Homework Assignment 4 – 600.445/645 Fall 2014

Instructions and Score Sheet (hand in with answers)

Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment ________________________________
Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment ________________________________

Please indicate whether you are taking 600.445 or 600.645 (Circle one)

Question	Points 600.445	Points 600.645	Points
1	5	5	
2	5	5	
3	10	10	
4	10	5	
5	10	10	
6	15	10	
7	5	5	
8	15	15	
9	10	10	
10	15	10	
Extra	10	10	
Total	**100+10**	**100**	

1. Remember that this is a **graded** homework assignment. It is effectively an exam.
2. You are to work alone or in a team of two people and are not to discuss the problems with anyone other than the TAs or the instructor.
3. Put your names and email address on each sheet and number the sheets.
4. You are encouraged to make free use of any published materials, the web, etc. in developing your answer but a) you must give full and proper citations to any references consulted and b) you may not consult, discuss, or otherwise communicate about this assignment with any human being except your lab partner, the course instructor, or the TAs. The one exception is that you should not refer to previous years' homework.
5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
7. Sign and hand in the score sheet as the first sheet of your assignment
8. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment.
9. Attach the grade sheet as the first sheet and attach all sheets together.
10. You must include a self-addressed, seal-able 8 ½ x 11 inch envelope if you expect to the homework to be returned (per JHU's interpretation of FERPA).

