NSF Engineering Research Center
for Computer Integrated Surgical
Systems and Technology

Validation of Statistical Atlases

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WHITING
SCHOOL OF
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Validation of Statistical Atlases

- Given an atlas with mean shape $\bar{S}$, modes U and a new shape instance S^{new}
- Compute λ using

$$\lambda = U^T (S^{new} - \bar{S})$$

- and estimate the new shape instance as follows

$$S_{est}^{new} = \bar{S} + U\lambda$$

Validation of Statistical Atlases

- Given S^{new} and S^{new}_{est}

A variety of error metrics can be computed

- Vector based metrics
  - L1 norm $\varepsilon = \sum_k \left| S^{new}[k] - S^{new}_{est}[k] \right|$
  - L2 norm $\varepsilon = \left\| S^{new} - S^{new}_{est} \right\|_2$
  - Mahalanobis distance $\varepsilon = \sqrt{(S^{new} - S^{new}_{est})^T \Sigma^{-1} (S^{new} - S^{new}_{est})}$
  - Angle between shape vectors $\theta = a\cos\left(\frac{S^{new} . S^{new}_{est}}{\left| S^{new} \right| \left| S^{new}_{est} \right|} \right)$

Validation of Statistical Atlases

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- Mesh based metrics
  - Vertex to vertex correspondence errors
  - Surface distance between meshes using ICP
- Volume based metrics – voxelize the mesh and compare the volumes
  - Volume overlap

Leave out validation

Given set of models $\{\cdots S^k \cdots\}$, do the following

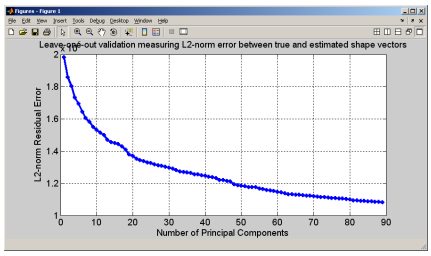
for $k = 1$ to N do

$\bar{S}$, **U** = Compute statistical atlas ($\{\cdots S^{k-1}, S^{k+1} \cdots\}$)

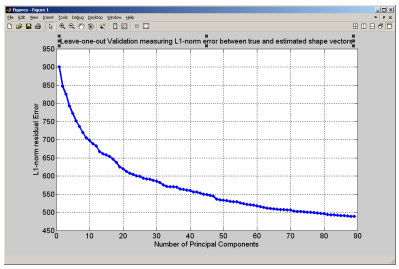
$\vec{\lambda} = \mathbf{U}^T (S^k - \bar{S})$

$E^k = S^k - (\bar{S} + \mathbf{U}\vec{\lambda})$

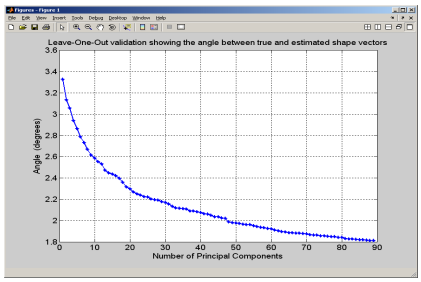
Compute statistics on the $\{\cdots E^k \cdots\}$



