Model Simplification

- Models used in CIS must be guaranteed to be accurate within known bounds
- But 3D models from medical images often are very complex, and require representations with large data structures.
- Algorithms using models are often computationally expensive, and computation costs go up with model complexity
- **PROBLEM:** reduce model complexity while preserving adequate accuracy

~350,000 triangles!

Model simplification

- Problem is also common in computer graphics
  - There is a growing literature
  - **But** many graphics techniques only care about appearance, and do not necessarily preserve accuracy or other properties (like topological connectivity) important for computational analysis
- Broadly, two classes of approaches
  - Top down
  - Bottom-up

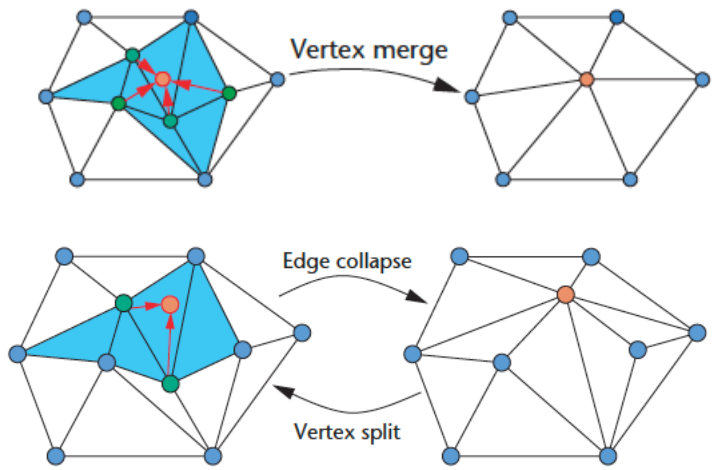
Top down

- Active surfaces used in segmentation
- Deformable registration of an atlas to a patient
  - E.g., skull atlas discussed in craniofacial lecture had about 5000 polygons (perhaps 15-20,000 triangles)
- Recursive approximations
  - E.g., Pizer *et al.* "cores"

Bottom-up methods

- Typically, start with very high detail model generated from CT images
  - Large number of elements a consequence of small size of pixels & way model is created
- Then merge nearby elements into larger elements
  - E.g., "decimation" (Lorensen, et. al.)
  - E.g., "superfaces" (Kalvin & Taylor)
  - E.g., Gueziec
- An excellent tutorial may be found in:
  - David Luebke; A Developer's Survey of Polygonal Simplification Algorithms; *IEEE Computer Graphics and Application*, May 2001
- Online resources
  - **Software package:** F. Cacciola, M. Rouxel-Labbé, and B. Şenbaşlar, "CGAL 5.1.1 - Triangulated Surface Mesh Simplification," https://doc.cgal.org/latest/Surface_mesh_simplification/index.html
  - **Stanford course (CS468) lecture notes:** http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/08_Simplification.pdf

Bottom-up merging

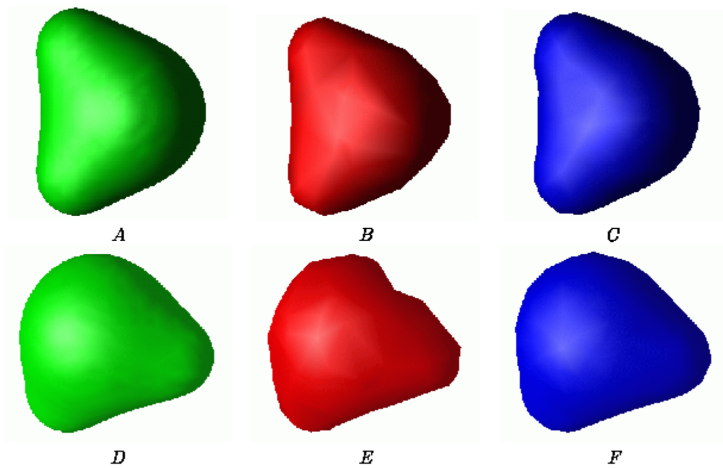


Source: David Luebke; A Developer's Survey of Polygonal Simplification Algorithms; IEEE Computer Graphics and Application, May 2001

