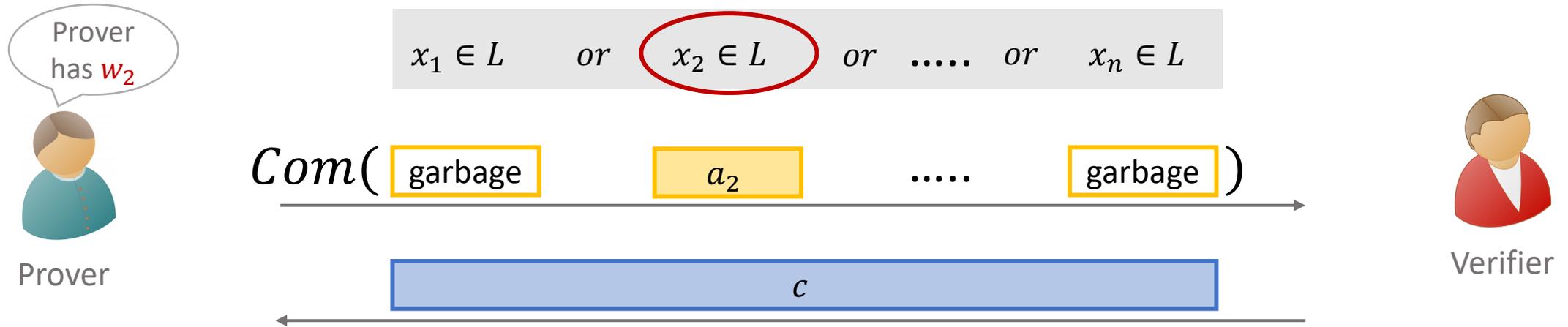


# Bulkiest Part of a $\Sigma$ -Protocol



w.l.o.g., Third round messages are the longest!

# Stacking $\Sigma$ -Protocol for Disjunctions



Compute  $z_2$  honestly

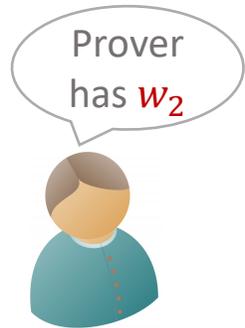
Simulate  $(a_1, z_1), (a_3, z_3) \dots, (a_n, z_n)$  using  $c$

$op = \text{Equivocate } com \text{ to } [a_1, a_2, \dots, a_n]$



Can we re-use the third-round message of the active branch?

# Stacking $\Sigma$ -Protocol for Disjunctions



Prover

$x_1 \in L$  or  $x_2 \in L$  or ..... or  $x_n \in L$

$Com(\text{garbage}, a_2, \dots, \text{garbage})$

$c$



Verifier

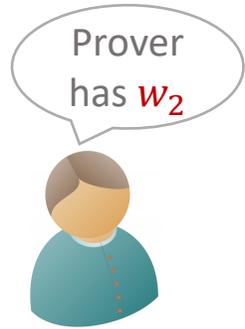
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Doesn't work generically. The underlying  $\Sigma$ -Protocols, must satisfy some properties

$z_2, op$

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Given a fixed challenge, the distribution of possible third round messages for any pair of statements in the language are indistinguishable from each other.