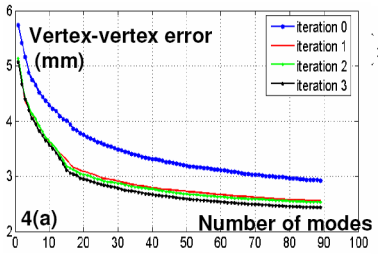
L2-norm

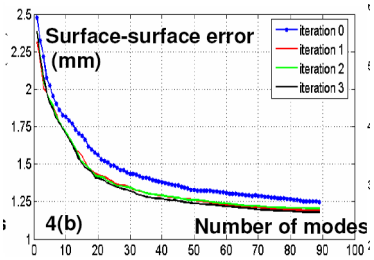


L1-norm



angle





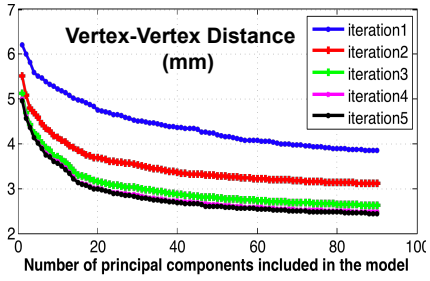
Leave-Out Validation Experiments

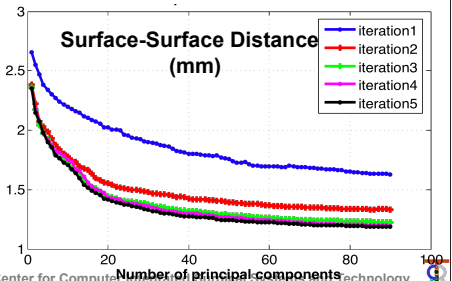
- # of iterations: 5
- # of data sets: 110
- # of data sets in atlas: 90
- # of data sets left out: 20
- Given a left-out dataset, s_j

compute the estimated shape from atlas using

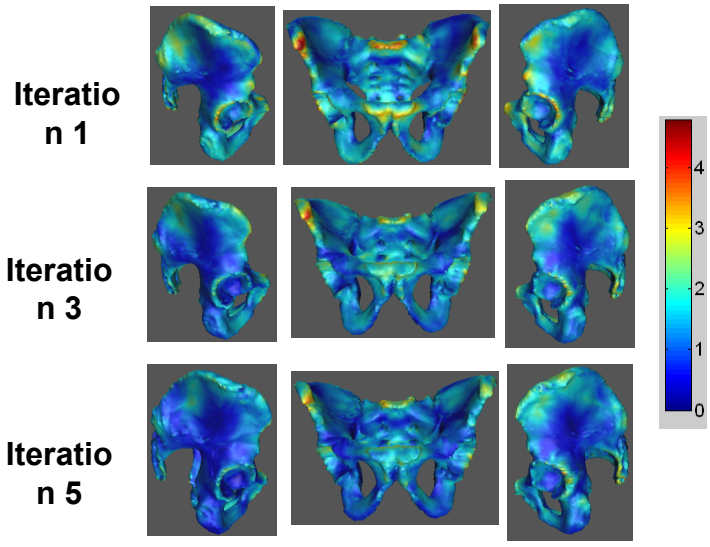
$$\lambda = U'^{*}(s_j - \bar{S})$$

$$s_j^{est} = \bar{S} + U\lambda$$





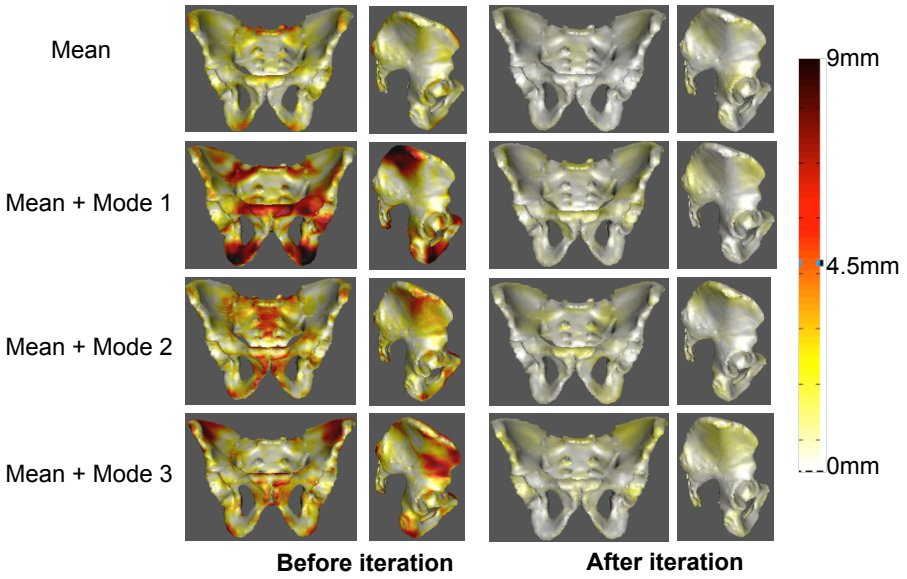
Distribution of Surface Registration Errors