A typical example: Gueziec's Method

- Reference
  - A. Gueziec, "Surface Simplification inside a tolerance volume", *IBM Research Report RC20440,* 5/20/97
- Essentially "triangle decimation" done correctly
  - Preserves topology
  - Preserves volume
  - Provable error bound

Simplified surfaces



Original
(1747 vertices)

Shroeder
(145 vertices)

Gueziec
(145 vertices)

Approximated bunny

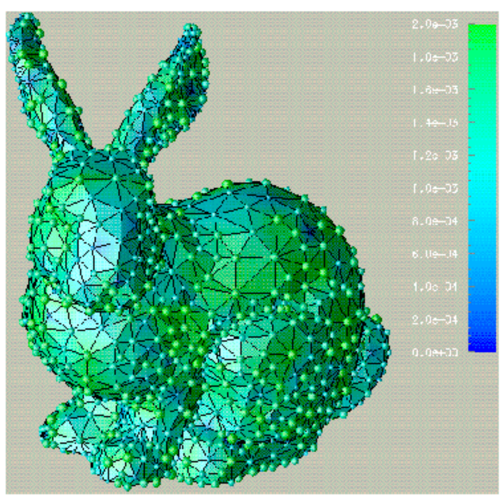
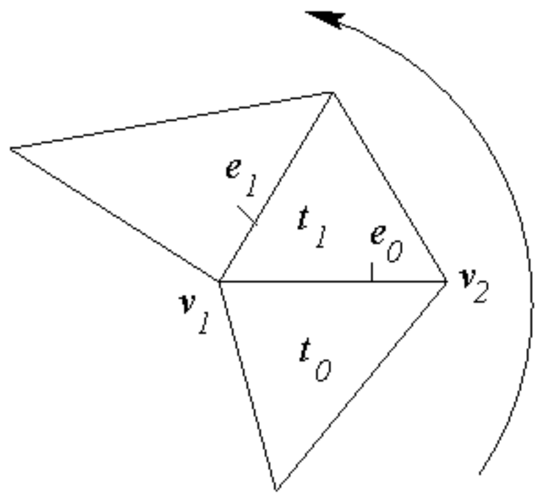


Figure 3: Error volume partially represented using color and spheres centered at surface vertices. The radius of each sphere equals the error value at the vertex, here bounded by .8 % of the bounding box diameter.

Building a vertex star



Notation: "star" v* of a vertex v

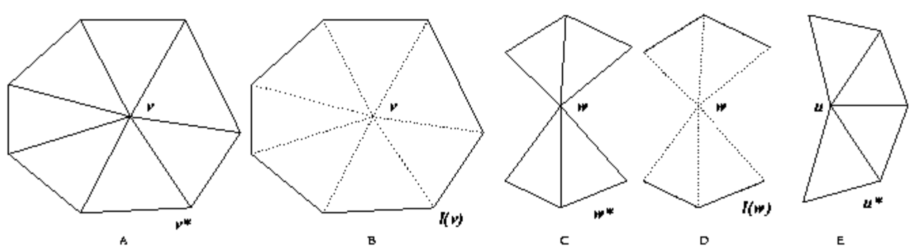
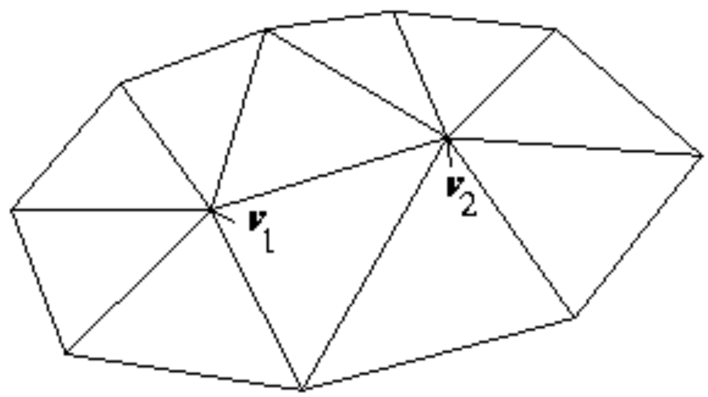
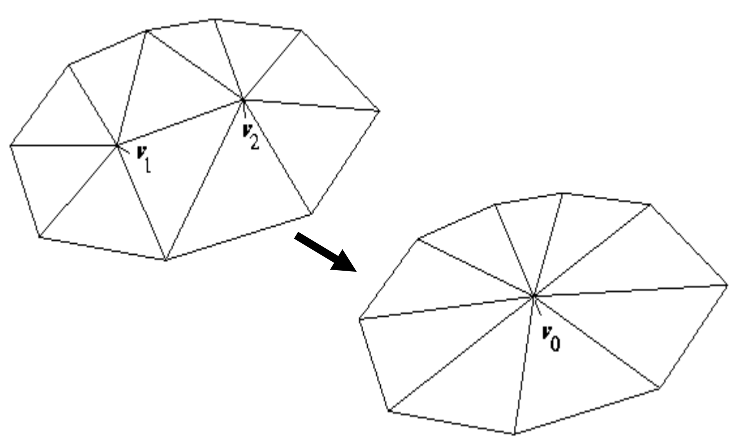


