CHomework Assignment 2 – 600.455/655 Fall 2017 (Circle One)

Instructions and Score Sheet (hand in with answers)

Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
Signature (required) I/We have followed the rules in completing this assignment ______________________________	Signature (required) I/We have followed the rules in completing this assignment ______________________________

Question	Points (445)	Points (645)		Totals
1A	2	2		
1B	3	3		
1C	5	5		
1D	10	10		
1E	10	10		
1F	15	15		
2A	5	5		
2B	5	5		
2C	10	10		
2D	10	10		
2E	10	10		
2F	15	15		
Total	100	100		

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
6. Sign and hand in the score sheet as the first sheet of your assignment.
7. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment

Question 1

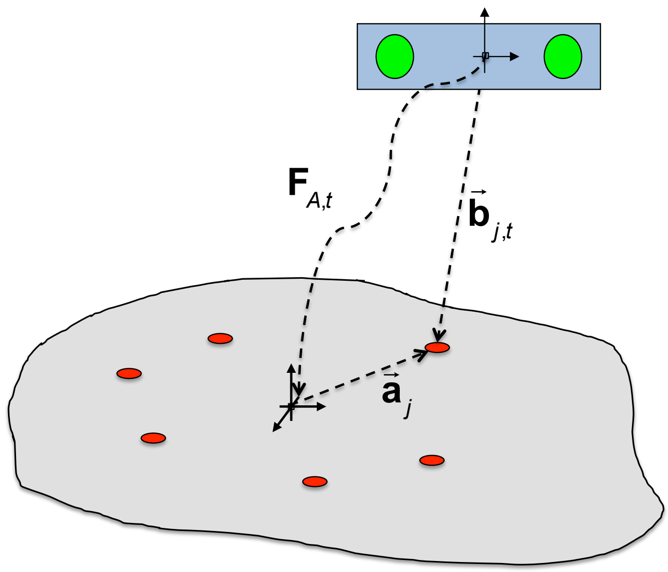

Fig. 1: Tracked object

Consider the "tracked object" shown in Fig. 1, above. An optical tracking device, such as a Northern Digital Polaris™ is used to track the 3D positions of a fairly large number of optical markers relative to the camera. At time t the position of marker j relative to the tracking camera reported by the tracking system is $\vec{\mathbf{b}}_{j,t}$. The position of the marker j relative to the object coordinate system is $\vec{\mathbf{a}}_j$, and the object's pose relative to the tracking system is $\mathbf{F}_{A,t}$. You can assume that the tracking system has a means for computing $\mathbf{F}_{A,t}$, given a set of values $\vec{\mathbf{a}}_j$ and corresponding values $\vec{\mathbf{b}}_{j,t}$. **NOTE:** This scenario frequently arises when one is making one's own tracking fiducials or when one is simply attaching markers to anatomy for the purpose of tracking.

A. Suppose that the object moves slightly, so that $\mathbf{F}_{A,t+1} = \Delta\mathbf{F}_A\mathbf{F}_{A,t}$, where $\Delta\mathbf{F}_A = [\Delta\mathbf{R}_A, \Delta\vec{\mathbf{p}}_A]$, What will be the new value reported for $\vec{\mathbf{b}}_{j,t+1}$?

B. Suppose that we can approximate $\Delta\mathbf{F} \approx [\mathbf{I} + sk(\vec{\alpha}), \vec{\varepsilon}]$. Give an approximate expression for $\vec{\mathbf{b}}_{j,t+1}$.

C. Suppose now that the values for $\mathbf{a}_j$ are not known, but that the values for the $\vec{\mathbf{b}}_{j,t}$ are accurately measured. Outline a procedure for defining a set of $\vec{\mathbf{a}}_j$ that can be used to compute a pose $\mathbf{F}_{A,t}$ such that $\mathbf{F}_{A,t}\vec{\mathbf{a}}_j = \vec{\mathbf{b}}_{j,t}$ and $\sum_j \vec{\mathbf{a}}_j = \vec{\mathbf{0}}$.