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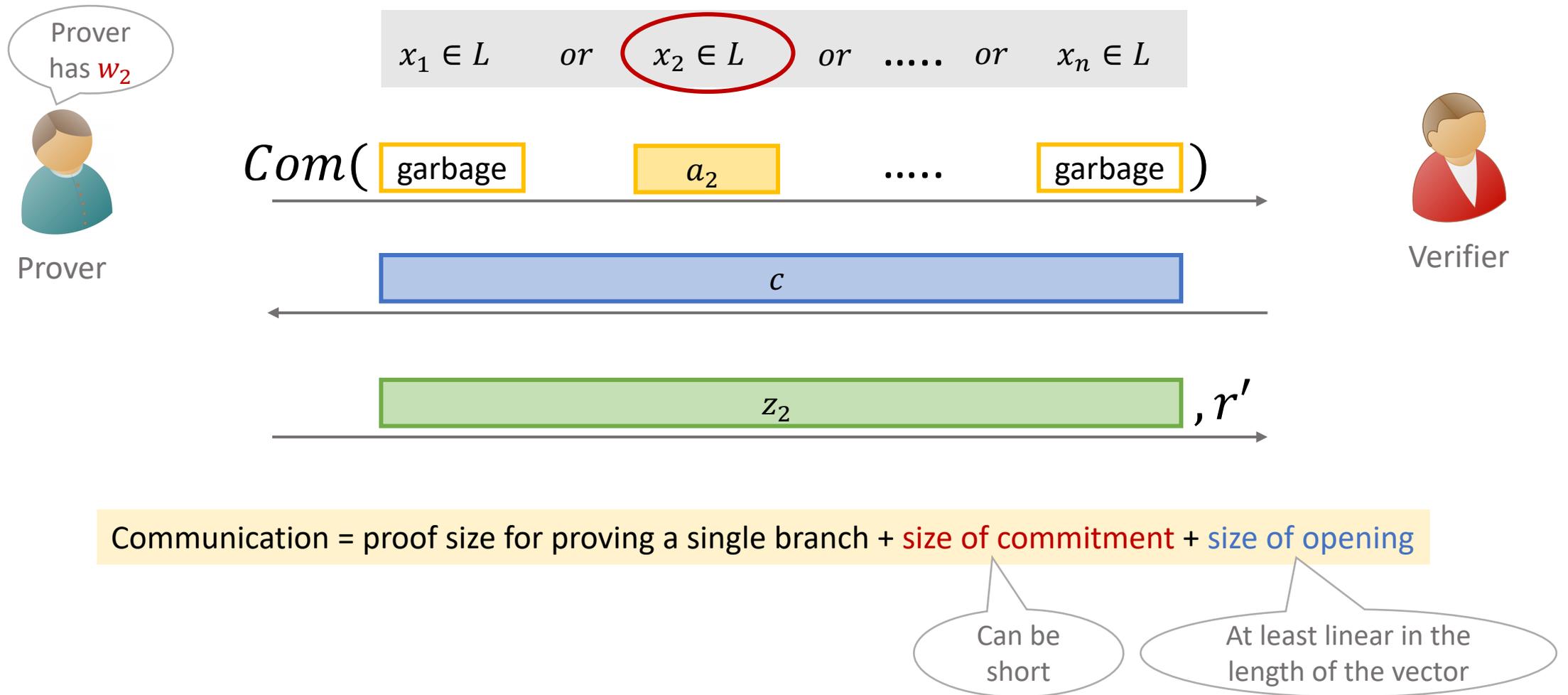
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=

$$\left\{ (a, z) \mid r^p \stackrel{\$}{\leftarrow} \{0, 1\}^\lambda; a \leftarrow A(x, w; r^p); z \leftarrow Z(x, w, c; r^p) \right\} \approx \left\{ (a, z) \mid z \stackrel{\$}{\leftarrow} \mathcal{D}_c^{(z)}; a \leftarrow \mathcal{S}^{\text{EHVZK}}(1^\lambda, x, c, z) \right\}$$

# Stacked $\Sigma$ -Protocol for Disjunctions



# Recursive Stacking

$x_1 \in L$     *or*     $x_2 \in L$     *or*    .....    *or*     $x_n \in L$

1 out of 2 disjunction

$\Sigma_2 = \text{Stack } \Sigma \text{ and } \Sigma$

Communication =  $|\Sigma| + \text{Commitment} + 1$

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1 out of 8 disjunction

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Communication =  $|\Sigma| + 3 \times \text{Commitment} + 1 + 1 + 1$

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$\Sigma_8 = \text{Stack } \Sigma_4 \text{ and } \Sigma_4$

Communication =  $|\Sigma| + 3 \times \text{Commitment} + 1 + 1 + 1$

.....