Figure 4: A: the star $\mathbf{v}^*$ of a regular vertex $\mathbf{v}$ of valence seven. B: the link $\ell(\mathbf{v})$ of the regular vertex $\mathbf{v}$, composed of one simple closed polygonal curve. C: the star $\mathbf{w}^*$ of a singular vertex $\mathbf{w}$ of valence four. D: the link $\ell(\mathbf{w})$ of $\mathbf{w}$, composed of two disconnected polygonal curves. E: the star $\mathbf{u}^*$ of a boundary vertex of valence five.

Notation: “star” $(v_1,v_2)^*$ of edge (v_1,v_2)



Fundamental operation: Edge Collapse



Book-keeping to remember hierarchy

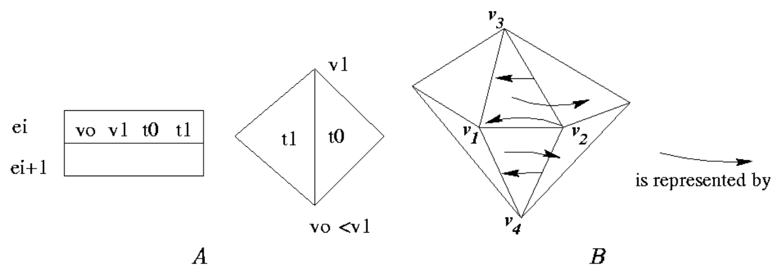
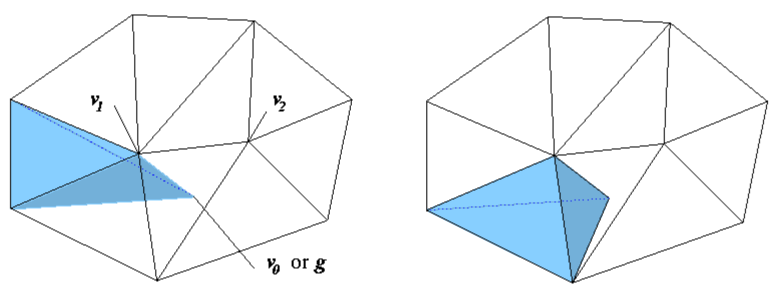


Figure 7: A: an edge refers to four indices. B: defining parents of surface elements during an edge collapse.

