Homework Assignment 4 – 600.445/645 Fall 2016 (Circle One)

Instructions and Score Sheet (hand in with answers)

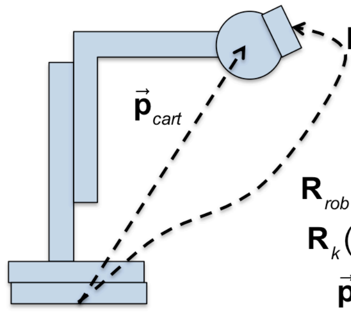
Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
Signature (required) I/We have followed the rules in completing this assignment _______________________________	Signature (required) I/We have followed the rules in completing this assignment _______________________________

Question	Points (445)	Points (645)		Totals
1A	6	2		
1B	4	2		
1C	4	3		
1D	6	3		
1E	15	10		
1F	15	15		
1G	See Note (15)	15		/ 50
2A	2	2		
2B	3	3		
2C	10	10		
2D	10	10		
2E	10	10		
2F	10	10		
2G	5	5		/ 50
3	See Note (15)	See Note (15)		/ 15
Total	100	100		

Note: Students may attempt any or all these problems for extra credit. We will award up to 15 extra points, but your total score will be limited to 100. I.e., if your total on the remaining problems is S and you score a total of E points on the extra credit problems, your net homework score will be $\min(100, S + \min(E, 15))$. They are good problems, and I would urge people to try them.

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
6. Sign and hand in the score sheet as the first sheet of your assignment.
7. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment

Question 1



$$\mathbf{F}_{rob} = [\mathbf{R}_{rob}, \vec{\mathbf{p}}_{rob}] = [\mathbf{R}_{rob}, \vec{\mathbf{p}}_{cart} + \mathbf{R}_{rob}\vec{\mathbf{p}}_{cr}]$$

$$\mathbf{R}_{rob}(\vec{\theta}) = \mathbf{R}_1(\theta_1)\mathbf{R}_2(\theta_2)\mathbf{R}_3(\theta_3)\mathbf{R}_4(\theta_4)$$

$$\mathbf{R}_k(\theta_k) = Rot(\vec{\mathbf{a}}_k, \theta_k)$$

$$\vec{\mathbf{p}}_{cart} = [x, y, z]^T$$

Fig. 1: Robot Kinematics

Consider the robot shown in **Fig. 1**. This robot consists of a Cartesian base, a multi-axis wrist mechanism, and a tool holder. The transformation between the robot's base and the tool holder is given by $\mathbf{F}_{rob}(\vec{\mathbf{q}})$, where $\vec{\mathbf{q}} = [\theta_1, \theta_2, \theta_3, \theta_4, x, y, z]$, as shown in the figure. We may also write this as $\vec{\mathbf{q}} = [\vec{\theta}^T, \vec{\mathbf{p}}_{cart}{}^T]^T$ or (abusing notation a bit) simply as $\vec{\mathbf{q}} = [\vec{\theta}, \mathbf{p}_{cart}]$ where the meaning is clear. For now, we can assume that the robot is very precise and accurate. The limits on the range of joint motion are given by $\vec{\mathbf{q}}_{min} \le \vec{\mathbf{q}} \le \vec{\mathbf{q}}_{max}$; the velocity limits of the joints are given by $-\vec{\mathbf{v}}_{max} \le \dot{\vec{\mathbf{q}}} \le \vec{\mathbf{v}}_{max}$.

The robot controller is able to retrieve the robot's joint positions and velocities at any time. Given a desired motion, the robot's controller computes new joint position goals at time intervals of Δt and sends them to the low level joint servo-controllers. For the purposes of this exercise, you may (unrealistically) assume that the joints respond instantaneously and always reach the desired goal after Δt time, and that the joint speed for an incremental joint motion during the time interval is $\Delta\vec{\mathbf{q}} / \Delta t$. The controller has available a very fast subroutine for solving least squares problems with linear constraints. I.e., it can solve problems of the general form

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \left\| \mathbf{Ex} - \mathbf{f} \right\|^2$$

such that

$$\mathbf{Ax} \le \mathbf{b}$$
$$\mathbf{Cx} = \mathbf{d}$$

very quickly for reasonable numbers of constraints. (Note that the symbols **x**, **E**, **f**, etc. here are simple place-holders to show the form of problems that can be solved. They have no relationship to the symbols used elsewhere in this assignment.)