Scenario

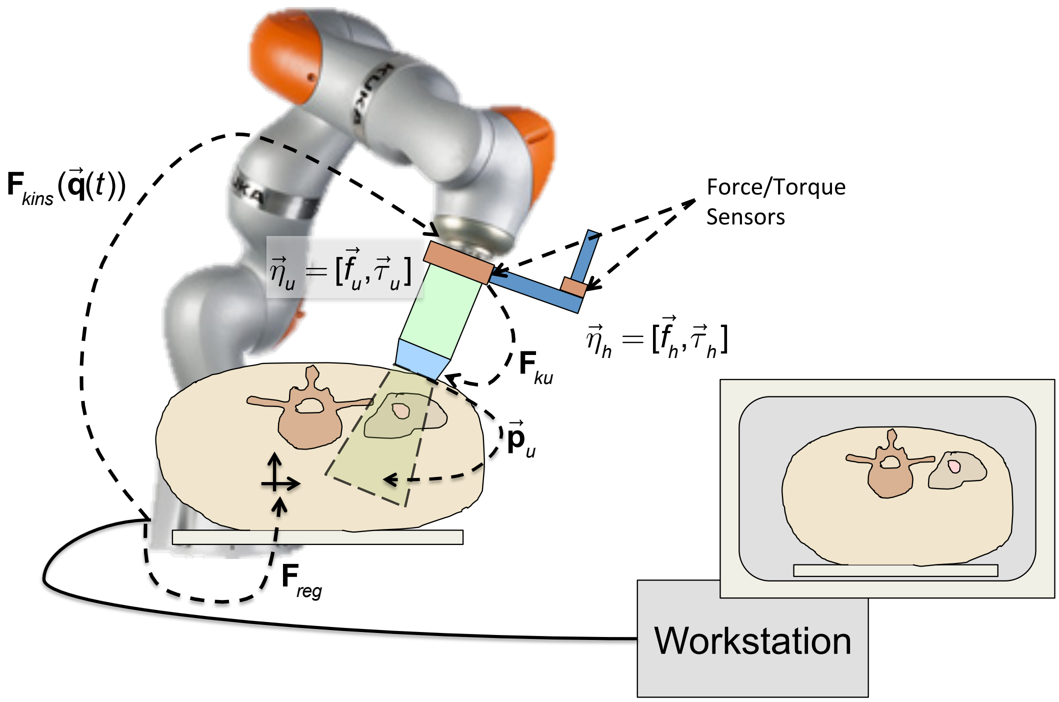

Fig. 1: Robotically-Assisted Ultrasound System

Consider the robotically-assisted ultrasound system shown in Fig. 1. This system has a workstation, an ultrasound system, and a robot and two force/torque (F/T) sensors. One of these sensors is attached to a handle, which, in turn is attached to the tooling attachment plate of the robot. When the human user exerts forces or torques on this handle, the F/T sensor senses these values and the workstation computes a corresponding F/T vector $\vec{\eta}_h = [\vec{f}_h, \vec{\tau}_h]$ resolved in the coordinate system of the robot's tooling attachment plate. The robot has another F/T sensor that is also attached to the tooling plate and to an ultrasound probe sensor. When forces or torques are applied to the ultrasound probe (e.g., when the probe is pressed against the patient) this sensor measures them and the workstation computes an F/T vector $\vec{\eta}_u = [\vec{f}_u, \vec{\tau}_u]$, also resolved in the coordinate system of the robot's tooling attachment plate.

The workstation is able to read the joint values $\vec{\mathbf{q}}(t)$ of the robot and has a function $\mathbf{F}_{kins}(\vec{\mathbf{q}})$ that computes the pose of the tooling plate relative to the base coordinate system of the robot. The workstation also has a function

$$\mathbf{J}_{kins}(\vec{\mathbf{q}}) = \begin{bmatrix} \mathbf{J}_{\alpha}(\vec{\mathbf{q}}) \\ \mathbf{J}_{\varepsilon}(\vec{\mathbf{q}}) \end{bmatrix}$$

such that for small changes $\Delta\vec{\mathbf{q}}$, the corresponding pose of the robot's tooling plate pose is given by $\mathbf{F}_{kins}(\vec{\mathbf{q}}+\Delta\vec{\mathbf{q}}) = \Delta\mathbf{F}_{kins}\mathbf{F}_{kins}(\vec{\mathbf{q}})$, where $\Delta\mathbf{F}_{kins} \approx [\mathbf{I} + sk(\vec{\alpha}), \vec{\varepsilon}]$ and

$$\vec{\xi} = \begin{bmatrix} \vec{\alpha} \\ \vec{\varepsilon} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{\alpha}(\vec{\mathbf{q}}) \\ \mathbf{J}_{\varepsilon}(\vec{\mathbf{q}}) \end{bmatrix} \Delta\vec{\mathbf{q}}$$

Note that we will adopt the notation $\vec{\xi} = [\vec{\alpha}, \vec{\varepsilon}]^T$ to indicate a set of small orientation and position variables. We will use $\Delta\mathbf{F}(\vec{\xi}) \approx [\mathbf{I} + sk(\vec{\alpha}), \vec{\varepsilon}]$ to indicate the corresponding pose change.

The workstation also has a model of the patient's anatomy, which has been registered to the robot, so that a position $\vec{\mathbf{p}}_{pat}$ in the patient coordinate system corresponds to $\mathbf{F}_{reg}\vec{\mathbf{p}}_{pat}$ in the robot coordinate system.

Similarly, a calibration has been performed so that a point at location $\vec{\mathbf{p}}_u = [x_u, 0, z_u]$ in an ultrasound image corresponds to a point $\mathbf{F}_{ku}\vec{\mathbf{p}}_u$ relative to the tooling plate of the robot. The pose of the ultrasound probe relative to the base frame of the robot is thus $\mathbf{F}_{Bu} = \mathbf{F}_{kins}\mathbf{F}_{ku}$. For the purposes of this exercise, you can assume that points $\vec{\mathbf{p}}_u = [x_u, 0, 0]$ of the ultrasound images system corresponds to points on the ultrasound probe where the surface of the probe is in contact with the surface of the patient. I.e., the ultrasound probe coordinate system has its "origin" where the probe is in contact with the patient.

Note: We sometimes describe haptic interfaces in which the human pushes on the robot and the robot moves accordingly as "admittance-type" interfaces. Similarly, we refer to interfaces where the robot pushes back on the human in response to motion by the human as "impedance-type" interfaces. The "steady hand" robot virtual fixtures described in class were of the admittance type.