Picking new vertex to preserve volume



Volume associated with edge star is sum of tetrahedra
v_0 = vertex associated with simplified vertex star
g_0 = centroid of edge star

Picking new vertex to preserve volume

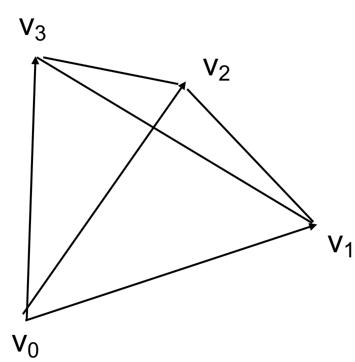
Given edge star $(v_1,v_2)^* = \{ v_3,\ldots,v_n\}$. Let T_{12} be the set of all triangles t in $v_1^* \cup v_2^*$ and let $vertices(t) = \{v_{t1}, v_{t2}, v_{t3}\}$ be the set of verticies associated with a triangle t. Compute the centroid $g = \sum_{i=3}^{n} \frac{v_i}{n-2}$ of $(v_1,v_2)^*$. Then the volume associated with the $(v_1,v_2)^*$ is

$$V_{1,2} = \sum_{t \in T_{12}} V_{tetra}(g, v_{t1}, v_{t2}, v_{t3})$$

We want to pick v_0 such that

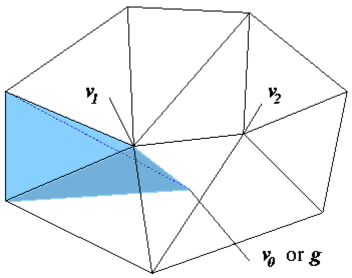
$$\sum_{i=3}^{n-1} V_{tetra}(g, v_0, v_i, v_{i+1}) = \sum_{t \in T_{12}} V_{tetra}(g, v_{t1}, v_{t2}, v_{t3})$$

Volume of a tetrahedron



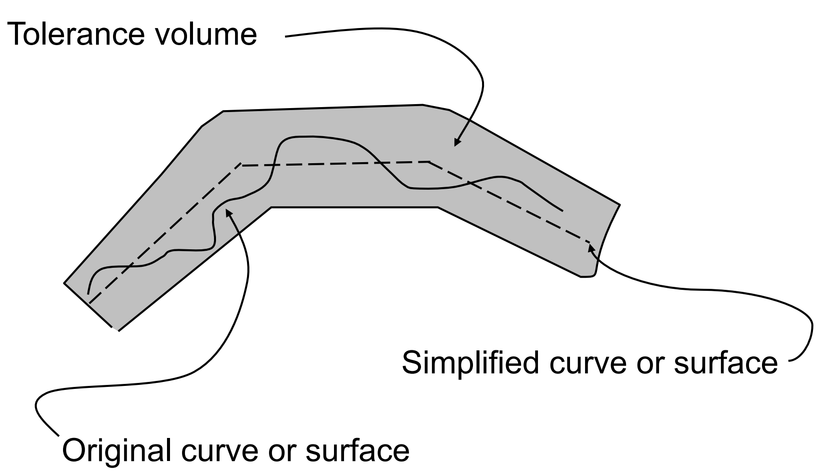
$$V_{tetra}(v_0, v_1, v_2, v_3) = \frac{1}{6}(v_1 - v_0) \bullet (v_2 - v_0) \times (v_3 - v_0)$$

Picking new vertex to preserve volume

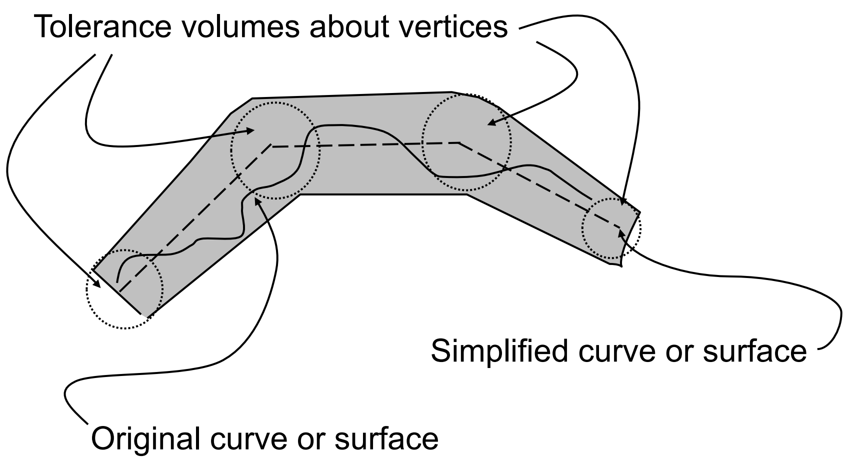


- Volume preservation constraint defines a plane on which v_0 must lie.
- Select the point on this plane that minimizes sum-of-squared distance to planes of all triangles being collapsed