A. Suppose that we move the rotational joints from $\vec{\theta}$ to $\vec{\theta} + \Delta\vec{\theta}$. Then we will have a new orientation $\mathbf{R}_{rob}(\vec{\theta} + \Delta\vec{\theta}) = \Delta\mathbf{R}_{left}\mathbf{R}_{rob} = \mathbf{R}_{rob}\Delta\mathbf{R}_{right}$. Develop formulas for $\Delta\mathbf{R}_{left}$ and $\Delta\mathbf{R}_{right}$ in terms of the $\mathbf{R}_k(\theta_k)$, $\vec{\mathbf{a}}_k$, and $\Delta\theta_k$. Here, you can write $\mathbf{R}_k(\theta_k)$ simply as $\mathbf{R}_k$. **Hint:** Look back at some of the identities in the lecture notes. Your answer will involve a product of factors that look like $Rot(\text{some\_axis}_k, \Delta\theta_k)$. You need to figure out expressions for the "some_axis_k".

B. Assume now that the $\Delta\theta_k$ values are sufficiently small so that you can ignore second order terms. Develop linear approximations for computing $\Delta\mathbf{R}_{left}$ and $\Delta\mathbf{R}_{right}$. I.e., I am looking for a linear approximation for computing $\vec{\alpha}_{left}$ and $\vec{\alpha}_{right}$, where

$\Delta\mathbf{R}_{left} \approx \mathbf{I} + skew(\vec{\alpha}_{left})$ and $\Delta\mathbf{R}_{right} \approx \mathbf{I} + skew(\vec{\alpha}_{right})$. In particular we are looking for matrices $\mathbf{M}_{left}$ and $\mathbf{M}_{right}$, where $\vec{\alpha}_{left} = \mathbf{M}_{left} \bullet \Delta\vec{\theta}$ and $\vec{\alpha}_{right} = \mathbf{M}_{right} \bullet \Delta\vec{\theta}$.

C. Given $\Delta\vec{\mathbf{q}} = [\Delta\vec{\theta}, \Delta\vec{\mathbf{p}}_{cart}]$ and $\mathbf{F}_{rob}(\vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}) = \Delta\mathbf{F}_{left}\mathbf{F}_{rob} = \mathbf{F}\Delta\mathbf{F}_{right}$, give formulas for $\Delta\mathbf{F}_{left}$ and $\Delta\mathbf{F}_{right}$ in terms of $\Delta\vec{\mathbf{p}}_{cart}$, $\Delta\mathbf{R}_{left}$, and $\Delta\mathbf{R}_{right}$.

D. Note that for small values of $\Delta\vec{\mathbf{q}}$, we can parameterize the corresponding small pose change $\Delta\mathbf{F}_{left}$ by $\vec{\delta}_{left} = [\vec{\alpha}_{left}{}^T, \vec{\varepsilon}_{left}{}^T]^T$, where $\Delta\mathbf{F}_{left} \approx [\mathbf{I} + skew(\vec{\alpha}_{left}), \vec{\varepsilon}_{left}]$, and similarly for $\Delta\mathbf{F}_{right}$. In question 2B, you have explained how to compute a linear approximation for $\Delta\mathbf{R}_{left}$ and $\Delta\mathbf{R}_{right}$. Now, extend this result to give matrices $\mathbf{J}_{left}$ and $\mathbf{J}_{right}$ so that $\vec{\delta}_{left} = \mathbf{J}_{left} \bullet \Delta\vec{\mathbf{q}}$ and $\vec{\delta}_{right} = \mathbf{J}_{right} \bullet \Delta\vec{\mathbf{q}}$.

E. Suppose now that the robot is at pose $\mathbf{F}_{rob}(\vec{\mathbf{q}})$ but you want to move the robot to a nearby pose $\mathbf{F}_{rob}^{goal}$. Describe a method for computing $\vec{\mathbf{q}}^{goal} = \vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}$ so that $\mathbf{F}_{rob}(\vec{\mathbf{q}} + \Delta\mathbf{q}) = \mathbf{F}_{rob}^{goal}$. For now, you may ignore joint position and velocity limits.

F. Note that the robot's kinematic structure is redundant. Describe a method for modifying your answer to Question 1.E so that the joint positions stay as close to their midpoints as possible.

G. Suppose that $\mathbf{F}_{rob}^{goal}$ is some significant distance from the current pose $\mathbf{F}_{rob}^{(t)}$ of the robot. Describe a method for producing a uniform motion of the robot so that it reaches $\mathbf{F}_{rob}^{goal}$ after some time ΔT. I.e., $\mathbf{F}_{rob}^{(t+\Delta T)} = \mathbf{F}_{rob}^{goal}$. During this motion, the position $\vec{\mathbf{p}}_{rob}$ of the robot should move along a straight path as a uniform velocity, and the orientation of the robot's tool plate should rotate at a uniform angular velocity about some axis $\vec{\mathbf{a}}_{rot}$. For now, you may ignore joint constraints, but you should attempt to keep the joint positions as close to their midpoints as possible.