1 out of n disjunction

$\Sigma_n = \text{Stack } \Sigma_{n/2} \text{ and } \Sigma_{n/2}$

Communication =  $|\Sigma| + \log(n) \times \text{Commitment} + \log(n)$

# Examples of Stackable $\Sigma$ -Protocols

Many natural sigma protocols are stackable

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$$R(x, w): x \stackrel{?}{=} g^x$$

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Prover

$$a = g^r$$


$$c$$


$$z = cw + r$$



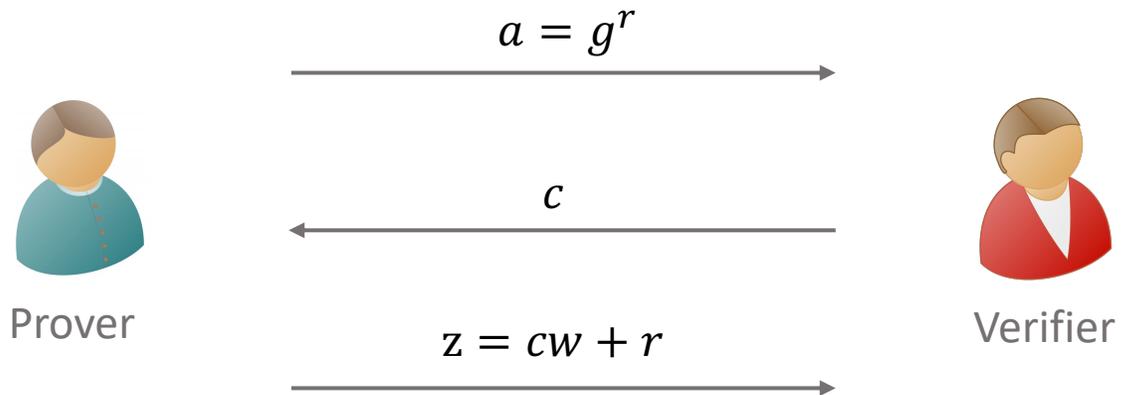
Verifier

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Example 1: Schnorr's  $\Sigma$ -Protocol

$$R(x, w): x \stackrel{?}{=} g^x$$



**Simulation Strategy:** Sample random  $z$ . Compute  $a = g^z x^{-c}$

Independent  
of instance

# Examples of Stackable $\Sigma$ -Protocols

Many natural sigma protocols are stackable

Example 1: Schnorr's  $\Sigma$ -Protocol

Example 2: Graph 3-coloring

Is a graph  $G = (V, E)$ , 3-colorable?