Choice of Initial Template

- Claim:
  - iterative method does not depend on the choice of template
- Criteria:
  - Mean shape converges
  - Modes exhibit similar deformation patterns
- Experimental setup:
  - Three random templates
  - Atlases with and without bootstrapping compared
- Result
  - All three atlases exhibit similar deformation patterns after bootstrapping

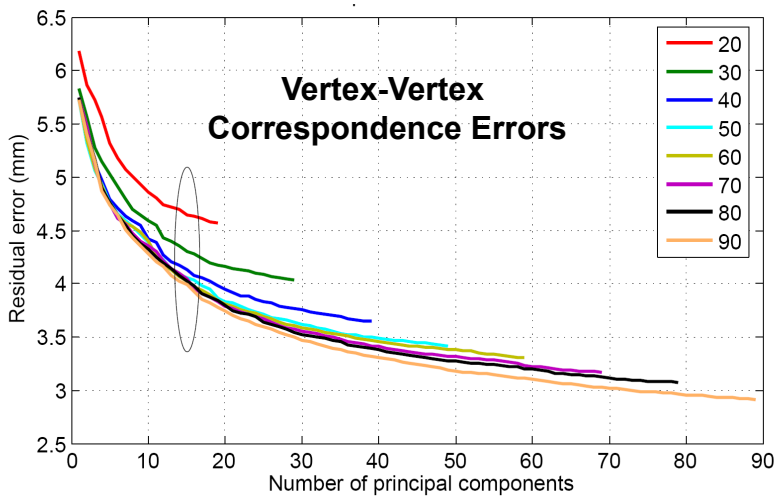
Average Difference between Atlases 1,2 and 3



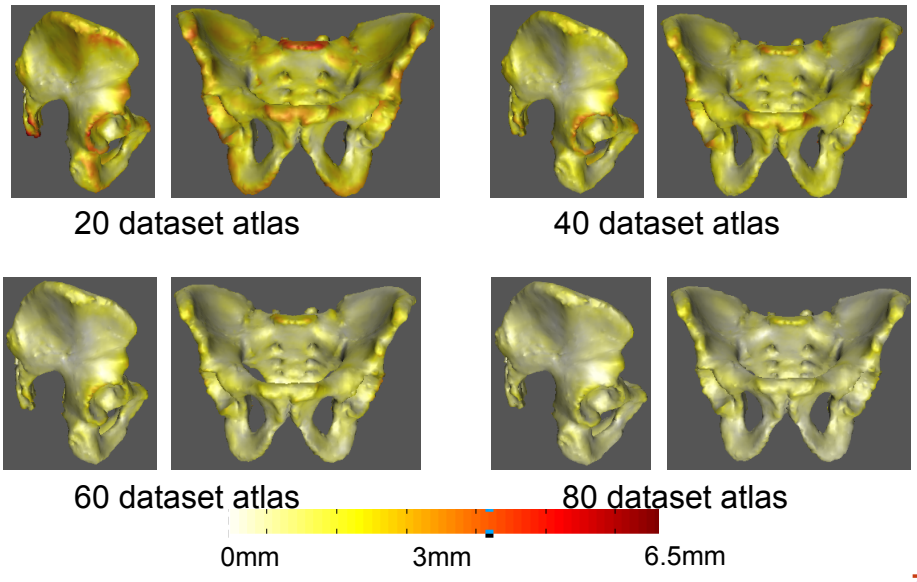
Training Sample Size

- Goal:
  - To determine the size of the training sample to build a stable statistical atlas
- Criteria:
  - Atlas is stable
  - No significant improvement in residual error
- Experimental setup:
  - Varying sample size 20, 40, 60, 80
  - Leave-20-out validation test
- Result:
  - Minimum of 50 data sets are required for pelvis atlas

Training Sample Size



Surface residual error using 18 modes for different sample set sizes



Stability Analysis – Mean Shape