Questions

1. Given a small change in the joint angles $\Delta\vec{\mathbf{q}}$, what is the corresponding change in the position of the ultrasound probe coordinate system relative to the patient model coordinate system?
2. If an anatomic feature is at position $\vec{\mathbf{p}}_0$ in the ultrasound image coordinate system when the robot's joints are at position $\vec{\mathbf{q}}$. What will be the position of the same feature when the robot's joints are at position $\vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}$?
3. Suppose that we have a gain matrix $\mathbf{K}_h$ such that handle F/T values $\vec{\eta}_h$ should produce an incremental motion $\Delta\mathbf{F}(\vec{\xi}_h = \mathbf{K}_h\vec{\eta}_h)$ expressed in the coordinate system of the tooling plate. Using the "admittance style" virtual fixtures discussed in class, express an optimization problem that will produce a set of incremental joint values $\Delta\vec{\mathbf{q}}$ that will produce this desired incremental motion.

4. How would you modify this optimization problem to ensure that the linear velocity of the ultrasound probe in any direction never exceeds a value $v_{\max}$. Here, I am looking for a quadratic constraint.
5. Suppose that your optimizer is only able to handle linear constraints, how would you deal with this?
6. Explain how you would further modify this optimization problem to ensure that the ultrasound probe never exerts a force greater than $f_{\max}$ in the direction into the patient. (Essentially, I am asking you to add a constraint or set of constraints).
7. Suppose that the current velocity of the robot's joints is given by $\dot{\mathbf{q}}$ and that the control loop of the robot runs every Δt milliseconds. What would you add to your optimization problem to ensure that the acceleration commanded for any joint is limited to $\ddot{q}_{\max}$.
8. Suppose that an interesting anatomic feature had been identified at location $\vec{\mathbf{p}}_a$ in ultrasound image coordinates. Define an optimization problem using an admittance-style virtual fixture to assist the ultrasonographer to place this feature at position $\vec{\mathbf{p}}_c$ in image coordinates while also ensuring that the ultrasound probe keeps the image in the same plane. (I.e., the xz plane of the ultrasound image should remain fixed.) This virtual fixture should also guarantee that forces exerted into the patient should not exceed $f_{\max}$
9. How would you modify your virtual fixture so that the robot attempts to ensure that force exerted on the patient stays close to f_{des} ? This may be important for food ultrasound image quality.
10. Suppose that an anatomic feature has been identified at location $\vec{\mathbf{p}}_b$ in patient model coordinates. Define an optimization problem using an admittance-style virtual fixture to assist the ultrasonographer to place this feature at position $\vec{\mathbf{p}}_c$ in image coordinates while also ensuring that the ultrasound probe keeps the image in the same plane. (I.e., the xz plane of the ultrasound image should remain fixed.)

Additional question for 600.645 students (extra credit for 600.445)