Question 2

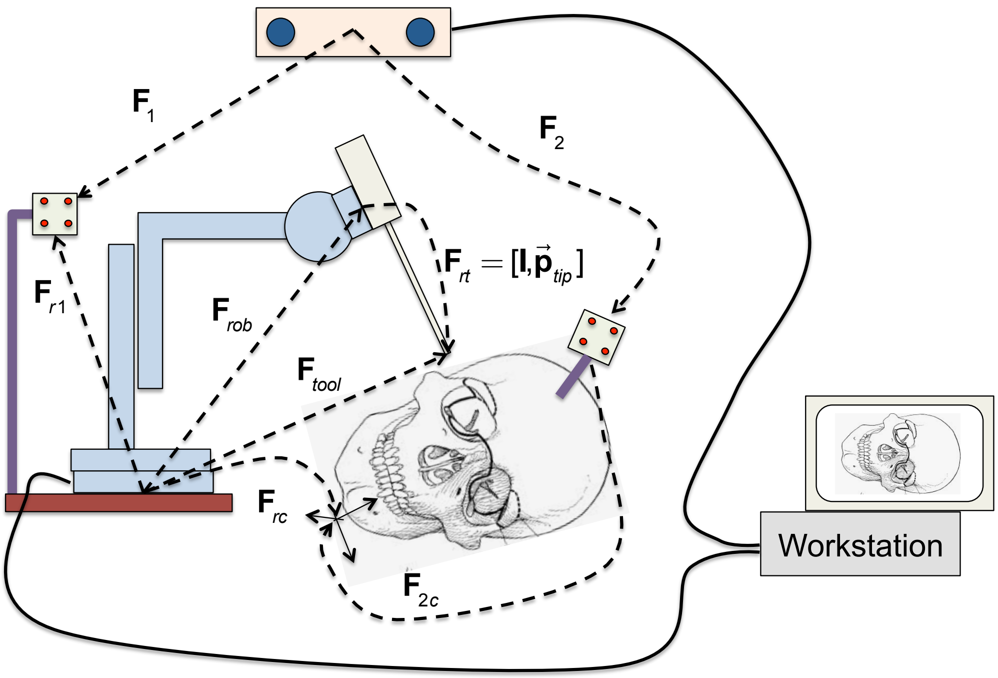


Fig. 2: Robot-Assisted Surgery

Consider now the situation shown in Fig. 2, in which the robot described in Question 1 has been integrated into a surgical application. Here, a surgical tool has been affixed to the robot's tool holder, so that the pose of the tool tip relative to the robot base is $\mathbf{F}_{tool} = \mathbf{F}_{rob}\mathbf{F}_{rt} = [\mathbf{R}_{tool}, \vec{\mathbf{p}}_{tool}]$, where $\mathbf{F}_{rt} = [\mathbf{I}, \vec{\mathbf{p}}_{tip}]$. The tool shaft runs from the tool coordinate system origin to the tip. I.e., it is parallel to $\mathbf{R}_{tool}\vec{\mathbf{p}}_{tip}$. The surgical system also has an optical tracking system, and markers have been attached to the base of the robot and to the patient. The optical tracking system is capable of determining the pose of these markers relative to the tracking sensor as $\mathbf{F}_1$ and $\mathbf{F}_2$, respectively. A calibration step has been performed such that the known pose of the robot's marker relative to the robot base is $\mathbf{F}_{r1}$. A preoperative CT scan of the patient is available. A registration step has also been performed, so that the position $\vec{\mathbf{c}}$ in the CT image relative to the patient marker is given by $\mathbf{F}_{2c}\vec{\mathbf{c}}$. Further, the robot's tool holder has an integrated force sensor that is capable of sensing forces $\vec{f}$ and torques $\vec{\tau}$ exerted by the tool onto the tool holder, resolved in the tool holder's coordinate system.

The surgical workstation includes the robot controller and has access to all information about the robot state, the forces sensed by the force sensor, and the information computed by the optical tracking system.

A. Give an expression for computing the transformation $\mathbf{F}_{rc}$ between the robot base coordinate system and the CT coordinate system. **Note:** In subsequent questions, you may use $\mathbf{F}_{rc}$, rather than repeating the expression for recruiting it.