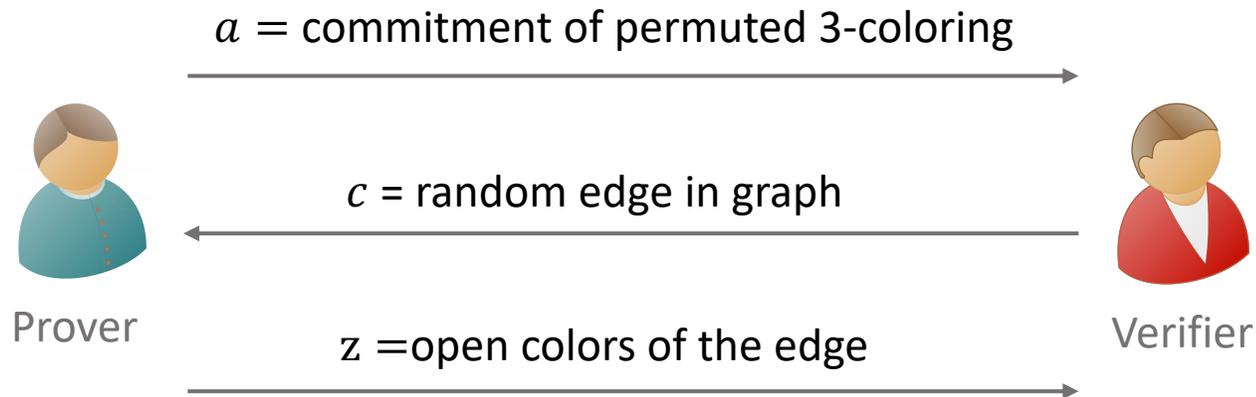
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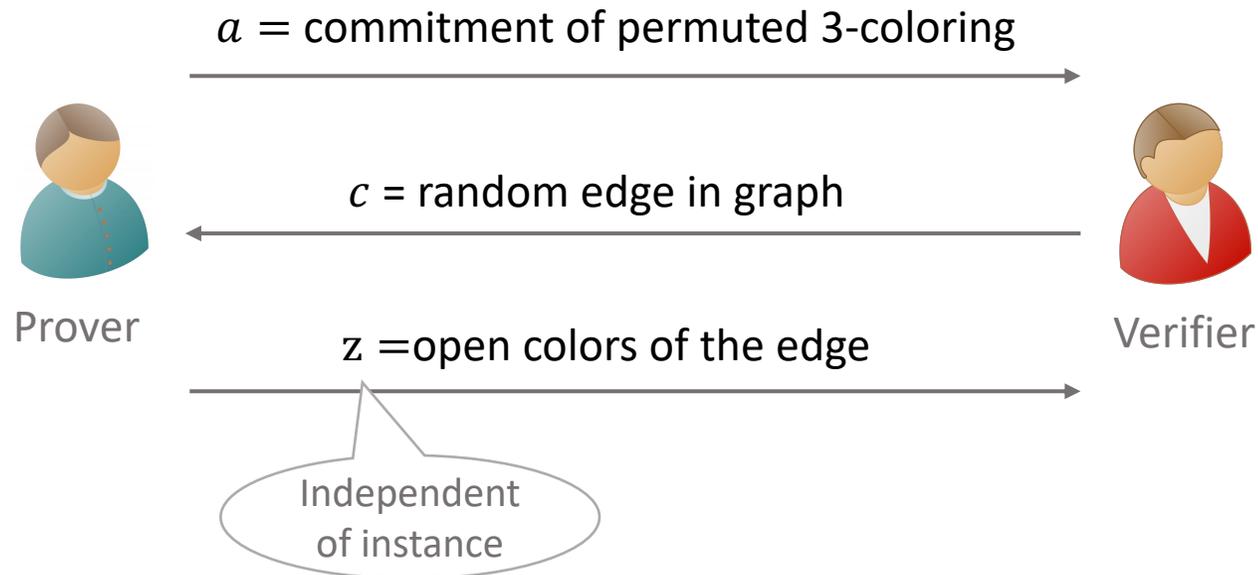
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Example 2: MPC-in-the-head [IKOS]

# [IKOS07] is Stackable?

For function  $R(x, \cdot)$ , that takes  $w$  as input

Run MPC in the head, commit to views of all parties



Prover



Verifier



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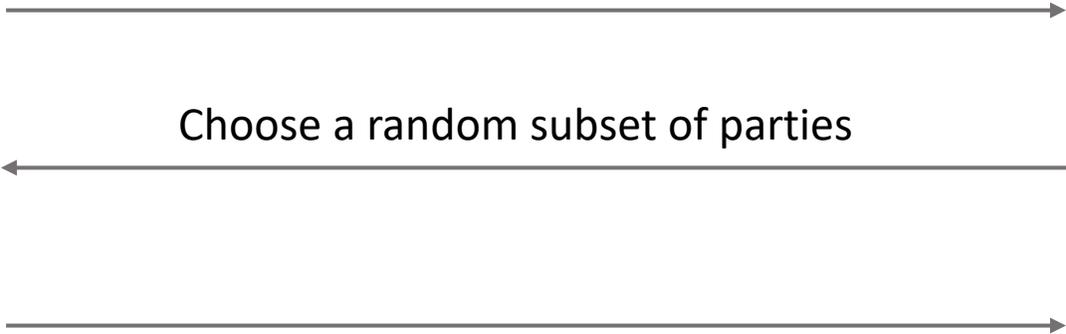
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Verifier



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Open views of the chosen parties



Prover



Verifier

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Verifier

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Choose a random subset of parties

Simulate the views of these parties' using simulator of the underlying MPC protocol

