Homework Assignment 3 – 601.455/655 (CIRCLE ONE)

Fall 2017

Instructions and Score Sheet (hand in with answers)

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NOTE: The total number of points in the questions exceeds 100. This is intentional. You can think of some of the questions as being (in part) extra credit. However, I have also limited the total number of points that will count toward your grade. If either student is enrolled in 601.655, we will use the limit of 100 points. For two students taking 601.455, we will set the limit at 105.
1. Remember that this is a graded homework assignment. It is effectively an exam.

2. You are to work alone or in a team of two people and are not to discuss the problems with anyone other than the TAs or the instructor.

3. Put your names and email address on each sheet and number the sheets.

4. You are encouraged to make free use of any published materials, the web, etc. in developing your answer but a) you must give full and proper citations to any references consulted and b) you may not consult, discuss, or otherwise communicate about this assignment with any human being except your lab partner, the course instructor, or the TAs. The one exception is that you should not refer to previous years’ homework.

5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.

6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.

7. Sign and hand in the score sheet as the first sheet of your assignment

8. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment.

9. Attach the grade sheet as the first sheet and attach all sheets together.

10. You must include a self-addressed, seal-able 8 ½ x 11 inch envelope if you expect to the homework to be returned (per JHU’s interpretation of FERPA).
Note

These questions involve workflow, algorithm and system design. Many of them relate to registration problems in some way or may otherwise involve iterations invoking methods discussed in class. You do not need to recite all the details of how you would compute a 3D-to-3D point cloud transformation, but you should be very clear about what method you plan to use and what the inputs and outputs will be. Similarly, if you plan to use a method like ICP, you don’t need to recite all the algorithmic details (spatial data structures, specific termination conditions, etc.) but it should be reasonably clear how you are setting up the problem, what the inputs and outputs will be, whether there are outliers and what those may be, and what the basic iteration will be. For other iterative processes invoking known methods, you should describe the basic steps of the process, what changes at each step, and (roughly) what the termination condition might be.

Question 1

Consider the stereo vision-based navigational tracker shown in Figure 1. There are two cameras with focal length \( h \) placed a distance \( 2d \) apart. The transformations between the coordinate system \( F_c \) associated with the tracker and the coordinate systems associated with the two cameras are \( F_{cl} = I, [0,0,-d] \) and \( F_{cr} = I, [0,0,d] \).

We define a simple pinhole projection formula, which we will use both for the cameras in this question and for the x-ray system in a later question:

\[
\text{proj}(\mathbf{b}, \mathbf{s}) = \begin{bmatrix} s_x + \frac{(b_z - s_z)s_z}{s_z - b_z} & s_y + \frac{(b_z - s_z)s_z}{s_z - b_z} & 0 \end{bmatrix}
\]

Note that this model may be a bit different from what you may be used to seeing in computer vision papers or notes, but it is what we are using here.

Now, suppose that we have a visible feature space with coordinates \( \mathbf{c} \) relative to the tracker system coordinate system \( F_c \). Then the image coordinates \( \mathbf{a} \) and \( \mathbf{b} \) of this point in the two cameras will be given by \( \mathbf{a} = \text{proj}(F_{cl}^{-1}\mathbf{c}[0,0,h]) \) and \( \mathbf{b} = \text{proj}(F_{cr}^{-1}\mathbf{c}[0,0,h]) \), where \( h \) is the focal length of cameras.

A. Give expressions for computing the coordinates \( c_x, c_y, c_z \) of \( \mathbf{c} \) from the coordinates \( [a_x, a_y, a_z] \) and \( [b_x, b_y, b_z] \) of \( \mathbf{a} \) and \( \mathbf{b} \).
B. Suppose that the actual values of \(d\) and \(h\) are \(d + \delta\) and \(h + \eta\), where \(\delta\) and \(\eta\) are small errors. What will be the error \(\Delta \vec{c}\) introduced into our calculation for \(\vec{c}\) ?