Basic idea of tolerance volumes



Generate volumes from vertex tolerances



Error "tube" around an edge

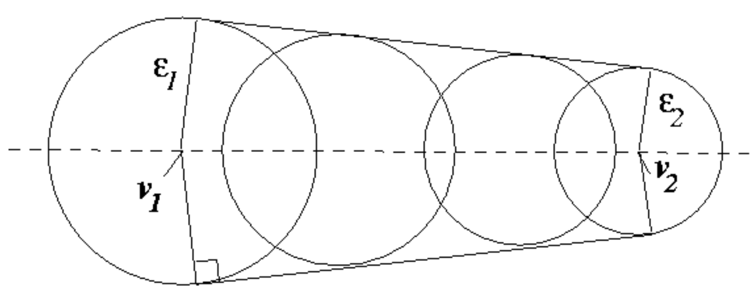


Figure 14: An edge tube

Triangle tube or "fat" triangle

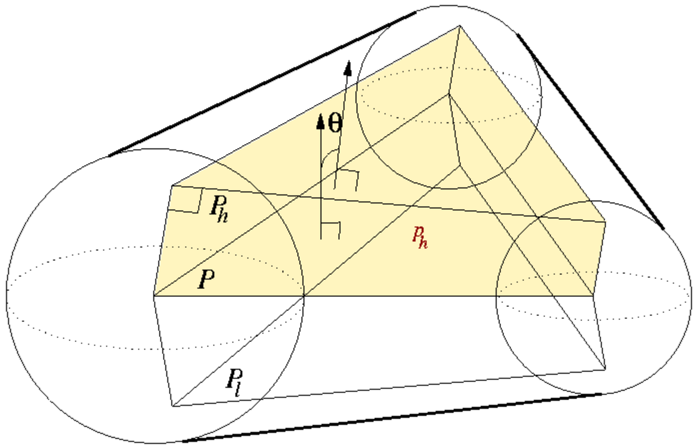


Figure 15: A triangle tube.

Local enforcement of tight tolerance

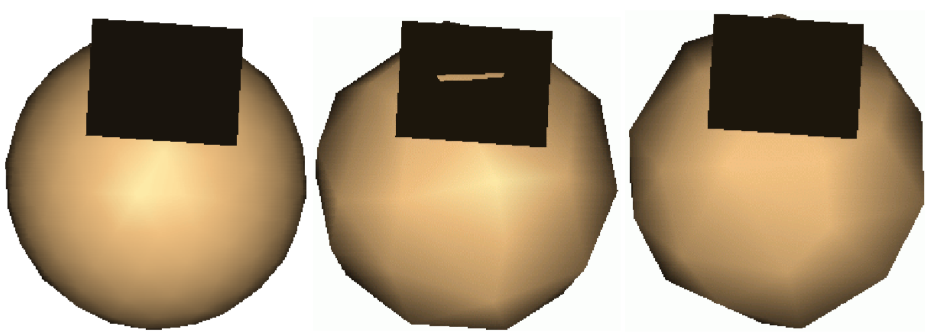
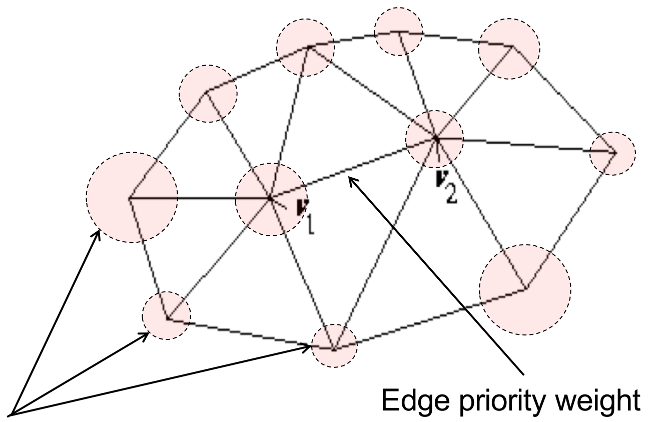


Figure 13: Using the tolerance volume to maintain tangency of two surfaces after simplification. From Left to Right, original sphere model (320 triangles) represented with tangent plane, simplified sphere model (82 triangles) with a tolerance equal to 10 % of the sphere diameter; the tangency condition is violated. Simplified sphere model (88 triangles) with the same global tolerance, but with a tolerance of 0 at the contact point.