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Verifier

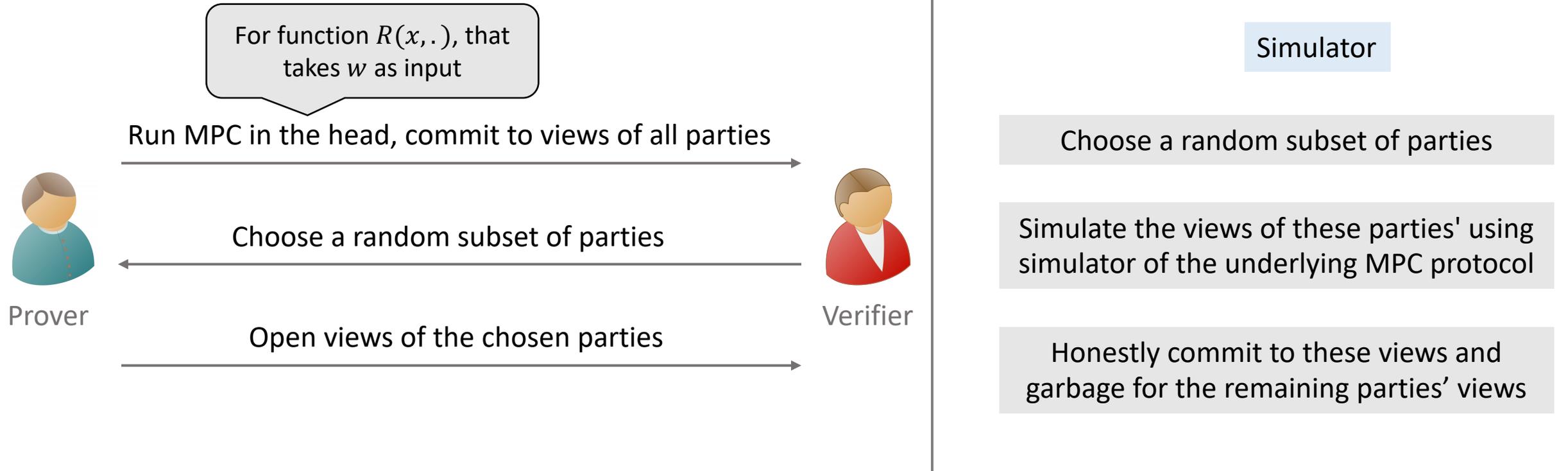
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Choose a random subset of parties

Simulate the views of these parties' using simulator of the underlying MPC protocol

Honestly commit to these views and garbage for the remaining parties' views

# [IKOS07] is Stackable?



It is naturally EHVZK. What about recyclable third round messages?

$F$ -Universally Simulatable MPC

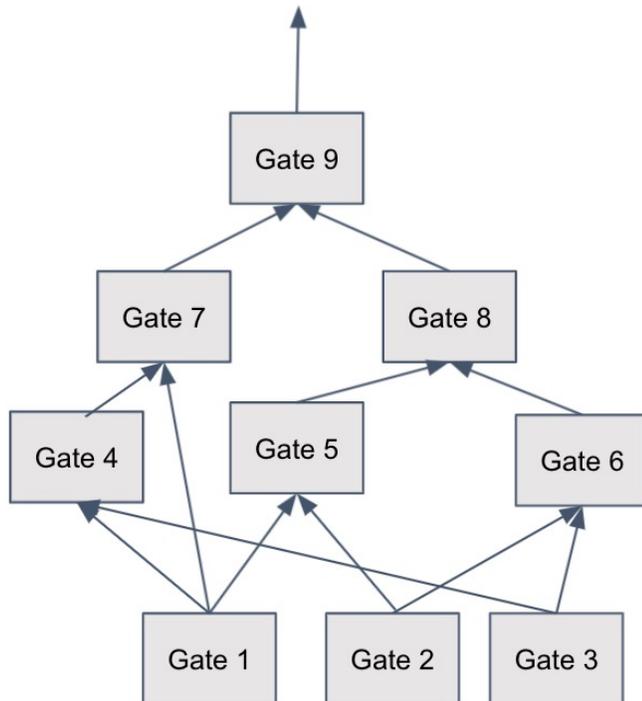
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Adversary's view in many MPC protocols can be condensed and decoupled from the structure of the functionality being evaluated