D. How would you modify your answer to Question 1.C if the values measured for $\vec{\mathbf{b}}_{j,t}$ are subject to some zero-mean random error $\Delta\vec{\mathbf{b}}_{j,t}$, so that $\left\|\sum_{1\le t\le T}\Delta\vec{\mathbf{b}}_{j,t}\right\|/T \le \rho / T$ as T gets big for some small value of ρ. For now, you can assume that the object is motionless relative to the tracking camera.

E. Now suppose that the object is moving slightly relative to the tracking camera. I.e., $\Delta\mathbf{F}_{A,t} \approx [\mathbf{I} + sk(\vec{\alpha}_{A,t}), \vec{\varepsilon}_{A,t}]$. These motions are small but are <u>not</u> zero mean. Assuming that there are also small random errors in tracking the markers relative to the camera, as in Question 1.D. Outline and justify a procedure for estimating $\vec{\mathbf{a}}_j$. Explain why it will produce the desired result. (**Hint:** Your answer will involve an iteration of some sort.)

F. Suppose that we have now accurately calibrated our values for $\vec{\mathbf{a}}_j$, but that there is still some bounded error in the values of the $\vec{\mathbf{b}}_{j,t}$ such that $\left\|\Delta\vec{\mathbf{b}}_{j,t}\right\| \le \rho$. Suppose that the system has computed a value for $\mathbf{F}_{A,t}$ with some error so that the actual value is $\mathbf{F}_{A,t}^{\ *} = \mathbf{F}_{A,t}\Delta\mathbf{F}_{A,t}$. Estimate the magnitude of the worst-case rotational error about some (unit vector) axis $\vec{\mathbf{n}}$ in object coordinates. (**Hint:** Recall $Rot(\vec{\mathbf{n}},\nu) \approx \mathbf{I} + sk(\nu\vec{\mathbf{n}})$ for small values of ν.