Priority Weighting



Vertex error weights ρ_i

Edge priority weight

$$w_{ij} = \left\| \vec{\mathbf{v}}_i - \vec{\mathbf{v}}_j \right\| + \rho_i + \rho_j$$

Algorithm outline

Until Queue/ Last Bucket Empty:

- Take edge with low(est) weight
- If edge can be safely collapsed
  1. If valence does not exceed maximum
  2. If simplified vertex is regular
  3. If triangle normal rotation is acceptable
  4. If triangle compactness is acceptable
  5. If error does not exceed tolerance

  □ Change neighboring configuration

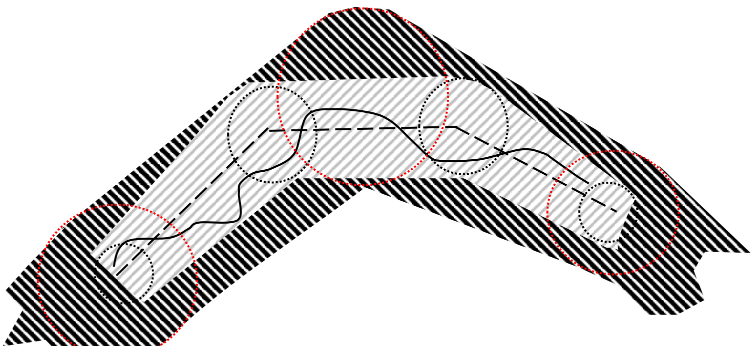
  □ Remove all edges of the star from the queue

  □ Reinstate new edges in the queue
- Else
  remove edge from queue

Figure 6: Simplification algorithm.

Merging Rule

When merge, assign (enlarge) vertex tolerances so that old surface shell is guaranteed to be completely inside the new surface shell



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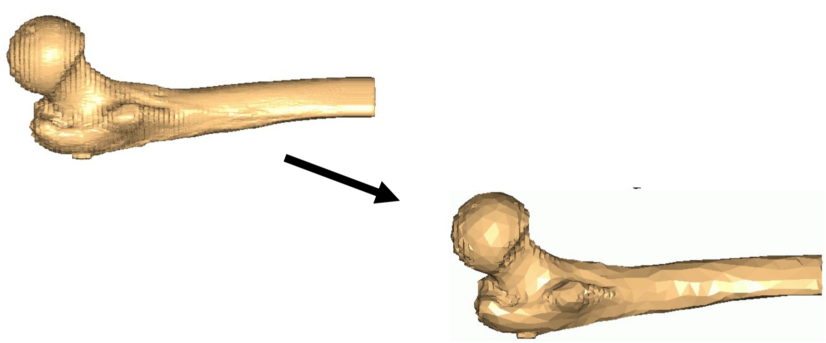
Volume preservation results

model	Femur	Buddha	Deino
original # of triangles	180,854	333,586	44,954
simplified # of triangles	3,124	49,106	19,490
original volume	233,462.7455 mm^3	23,048,568.98 $pixel^3$	230,276.599 mm^3
vol. after simplification	233,462.7452 mm^3	23,048,569.03 $pixel^3$	230,276.600 mm^3



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Triangle compactness improvement

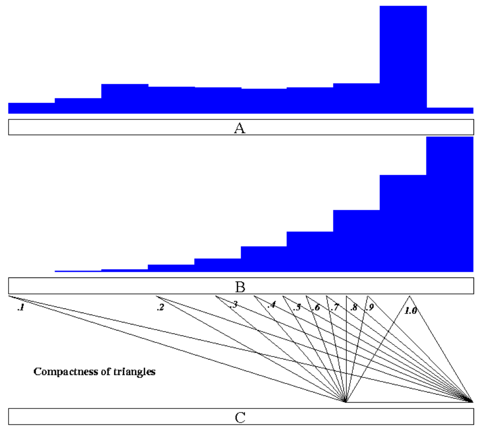


Figure 10: A: histogram of compactness values before simplification for the Femur example. B: histogram of compactness values after simplification. C: example of triangles of compactness values of .1 to 1 in .1 increments, corresponding to boundaries between the histogram bins. A flat triangle has a compactness of zero.