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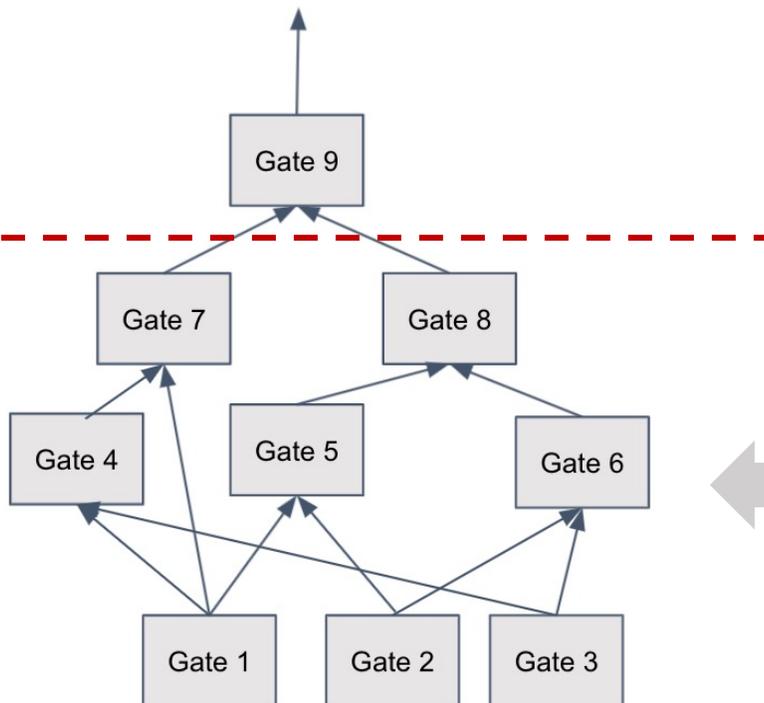
Example: Many secret sharing-based MPC (e.g. [BGW88])



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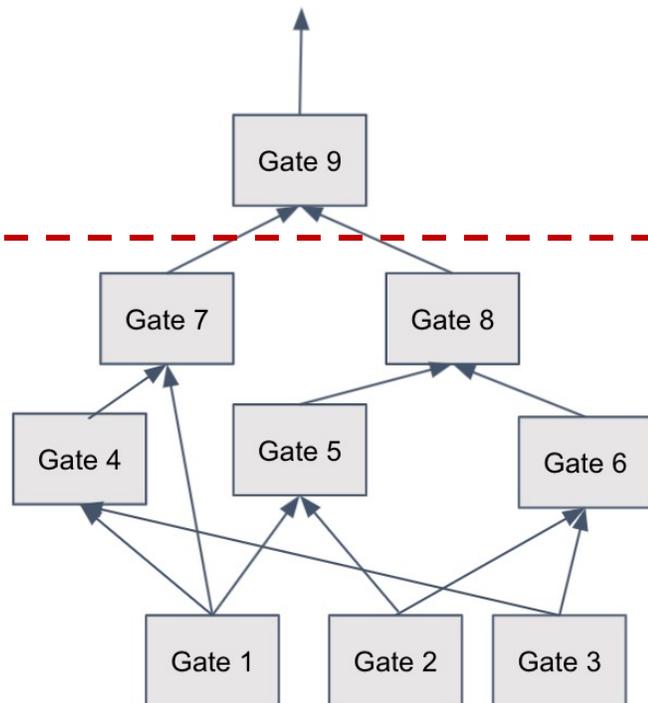