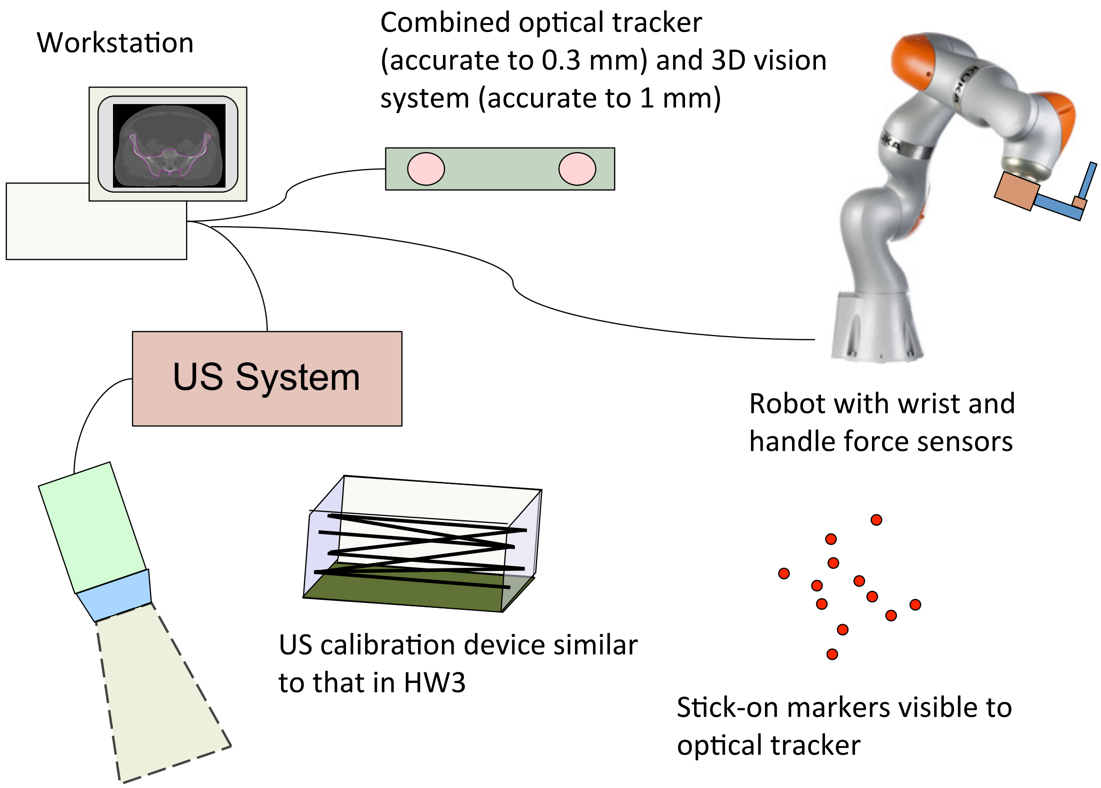

Fig. 2: Parts available for the problem

This question concerns how you would actually implement the system in Fig. 1. Suppose that you have the parts shown in Fig. 2. For ultrasound calibration, you have a fixture similar to that in Homework #3. You can assume that a rapid prototyping system is available to make any mounting brackets, fixtures, etc., but that there are limits to the available precision. In particular, you can assume that you can fabricate some adapter permitting you to attach the ultrasound probe to the robot, but that you cannot assume that this will be accurate enough to produce a good value for $\mathbf{F}_{ku}$. You can assume that the force sensor calibration has been done.

The workstation has software for combining a multiple set of 2D ultrasound images to form a 3D ultrasound volume, provided that the pose of the ultrasound sensor is known for each 2D image relative to some known patient coordinate system.

The value of $\mathbf{F}_{reg}$ is not known, but the workstation software includes a registration module that is capable of performing CT-to-MRI and CT-to-3DUS registration.

Your problem, again, is to describe a system for implementing this system using the components provided. There is no single correct solution to this problem using the equipment provided, but it is important that you explain clearly what your solution is and justify your approach.

1. Describe the system setup, possibly including sketches and verbal descriptions for

- Any brackets or other fabricated apparatus, with rough dimensions showing where any markers are to be attached.
- Description of where you will attach any other markers (on patient, on ultrasound probe, robot, OR table, or wherever)
- Description of any calibration procedures that must be done, and when they should be done.

Here, we are not seeking beauty or actual mechanical design drawings. Rather, the goal is simply to provide enough information to enable one of ordinary skill in the art to implement your design.

2. Describe the step-by step procedural flow, giving a clear statement of what is to be done at each step, what information is to be sensed, what is to be computed, and how this information is to be used. Here, you should also clearly define any variables that you may need to use in Part 3. Note that there are actually 3 workflows: 1) system calibration and configuration; 2) preoperative imaging; and 3) intraoperative imaging, registration, and visualization.

3. For any steps in Part 2 that involve computations, give sufficient formulas so that it is clear how the computations are to be done. Here, you can use the names of known methods (e.g., Arun's or Horn's method for point cloud to point cloud registration) with suitable citations to papers or lecture notes. But the formulations should make very clear what the inputs and outputs are in terms of your defined variables.