C. Suppose that you have available a fiducial structure consisting of a number of visible landmarks located at the corners of an otherwise transparent cube whose sides are of length \(w\) and positioned so that all landmarks are visible in both images. Describe a calibration procedure for determining \(\delta\) and \(\eta\). Provide enough algorithmic detail and mathematical formulation so that a reasonably intelligent programmer who does not have your excellent knowledge of CIS could write a program from your specification to compute \(\delta\) and \(\eta\).

D. Assume now that your program has been run and exact values of \(d\) and \(h\) are known. However, the image processing algorithm used to locate the image coordinates \(\vec{a}\) and \(\vec{b}\) of \(\vec{c}\) are subject to small errors \(\Delta \vec{a} = [\Delta a_x, \Delta a_y, 0]\) and \(\Delta \vec{b} = [\Delta b_x, \Delta b_y, 0]\). What will be the resulting error \(\Delta \vec{c}\) in your calculation of \(\vec{c}\) ?

E. For this tracker system, we also know that \(d = 250\) mm and \(h = 100\) mm. These points will be combined with other information (e.g., other point observations) to compute the transformation \(\text{FC}_T\) between tracker body and tracker system coordinates. Suppose that we know that \(\|\Delta \vec{a}\|\) and \(\|\Delta \vec{b}\|\) are both less than approximately 0.02 mm for any single marker. Here you may use either the usual \(L^2\) norm or the \(L^\infty\) norm. Provide a numerical estimate for bounds on the on the components \(|\Delta c_x|\), \(|\Delta c_y|\), and \(|\Delta c_z|\) of the error \(\Delta \vec{c}\) in your calculation of \(\vec{c}\).

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**Question 2**

Consider the x-ray system and tracking setup shown in Figure 2. The source of the x-ray system is located at a position \(\vec{s}\) relative to the detector coordinate system \(\text{FD}_D\). For the moment, you may assume that the detector is a modern flat-panel detector with no significant image distortion. Also present is an optical tracking system similar to that described in Question 1. There is an object placed in the field of view of both the x-ray system and the optical tracking system. There are a large number of features visible all over the surface of the object (shown in the figure as the intersections of many lines on the surface of the object).
object. The optical tracking system is able determine the coordinates \( \mathbf{c}_j \) of these features relative to the optical tracking system coordinate system \( \mathbf{F}_c \). Embedded in the object are a number of small spheres that are visible in x-rays, but are not visible to the optical tracking system. The image of the \( k'th \) sphere in the x-ray image appears at coordinates \( \mathbf{f}_k = [u_k, v_k, 0] \) relative to the detector coordinate system \( \mathbf{F}_d \). The object is rigid and the coordinates \( \mathbf{a}_k \) and \( \mathbf{b}_j \) of the spheres and surface features relative to any arbitrary object coordinate system \( \mathbf{F}_A \) associated with the object do not change if the object is moved.

In Question 2, the tracking system has a fixed and unchanging relationship to the x-ray detector, but the transformation \( \mathbf{F}_{cd} \) between x-ray detector and tracking system coordinates is not known. However, the object itself may be moved.

A. Given a “snapshot” of surface feature locations \( \{\cdots, \mathbf{c}_j(0), \cdots\} \) at some time \( t = 0 \), describe how you would define a coordinate system \( \mathbf{F}_A \) and associated body feature coordinates \( \mathbf{b}_j \) such that \( \mathbf{c}_j(t) = \mathbf{F}_{ca}(t) \mathbf{b}_j \) for any arbitrary subsequent snapshot \( \{\cdots, \mathbf{c}_j(t), \cdots\} \) of surface feature locations, where \( \mathbf{F}_{ca}(t) \) is the transformation between body and tracker coordinates at time \( t \). Give sufficient formulas so that that a reasonably intelligent programmer who does not have your excellent knowledge of CIS could write a program from your specification to compute the \( \mathbf{b}_j \).

B. Given the \( \mathbf{b}_j \) values and a “snapshot” of the observed feature locations \( \{\cdots, \mathbf{c}_j(t), \cdots\} \), provide a succinct summary of how you would compute \( \mathbf{F}_{ca}(t) \). Here you should assume that the computer does not know the pairing of the \( \mathbf{b}_j \) values and any of the observed feature locations \( \{\cdots, \mathbf{c}_k(t), \cdots\} \). For example, the features could be indistinguishable dots sprinkled at random over the surface. But you can also assume that the pairwise distances between the features are different.