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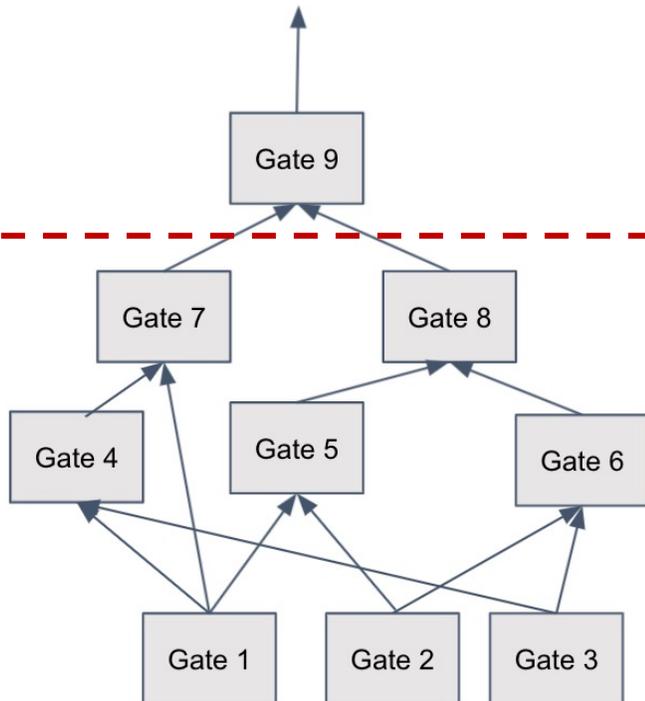
Given previously simulated shares and the output, simulate the final message

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Deterministic computation

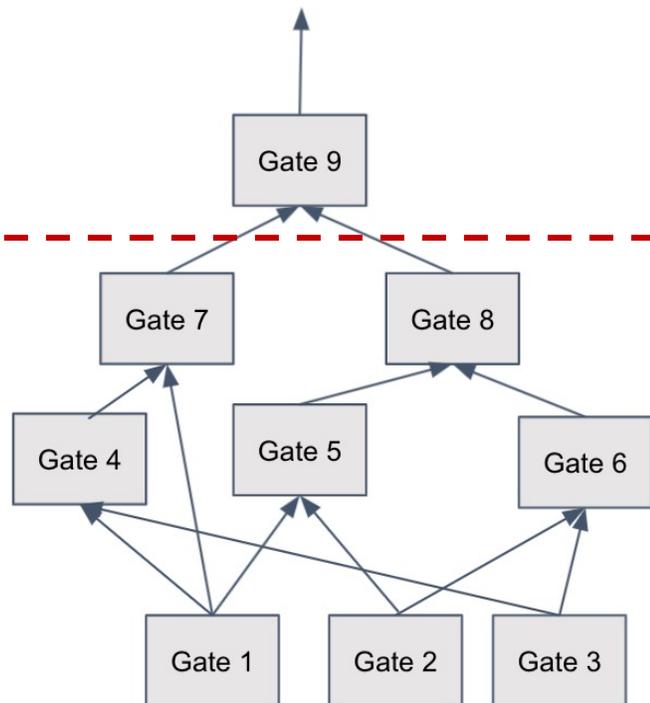
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Independent of the function/circuit!

# $F$ -Universally Simulatable MPC

Adversary's view in many MPC protocols can be condensed and decoupled from the structure of the functionality being evaluated

Example: Many secret sharing-based MPC (e.g. [BGW88])



Given previously simulated shares and the output, simulate the final message

Expanded Views

Deterministic computation

Simulator simulates random shares for the adversary for each of these gates

Condensed Views

Independent of the function/circuit!

# Modified [IKOS07] for $F$ -Universally Simulatable MPC

For function  $R(x, \cdot)$ , that takes  $w$  as input

Run MPC in the head, commit to views of all parties



Prover

Choose a random subset of parties



Verifier

**Condensed views** of the chosen parties and randomness used in corresponding commitments

# Modified [IKOS07] for $F$ -Universally Simulatable MPC

For function  $R(x, \cdot)$ , that takes  $w$  as input

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Verifier can expand condensed views assuming output is 1, check if commitments are valid and perform all other consistency checks



Prover



Verifier

# Modified [IKOS07] for $F$ -Universally Simulatable MPC

For function  $R(x, \cdot)$ , that takes  $w$  as input

Run MPC in the head, commit to views of all parties

Choose a random subset of parties

**Condensed views** of the chosen parties and randomness used in corresponding commitments

Since condensed views are independent of the functionality, this protocol now has recyclable third-round message

Verifier can expand condensed views assuming output is 1, check if commitments are valid and perform all other consistency checks



Prover



Verifier

# Disjunctions with Different Languages

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If third round messages are over different fields/rings – represent as bits and see what parts can be re-used

Thank You!