Triangle compactness improvement

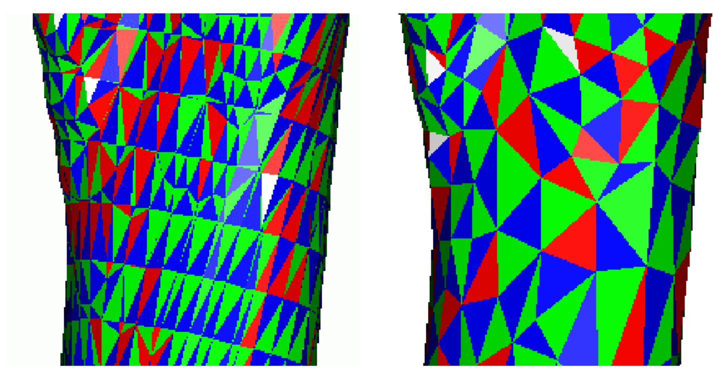


Figure 11: A visual inspection of these triangles extracted from the shaft of the Femur model shows that facets are more regular in the simplified femur model produced by the our algorithm (Right) than they are in the original output of an iso-surface algorithm (Left). In particular, most "sliver" (very narrow) triangles have been removed. Histograms of triangle compactness presented in Fig.10 confirm this observation.

Buddah

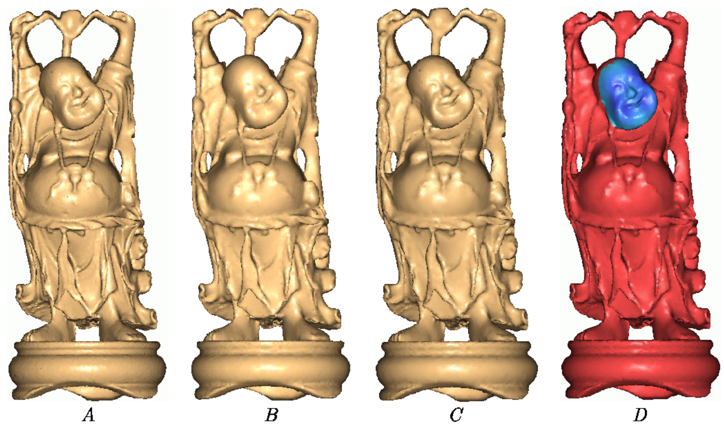
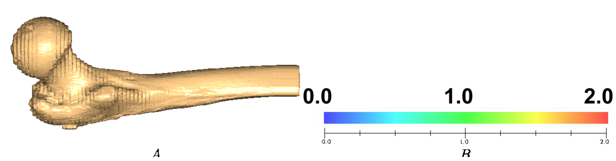
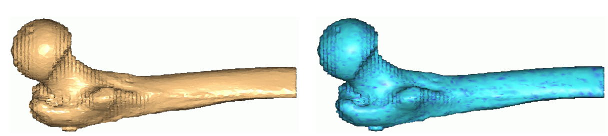


Figure 24: A: Buddha model of 334 K triangles. B: simplification with 46 K triangles using a uniform tolerance. C: simplification with 49 K triangles using a variable tolerance. D: coloring of the tolerance volume for the surface of C, with increasing values from blue to red in a Rainbow colormap.