C. Suppose that the entire surface of the object is covered with potential features. At any given time, not all are visible in to the optical tracker, although well more than enough are visible so that your answer to Question 2.B will produce a valid answer. Describe how you would find a complete collection of \( \mathbf{b}_j \) values to cover the entire object surface.

D. Assume that you have found answers to questions 2.A, 2.B, and 2.C (or that a helpful muse has disclosed the answer in your sleep). At this point you will have a set of \( \mathbf{b}_j \) values corresponding to indistinguishable features to cover the object surface and associated with some coordinate system \( \mathbf{F}_A \), together with a way to compute a coordinate transformation \( \mathbf{F}_{ca}(t) \) from an observed set of \( \{\cdots, \mathbf{c}_j(t), \cdots\} \). Again, you may assume that the pairwise distances between the features are distinct. Describe a procedure for determining the locations \( \mathbf{a}_k \) of the spheres relative to coordinate system \( \mathbf{F}_A \). Give sufficient formulas so that
that a reasonably intelligent programmer who does not have your excellent knowledge of CIS could write a program from your specification to compute the $\mathbf{a}_k$.

E. Describe a method for determining the transformation $\mathbf{F}_{CD}$ between x-ray detector and optical tracker coordinates. Once this relationship is known, then the relationship between detector and body coordinates can be computed from $\mathbf{F}_{CA} = \mathbf{F}_{CD} \mathbf{F}_{DA}$, so that $\mathbf{f}_k = \text{proj}(\mathbf{F}_{DA} \mathbf{a}_k, \mathbf{s})$.

Again, give sufficient procedural, algorithmic, and mathematical detail so that our hypothetical (and now tiring, but still enthusiastic) programmer can implement your specification.

**Question 3**

Suppose now that the x-ray system of Question 2 has been calibrated successfully, but that the tracking system has been moved so that $\mathbf{F}_{CD}$ is no longer known. However, four markers have been put at known positions $\mathbf{d}_j$ relative to the detector. The optical tracker can locate these markers at locations $\mathbf{c}_j$ relative to the tracking system.

The calibration body of Question 2 has been replaced in the field of view of the x-ray system with an anatomic object with some vascular structures. Both the tracking system and the x-ray system may move during the procedure. In fact, in answering this question, you may ask the x-ray technician to move the x-ray c-arm during the procedure to approximately defined positions relative to the anatomic object (e.g., “please rotate the c-arm by approximately 45 degrees and re-center it so that the anatomical object is in the field of view). The tracker motions may be due to room vibrations or may be intentional.

In addition, there are available up to 4 markers that may be stuck onto the anatomic object at fixed (but unknown) positions. The tracking system can track these markers, returning values $\mathbf{g}_j$ for the marker positions relative to the tracking system.

A CT scan of the object has been made and an image segmentation process has been performed to determine the centerlines of the vascular structures. When contrast is injected into the vasculature, the structures become visible in x-ray images. Image processing software is available to find the centerlines of these structures in the 2D x-ray images. Although not all of the structures may be visible in the x-ray images, most of them will be. The CT segmentation program represents the segmented vessels as a very dense set of 3D points $\mathbf{a}_k$ on the centerlines of the vessels. Similarly, the x-ray segmentation represents the images in the vessels as dense sets of points $\mathbf{d}_i$. The segmentation programs are not able to distinguish individual points. I.e., they cannot say which $\mathbf{d}_i$ is associated with any particular $\mathbf{a}_k$. 

![Figure 3](image-url)
Describe a method for computing the registration transformation $F_{CA}$ such that any point $\tilde{a}_k$ in CT coordinates will have the corresponding tracker coordinates $F_{CA} \cdot \tilde{a}_k$. Again, give sufficient procedural, algorithmic, and mathematical detail so that our hypothetical (and now extremely tired, but still enthusiastic) programmer can implement your specification.