Question 2

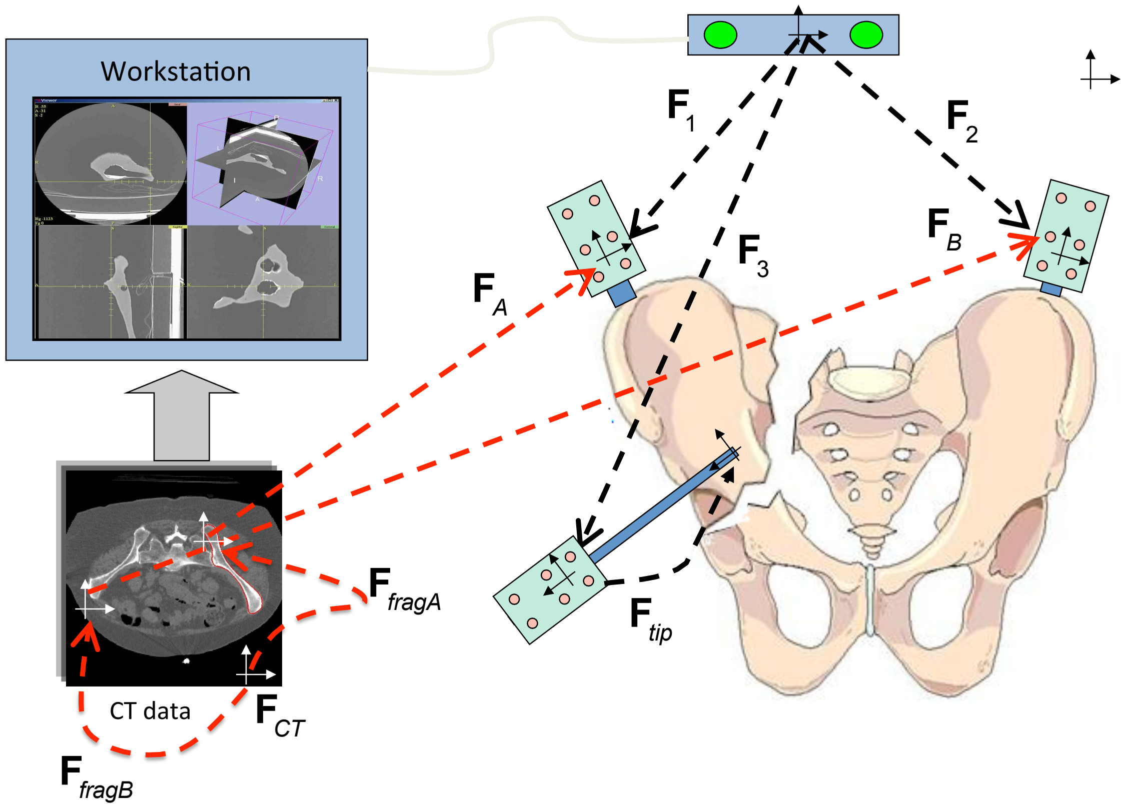

Fig. 2: Computer-Assisted Fracture Repair System

Consider the computer-assisted fracture repair scenario illustrated in Fig. 2, above. The patient has a severely broken bone (here, a pelvis). The goal is to reduce the fracture (i.e., realign the bone fragments) and bolt them together using screws and plates as shown in Fig. 3. The normal procedure goes roughly like this:

1. Reduce the fracture.
2. Align the plates with the bone fragments.
3. Drill pilot holes using the plates as a drill template.
4. Insert bone screws through the plates to secure the bones together.

Those of you who have noticed the similarity to carpentry should not be surprised. There are many ways to use a surgical navigation system to improve this procedure. In the particular embodiment we will consider, we will assume that we use the navigation system to help the surgeon <u>pre-drill</u> the pilot holes into the bone at the right place so that the bones are aligned correctly when the screws are inserted into the plates.