Femur Simplification

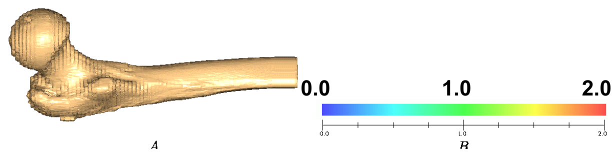


Original (181 K triangles)

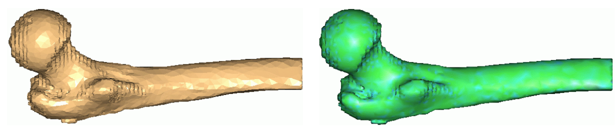


0.5 mm tolerance (26.8 K triangles)

Femur Simplification

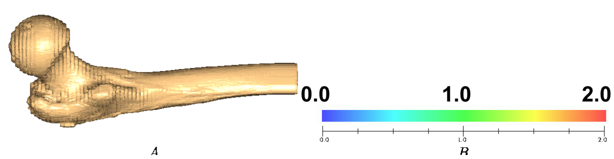


Original (181 K triangles)

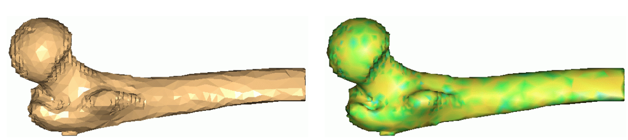


1.0 mm tolerance (9,592 triangles)

Femur Simplification

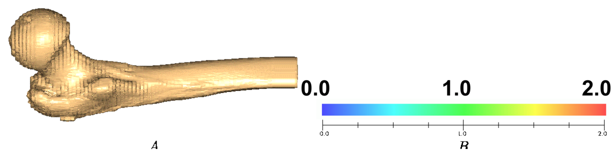


Original (181 K triangles)

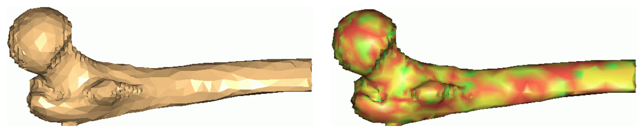


1.6 mm tolerance (4,618 triangles)

Femur Simplification



Original (181 K triangles)



2.0 mm tolerance (3,124 triangles)

Carotid artery

Figure 27: A: Carotid Arteries (57 K triangles). B: Simplification (5.6 K triangles) with a maximum error of 0.8%. C: Superimposition of A and B.

Carotid artery

Original (57 K triangles)

Simplified to 0.8% tolerance (5.6 K triangles)

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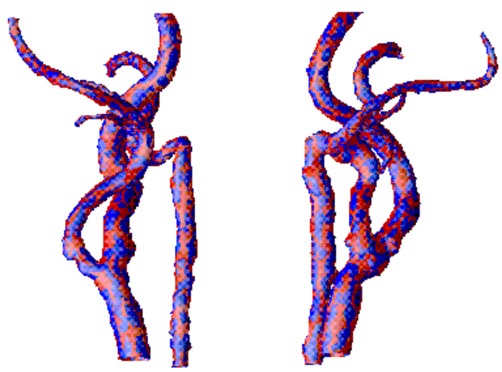
Carotid artery



Original and simplified superimposed

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Bunny Simplification

Original

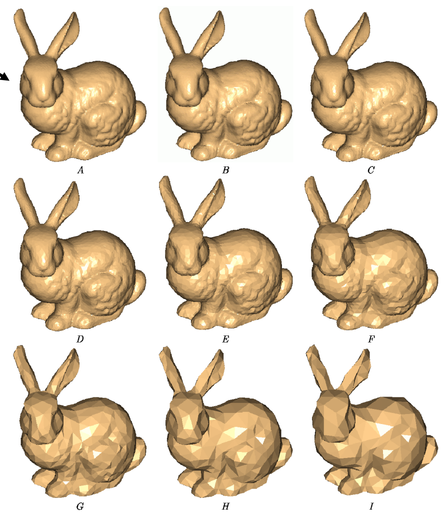

Figure 32: Successive simplifications of the Bunny model, flat shaded. A: original. B: tolerance of 1/32% of bounding box diameter. C: tolerance of 1/16%. D: 1/8%. E: 1/4%. F: 0.5%. G: 1%. H: 1.5%. I: 2%.

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