B. Suppose, further, that it is desired to position the robot so that the tool tip is at pose $\mathbf{F}_{tool}^{goal}$ in CT coordinates. What would be the corresponding pose $\mathbf{F}_{rob}^{goal}$ of the robot relative to its base?

C. Suppose that the patient may make some small motions, which can be detected by the robot workstation. Suppose further that the robot is currently at a pose $\mathbf{F}_{rob}^{(0)}$ with joint positions $\vec{\mathbf{q}}^{(0)}$. The corresponding pose of the tool is (of course) $\mathbf{F}_{tool}^{(0)} = \mathbf{F}_{rob}\mathbf{F}_{rt}$. Describe a strategy for moving the robot along a uniform path (relative to the patient) so that the tool winds up at pose $\mathbf{F}_{rob}^{goal}$ and the tool tip moves at speed s, if possible. In making this motion, the tool tip should move in a straight line, and the tool orientation relative to the patient should rotate about a uniform axis. Joint velocity limits should be respected, and the joint positions should stay as close to their midrange as possible, though this is less important than maintaining a uniform speed if you can. **Hint:** You may want to recall that the robot controller runs at a unified sample interval Δt. One possible strategy would be to consider what to command the robot to do in the time interval $t = 0$ to $t = \Delta t$, and then plan to repeat this strategy at every interval until the goal is reached. Your solution will involve an optimization problem with inequality and equality constraints.

D. How would you modify your solution to Question 1.C so that the robot moves along the path above, but in response to forces exerted by the surgeon's hand on the handle of the surgical tool? Here the robot's controller should execute a modified admittance control. In the absence of other constraints, the desired incremental motion (relative to the tool holder) would be given by

$$\begin{bmatrix} \vec{\alpha}_{right} \\ \vec{\varepsilon}_{right} \end{bmatrix} = \mathbf{A} \bullet \begin{bmatrix} \vec{f} \\ \vec{\tau} \end{bmatrix}$$

where **A** is an appropriate gain matrix. Here, of course, you have constraints on the desired motion, so you need to formulate some sort of constrained optimization problem.

E. Suppose that the tool is some sort of surgical cutter, with a spherical cutter tip of known diameter. For simplicity, let us assume that the patient's anatomy has been restrained so that motion relative to the robot base may be ignored. Suppose that we wish to use constrained hand guiding to machine out a volume of material in a cylindrical shape in a pose $\mathbf{F}_{cyl}$ defined relative to the patient's CT scan. After allowing for the radius of the spherical cutter tip, the coordinates $[x, y, z]$ of positions corresponding to the shape to be cut (in CT coordinates) are bounded as follows: $x^2 + y^2 \le \rho^2$; $-\psi \le z \le 0$. However, analysis of the anatomy nearby to this shape confirm that some undercutting is allowed, so long as the cutter tip eventually covers all the volume within 0.95ρ of the center axis of the shape. The surgeon will rely on visual feedback to guide the cutting, but will rely on the robot to enforce constraints. During cutting, the tool axis must remain aligned within 30 degrees of the axis of the cylinder. This may be desirable to help the surgeon see the anatomy being operated on, to facilitate irrigation during cutting, or for some other purpose. Initially, the cutter is aligned so that the cutter tip is located at the origin of the cylinder coordinate system and the tool axis is aligned parallel to the cylinder axis. How would you extend your answer to Question 1.D in order to assist the surgeon in performing this task?

F. How would you modify your answer to Question 2.E to impose very tight bounds on the shape cut (thus very greatly reducing the undercutting) without dramatically increasing the number of constraints on the system?

G. How would you modify your answer to Question 2.E so that the robot begins to resist the surgeon's hand motion while moving the cutter tip toward the bottom of the cylinder when the cutter tip is within a distance η of the defined cylinder bottom?

Question 3

Conduct a simplified Failure Modes and Effects Analysis for the system in Question 2. Here, I would like you to identify some of the more important things that could possibly go wrong with the system that could affect its performance or safety. Typical failures can run the gamut from bothersome (robot will not turn on) to severe. For each failure mode you identify the following:

- What the failure is
- How severe the consequences of the failure will be
- How would you detect the failure (i.e., what the observable symptoms of the failure might be)
- What the causes of the failure might be
- How you would mitigate the consequences of or prevent the failure from occurring.

In real clinical systems, these analyses may run for tens (or hundreds) of pages. We are not looking for anything that detailed here. There is no quota, but a typical answer might have from 5 to 10 failures. Mostly, we will be looking for thoughtful analyses.