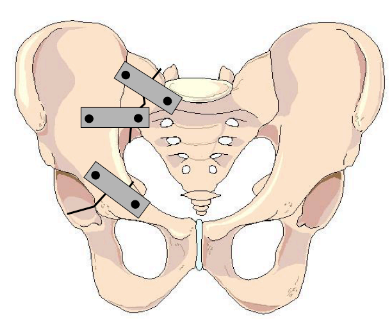

Fig. 3: Repaired Fracture

Procedural Flow

1. Presurgical Planning
    a. A CT scan of the patient is made, and the data is loaded into the workstation
    b. Image segmentation is performed to identify the bone fragments (which we will call "A" and "B"). It will define coordinate systems $\mathbf{F}_A$ and $\mathbf{F}_B$ associated with these fragments and it will determine the positions of anatomical features $\{\cdots\vec{\mathbf{f}}_{A,k}\cdots\}$ and $\{\cdots\vec{\mathbf{f}}_{B,k}\cdots\}$ defined with respect to bone fragment coordinate systems $\mathbf{F}_{fragA}$ and $\mathbf{F}_{fragB}$ that can be used to "register" these bone fragments intraoperatively. Here, $\mathbf{F}_{fragA}$ and $\mathbf{F}_{fragB}$ may be viewed as coordinate transformations from CT coordinates to local coordinate systems associated with the bone fragments.
    c. The workstation will help the surgeon determine how to reduce the fracture, i.e., how to manipulate the bone fragments so that they are properly aligned. I.e., it will help determine a transformation $\mathbf{F}_{reduce}$ such that $\mathbf{F}_{reduce}\mathbf{F}_{fragA}=\mathbf{F}_{fragB}$ when the fracture is reduced.
    d. It will help the surgeon determine the desired locations $\{\cdots\vec{\mathbf{h}}_{A,k}\cdots\}$ and $\{\cdots\vec{\mathbf{h}}_{B,k}\cdots\}$ relative to the bone fragment coordinate systems $\mathbf{F}_{fragA}$ and $\mathbf{F}_{fragB}$ for placing the pilot holes for drilling the fixation plates.
2. Intraoperatively
    a. Reference markers are affixed rigidly to the bone fragments. Marker 1 is attached to bone fragment A and Marker 2 is attached to bone fragment B.
    b. The surgeon has a drilling tool whose tip can also be used as a pointer. There is a navigational marker 3 attached to this drill, and the coordinate system of the tip relative to this marker is given by $\mathbf{F}_{tip}=[\mathbf{I},\vec{\mathbf{p}}_{tip}]$ and $\vec{\mathbf{p}}_{tip}=[0,0,d_{tip}]$. The tracking system is able to compute the coordinate systems of the navigational markers. At any given time t, the values computed for these markers will be given by $\mathbf{F}_1(t)$, $\mathbf{F}_2(t)$, and $\mathbf{F}_3(t)$.
    c. The drill/pointer is used to locate the anatomic features on the bone fragments, and "registration" calculations are performed to produce transformations $\mathbf{F}_A$ and $\mathbf{F}_B$, so that $\mathbf{F}_1(t)\vec{\mathbf{a}}$ and $\mathbf{F}_A\vec{\mathbf{a}}$ refer to the same anatomic point for any 3d coordinate $\vec{\mathbf{a}}$ on fragment A defined relative to $\mathbf{F}_{fragA}$ coordinates. Similarly $\mathbf{F}_2(t)\vec{\mathbf{b}}$ and $\mathbf{F}_B\vec{\mathbf{b}}$ refer to any anatomic coordinate $\vec{\mathbf{b}}$ on fragment B defined relative to $\mathbf{F}_{fragB}$.
    d. The surgeon uses the drill to drill the pilot holes while the computer performs appropriate graphics to inform the surgeon where the drill tip is relative to the designated bone fragment.

e. Surgery then proceeds manually to complete the procedure.

Questions

A. Give an expression for computing the position and orientation of the drill tip relative to bone fragment A, based on the tracking data.

B. Give an expression for computing the displacement of the drill tip from a desired hole location $\vec{\mathbf{h}}_{A,k}$ on bone fragment A.

C. Suppose that the tracking system has some error. So that the actual pose $\mathbf{F}_m^*(t)$ of each marker m is $\mathbf{F}_m^* = \mathbf{F}_m(t)\Delta\mathbf{F}_m(t)$. Give an expression for the error in your answer for Question 2.B in terms of the $\Delta\mathbf{R}_m$ and $\Delta\vec{\mathbf{p}}_m$ values. (For this problem, you can leave off the "(t)" part, since this is for one time frame.

D. Produce a linearized estimate of your answer to Question 2.C using the approximation $\Delta\mathbf{F}_m \approx [\mathbf{I} + sk(\vec{\alpha}_m), \vec{\varepsilon}_m]$.

E. Suppose, also that there also is a registration error $\Delta\mathbf{F}_A$ so that the correct registration value should be $\mathbf{F}_A^* = \mathbf{F}_A\Delta\mathbf{F}_A$. Assume that we can assume that this error is small, so that $\Delta\mathbf{F}_A \approx [\mathbf{I} + sk(\vec{\alpha}_A), \vec{\varepsilon}_A]$. Produce a linearized estimate of your answer to Question 2.D with this additional assumption.

F. The preformed plates used to secure the bone fragments have holes corresponding to holes to be drilled into bone fragments A and B, respectively. The holes will have somewhat larger diameter than the diameter of the screws, in order to accommodate small alignment and registration errors. Suppose that we have the following bounds on our tracking and registration errors: $\|\vec{\varepsilon}_n\| \leq \gamma$; $\|\vec{\alpha}_n\| \leq \gamma / 100$ for $n \in \{1,2,3,A,B\}$. How much clearance do you need to have for the holes to be sure that the screws will go in? I.e., how much larger do the holes need to be.